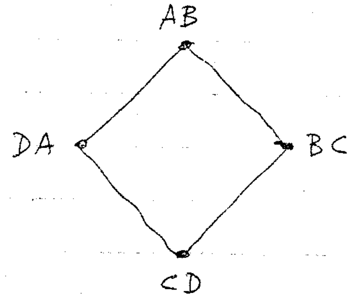
P.S.

2 cycles in the hexeny

(14) Tetragram:

A	B	C	D
A			D
A	B		
	B	C	
		C	D

To Beginning



There are 3 permutations.

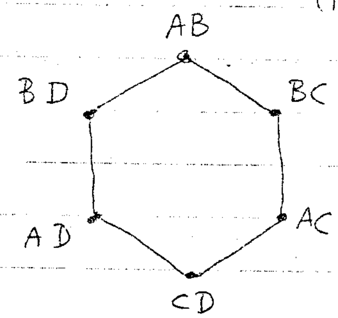
Hexagram:

(15)

A	B	C	D
	B		D
A	B		
	B	C	
A		C	
		C	D
A			D

To beginning

(15)a

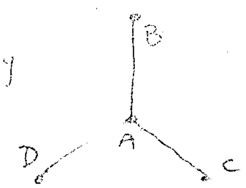


There are 4 permutations. The one shown is "A" oriented, polarized toward the A & $\bar{A}$ functions. This block (15) is very helpful in mapping the hexeny over the hexagon (15)a!

The corresponding tetrads to (15)a are thus:

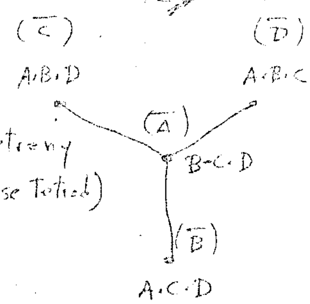
(16) a

1) 4 Tetreny (or Tetrad)



(16) b

3) 4 Tetreny (or inverse Tetrad)

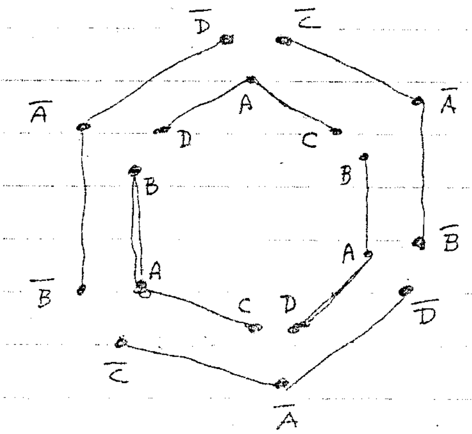


It may be visually confirmed that the dyads occurring in the Tetrad $(16)_a$ my be re-arranged (whithout changing angular orientation!) to construct the hexagram $(15)_a$, each dyad occurring twice.

It may be, further, confirmed that the collective dyads ocurring ~~th~~ in the Tetrads $(16)_a$ & $(16)_b$ may be re-arranged to construct The hexagram $(15)_a$, each dyad occurring once.

Forther, the collective Triads of $(16)_{a,b}$ may be rearranged to form the Hexagram $(15)_a$, each triad occurring once. The triads of $(16)_a$ & $(16)_b$ alternate and overlap, by a dyad, as they progress around The cycle $(15)_a$, Thus:

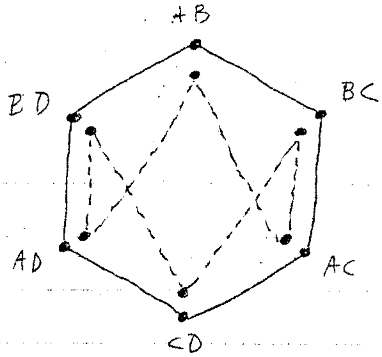
(17)



Each alternate point is the A-function, and "center" of a triad; the other alternate point is the $\bar{A}$-function, and center of an inverse triad. The hexagram is A-biased, (15).

A hexagram may intersect with any one of its (remaining) 3 permutations on 6 points, or on 2 antipodal dyads, +2 monads, as shown (18).

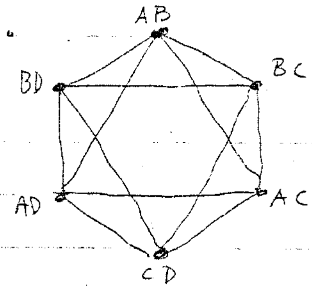
(18)



~~A~~ cycles of hexagrams, rotating the dotted hexagram in 60° 120°, of 180° increments about the solid hexagram, ~~is~~ are easy enough to visualize.

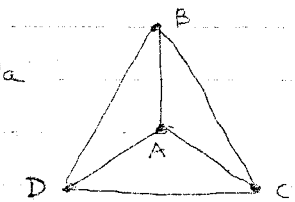
The aggregate of The 4 Hexagram permutations are linked in the full eikosany lattice, (19).

(19)

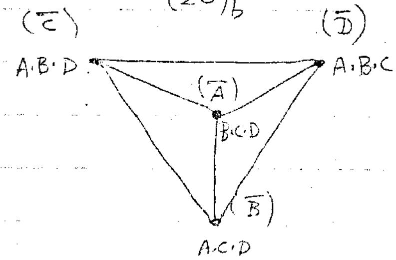


The associated Tetrads must show complete delineation of the 6 dyads, $(20)_a$ & $(20)_b$

$(20)_a$



$(20)_b$

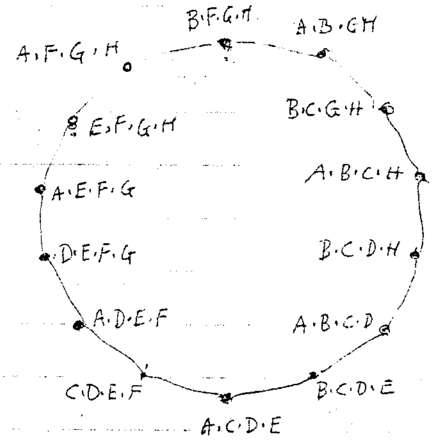


It is interesting to note that the same no. of lines, lengths & angularity, used to construct $(20)_a$ & $(20)_b$ can be rearranged to construct (19). Not only each of the collective dyads of $(20)_{a,b}$, but also each of the collective triads, reappear in a transformed relationship in the hexany (19).

2 cycles from the Heptakontany

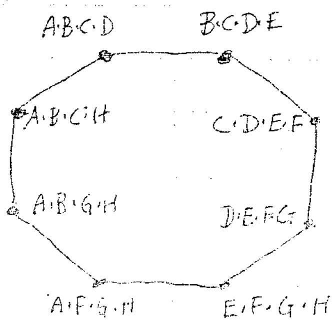
14-gon:
~~Octagon:~~

A	B	C	D	E	F	G	H
	B				F	G	H
A	B					G	H
	B	C				G	H
A	B	C					H
	B	C	D				H
A	B	C	D				
	B	C	D	E			
A		C	D	E			
		C	D	E	F		
A			D	E	F		
			D	E	F	G	
A				E	F	G	
				E	F	G	H
A					F	G	H



Octegon:

A	B	C	D	E	F	G	H
A	B	C	D				
	B	C	D	E			
		C	D	E	F		
			D	E	F	G	
				E	F	G	H
A					F	G	H
A	B					G	H
A	B	C					H



Well, That's the end of this note-pad. Yours,
Ern

A cube in eikosany context

The set, abcdef, may be partitioned into 3 cells of 2 elements.

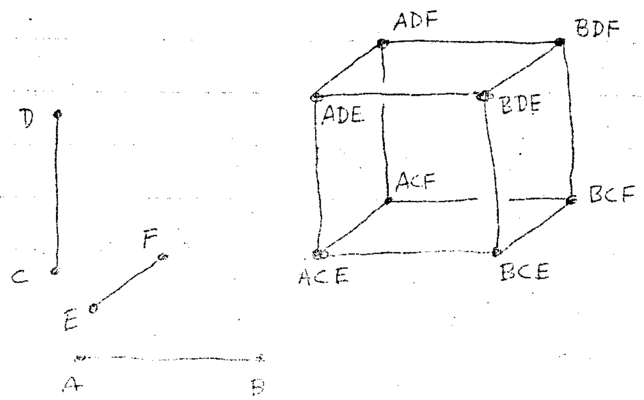
Example: ab, cd, ef

The number of such partitions may be determined thus;

$$\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{6\cdot 5}{1\cdot 2}\cdot\frac{4\cdot 3}{1\cdot 2}\cdot\frac{2\cdot 1}{1\cdot 2} = 90$$

The cells of the partition may be treated as dyads, or $\binom{2}{1}$ dyanies, and multiplied. The number of tones resulting from such a multiplication will be $\binom{2}{1}\binom{2}{1}\binom{2}{1} = 8$

Example: The $\binom{2}{1}$ ab dyany has 2 members (a & b); as do the cd and ef dyanies. The members of the dyany may be considered a set and the 3 sets multiplied; $\{a,b\}\times\{c,d\}\times\{e,f\}$. In this case, a 3-dimensional matrix also becomes the lattice.

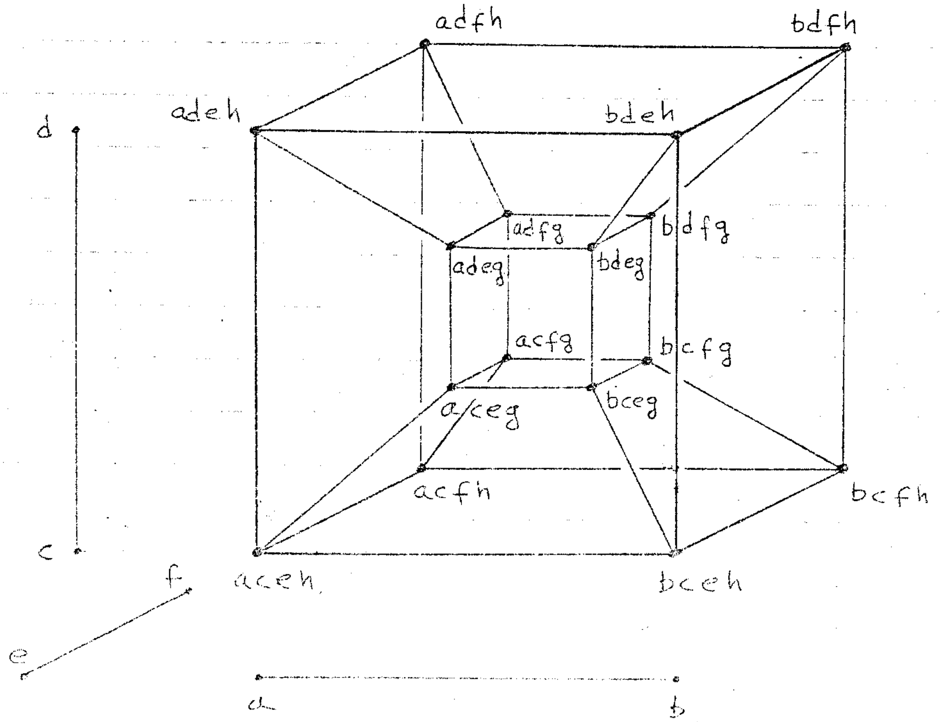


There can be no hexanies in this figure, but there are 4 hexagon cycles (ref page 8). Can you locate and identify them?

A hyper-cube in Heptakontany context

The ogdoadic set, abcdefgh, may be partitioned into 4 cells of 2 elements each. The number of such partitions is determined Thus: $\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{8\cdot 7}{1\cdot 2}\cdot\frac{6\cdot 5}{1\cdot 2}\cdot\frac{4\cdot 3}{1\cdot 2}\cdot\frac{2\cdot 1}{1\cdot 2} = 2{,}520$ partitions, good heavens! Each such partition may be treated as a set of 4 $\binom{2}{1}$ dyanies, multiplied in sequence. This results in a set of 16 members, $\binom{2}{1}\cdot\binom{2}{1}\cdot\binom{2}{1}\cdot\binom{2}{1} = 16$), which may be mapped over the hyper-cube.

Example; ab, cd, ef, gh :.

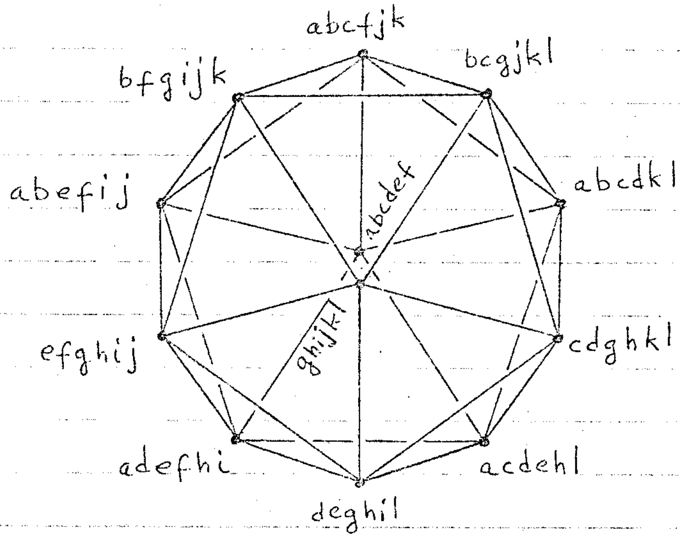


radial axis; (in) — (out)
g h

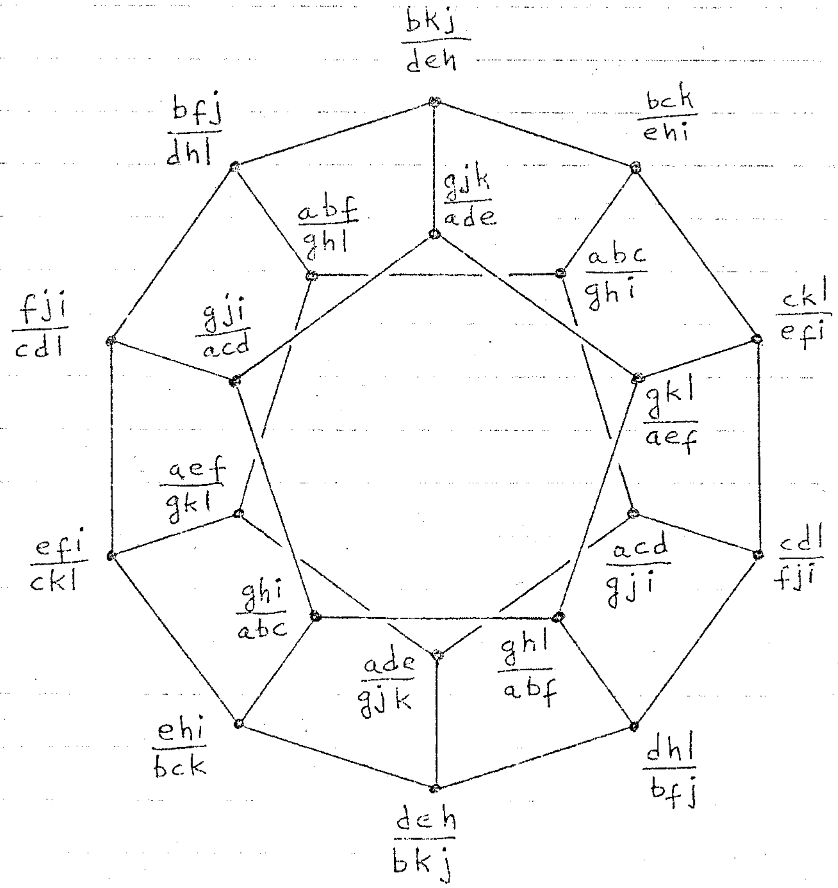
How many hexagon cycles are enmeshed in this figure? How many octagon cycles, (ref p.32)? How many congruent pair of cubes?

(To be continued p.35) FINIS E.W.

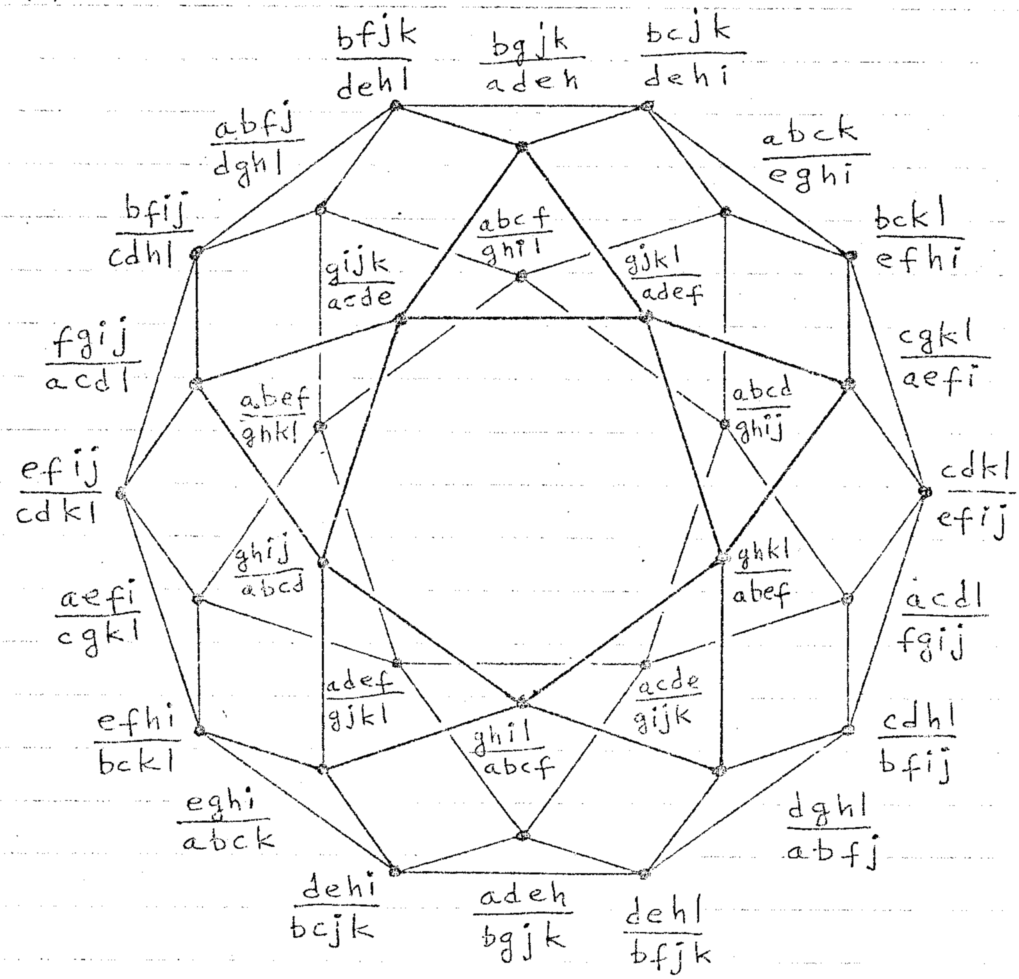
(1) Icosahedron



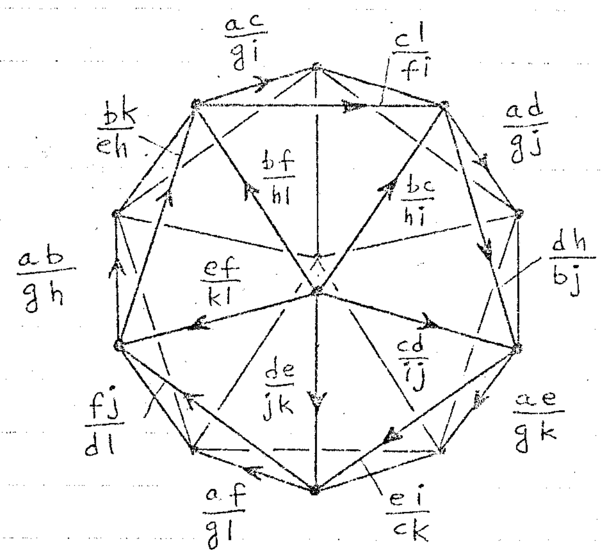
(2) Pentagonal Dodecahedron



(3) Dodeca-Icosahedron

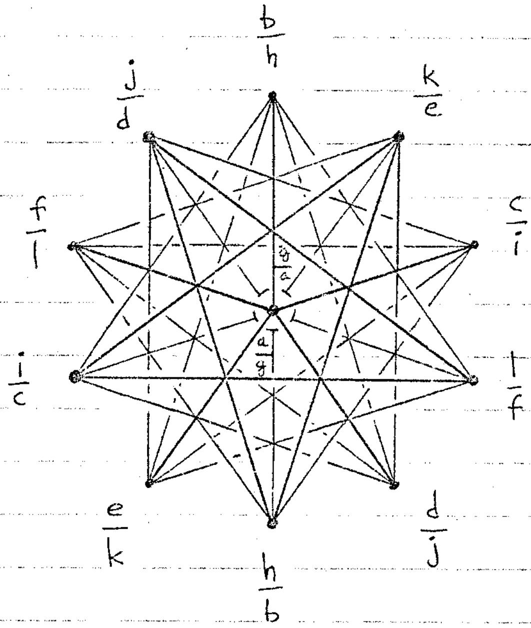


(4) The 15 different lines of the Icosahedron, (1).

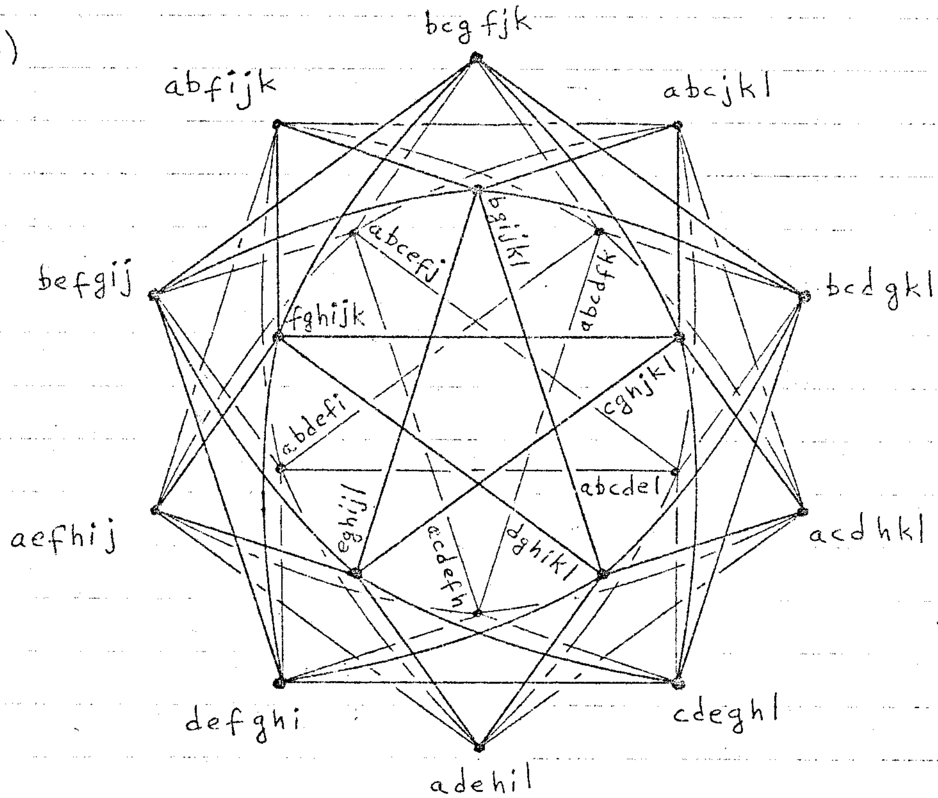


Stellate Dodecahedron:

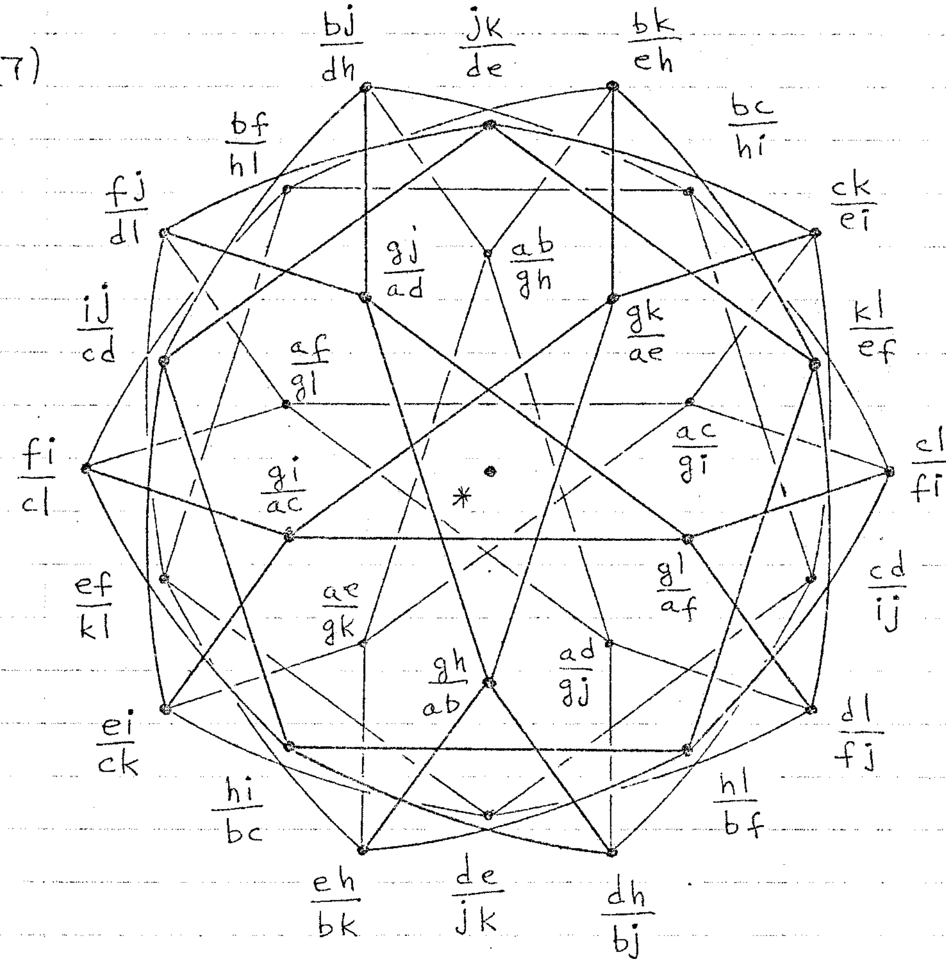
(5)



(6)



(7)



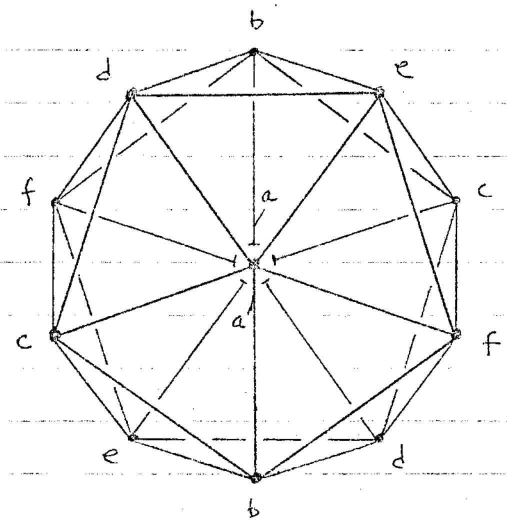
$* = \frac{abcdefghijkl}{abcdefghijkl}\ (=1)$

This figure is centered by drawing lines from each of its 30 surface nodes, ~~and~~ to the center (*) node. These lines are parallel to, and of the same length as the 15 ~~B~~ different lines occurring on the surface. These lines are identified in fig. (4).

The Icosahedron

There are 15 lines in the Icosahedron, each occurring twice, (4). There are also 15 dyads in the hexad, $\binom{6}{2} = 15$, each occurring once. Might it not be possible to map these 15 dyads over the 15 lines so that each dyad occurs twice? If I attempt to do so I get this arrangement:

(8)



I end up with the elements of the original hexad, each duplicated at opposite nodes. In fact, each subset of the hexad is duplicated on the antipode. This figure is interesting because it satisfies the requirement that each element of the hexad ~~is~~ be connected to each of the other elements by ~~a~~ lines of equal length. However, I would hate to have to construct an eikosany from such a module of elements. It can be observed that each pair of parallel lines, example: $\overset{f}{\underset{c}{|}}$ and $\overset{c}{\underset{f}{|}}$, are reciprocals.

This geometry (8) would have to have a very special set of rules to make this true, and so cannot be included with the geometries generally used for latticing. Worst of all, 6 identities or elements can hardly be said to identify the icosahedron with any clarity. So I guess theres not much else to do except chuck it (8), along with our hopes of connecting the nodes of the icosahedron with simplex $\left(\frac{a}{b}\right)$ intervals.

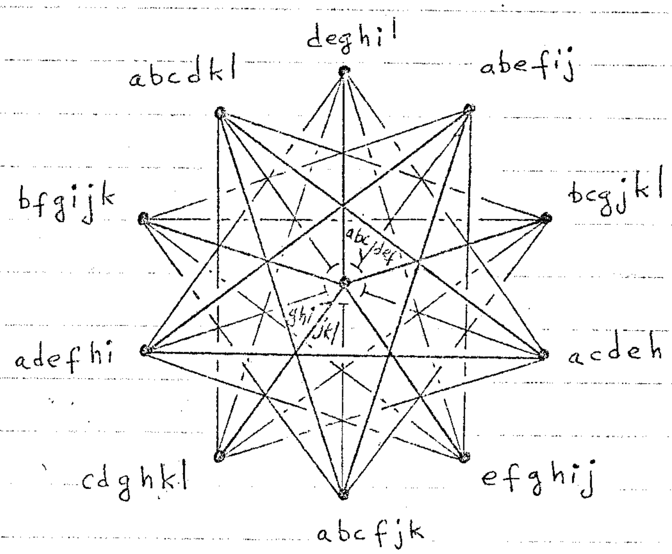
The icosahedron (1) is constructed of 12 elements. Duplex $\left(\frac{a\,b}{c\,d}\right)$ intervals connect each node with its neighbors. [There are 15 different intervals. These are identified in fig (4), and reappear, rearranged in figures (2), (3), (5), (6), & (7).] Nodes twice removed are related by quadruplex intervals $\left(\frac{a\,b\,c\,d}{e\,f\,g\,h}\right)$ and nodes thrice removed are related by sextuplex $\left(\frac{a\,b\,c\,d\,e\,f}{g\,h\,i\,j\,k\,l}\right)$ intervals. This exquisite hierarchy utterly defines the icosahedron, and avoids any implication of any other figure*. Its regrettable, but since it contains no simplex intervals, it is, in itself, quite useless for harmonic purposes. No need to discard it tho; it will make an excellent skeleton for a grand tour of modulations across and about the "924-any". Six out of Twelve $\binom{12}{6} = 924$. How many such figures will be found in $\binom{12}{6}$ context?

* The members of icosahedron (1) can be mapped over the points of a stellate dodecahedron (9). The lines connecting the points are deprived of any special meaning where they cross thru each other, tho normally these crossings are seen as the

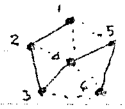
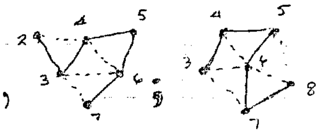
points of the dodecahedron. The stellate dodecahedron (9) is apparently the transform and topological equivalent of the ~~dodecahedron~~ icosahedron (1):

Stellate Dodecahedron:

(9)



The connecting intervals (lines) of icosahedron (1) have been reoriented [and, incidentally, considerably enlarged in scale], but not recombined, in such a manner that each of the 12 pentagonal, cycles in icosahedron (1) is transformed into a pentagram cycle, . I should point out, here, an interesting comparison with the stellate dodecahedron (5); in (5) each of the connecting lines of icosahedron (1) retains ~~their~~ its orientation, but ~~are~~ is recombined in a new set of nodal connections. Figures (5) & (9) of the stellate dodecahedron are quite distinct species. I will discuss fig (5) further, later.

Over the Icosahedron is found a delightful cycle(10), passing by a line thru each of the 12 nodes, forming, alternately, a 30° angle and a 144° angle. Sequential sets of 6 nodes in sequence (that is: 1 2 3 4 5 6, 2 3 4 5 6 7; etc) will pass, alternately, thru the members of a centered pentagon, , and the member of a stellate triangle,  (These being on a ~~sphericet~~ spherical surface).

Icosahedron Cycle: (basic form, see (11))

	a	b	c	d	e	f	g	h	i	j	k	l
(10)	a			d		f		h	i			l
	a			d		f	g			j		l
	a		c			f	g			j	k	
	a		c			f	g		i			l
	a		c		e			h	i			l
	a		c		e		g		i		k	
		b	c		e		g			j	k	
		b	c		e			h	i		k	
		b		d	e			h	i			l
		b		d	e			h		j	k	
		b		d		f	g			j	k	
		b		d		f		h		j		l

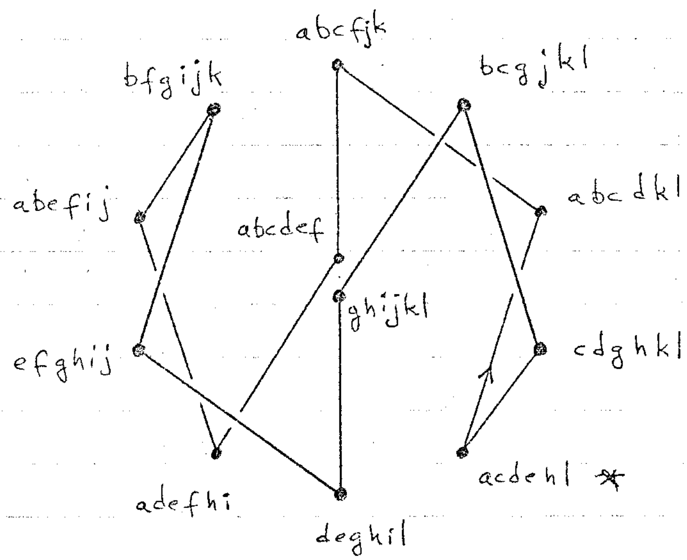
This cycle applies, equally well, to the stellate dodecahedron. x What will the analogs to the alternating, centered-pentagons & stellate-triangles be?

(11) Permutation of icosahedron cycle applicable to icosahedron (1) and to stellate dodecahedron (9):

a	g	f	l	i	c	b	h	e	k	j	d	
a			l		c		h	e			d	*
a			l		c	b			k		d	
a		f			c	b			k	j		
a		f			c	b		e			d	
a		f		i			h	e			d	
a		f		i		b		e		j		
	g	f		i		b			k	j		
	g	f		i			h	e		j		
	g		l	i			h	e			d	
	g		l	i			h		k	j		
	g		l		c	b			k	j		
	g		l		c		h		k		d	

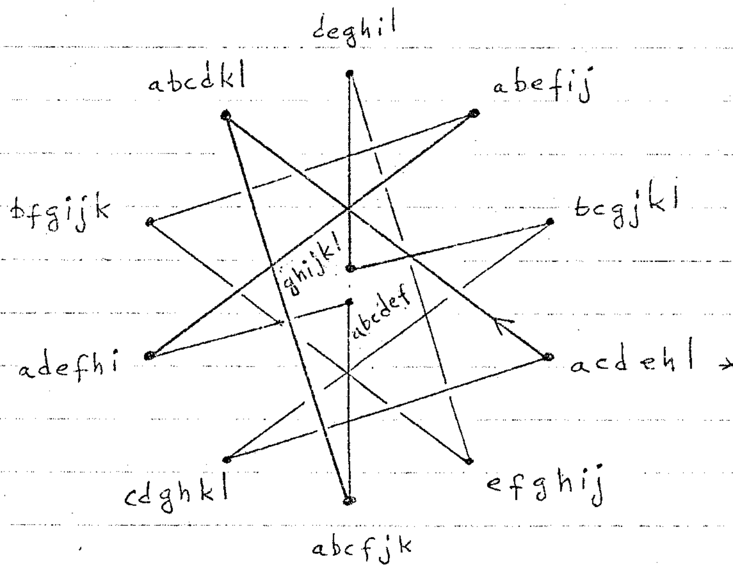
Pattern of icosahedron cycle (11) over icosahedron (1):

(12)



Pattern of cycle (11) over stellate dodecahedron (9):

(13)



Pattern (12), over the icosahedron, alternately forms 30° and 144° angles. What angles will be analogously formed as pattern (13) proceeds over the stellate dodecahedron?

The geometric structure of fig. (12) has a degree of symetry. How many ways may this structure be rotated about the surface of the icosahedron? (without falling exactly over itself.) Does this set of rotations contain its own reciprocals? Its own mirrors?

Let the reader construct an utterly asymetric ~~pattern~~ structure (14) passing thru the 12 nodes. How many ways may this asymetric structure be rotated about the surface of the icosahedron? Does this set of rotation contain its own reciprocals? Its own mirrors? What is the ~~diffence~~ difference between a reciprocal and a mirror? Axes of rotation may be found thru antipodal Triangular sides; thru antipodal nodes; and thru antipodal edges (lines). Thru ~~w~~which of these axes is the icosahedron superposable on its own mirror image?

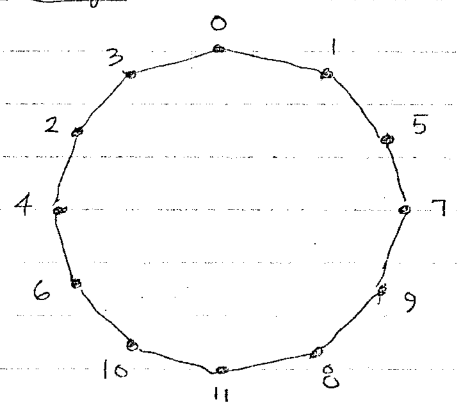
On mapping the 12-tone scale over the dodecagon and the Icosahedron

The image I form of my acoustic materials inevitably influences how I use it, however artfully or guilelessly this may be. ~~The word image~~ "Scale" is certainly a collective image, and, quite possibly, archtypal. Without thinking, my first association with "scale" is a series of steps moving upward. I start at "c" and, quite magically, arrive at "c" again when I reach the top. This leads to a "cycle" image which connects the top of the stairs with the bottom. But this image, now, begins to resemble the numerals on the face of a clock, which hurl themselves in a clockwise direction about the center. This cycle is intimately allied with the "cycle-of-Fourths". Both of these cycles have been enlightening and helpful in the development of my craft. (a) They both proceed by equal intervals. (b) They are both dependent upon circulation about a center for their identification. (c) And each pair of antipodes is separated by an interval of 6 units. (d) Further, parallel lines yield reciprocal intervals. (c) & (d) result from (a) & (b).

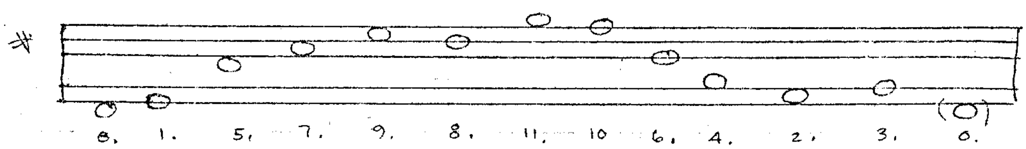
However, I may arrange the 12 members of the set in a geometrically consistent, centerless dodecagon, where parallel lines have equal values. Numbering the members, "0" thru "11", (0=12), I partition them into six pair in such a way that the sum of each pair is, first, an odd number, and second, identical to the sum of each other pair. The members of each pair are then assigned, in any sequence I may choose, to antipodal nodes of the dodecagon. Example: Partitioning so the sum is 11 yields 0,11; 1,10; 2,9; 3,8; 4,7; & 5,6. Now, I assign these pair at random, if I wish, across the antipodes!

<u>12-tone Dodecagon</u>

(15)



$(15)_b$



Hence, a mirroring cycle. How many such cycles will there be in the 12-tone scale? (excluding modulation)

*Think of the staff-lines as black keys.

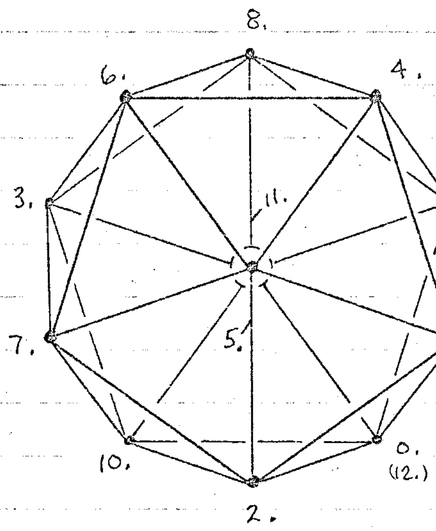
If I subtract "1" from each member of the dodecagon (15), I have modulated the entire structure downward by 1 unit (one semitone). However the sum across the antipodes is no longer 11; it is now, in each case, 9. If I subtract 2 it is 7, and so on. Altering the sums across the antipodes, then, to derive novel material, produces, only, a multiplicity of keys, (modulations).

Consider the intervals separating the antipodes; they are 1, 3, 5, 7, 9, 11 units (or their complements modulus 12). These may be grouped into complementary pair (mod 12) thus: 1,11 3,9 5,7. Each member of a pair may function as its complement. Example: subtract 6 from the entire cycle; the "1" unit interval between members 5 and 6 now appears as an 11-unit interval ~~between~~ (in the reciprocal direction) between members 0 and 11. Conversely, the 11-unit interval between members 0 and 11 now appears as a 1-unit interval (in recip. direction) between members 5 and 6. With this understanding of complements it might be said that there are only 3 different intervals across the antipodes, each occurring twice. ~~and~~ Each ~~having~~ ~~being~~ has an odd number of units; one member of each odd-numbered complementary pair must be represented. ¶ With the cautions & considerations described on this page, I must admit the question, "How many such cycles?" still boggles me! Any suggestion?

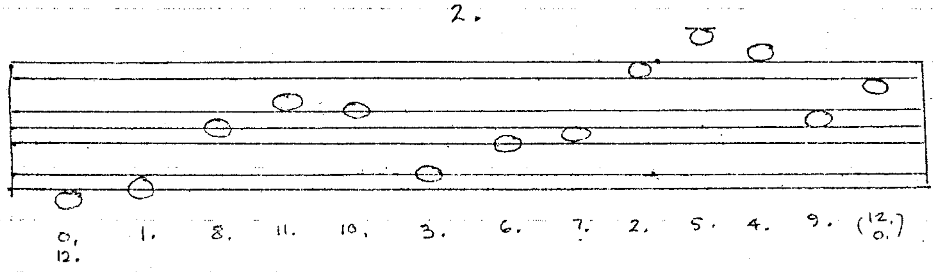
The 12-tone scale may be mapped over the points of the icosahedron in 2 ways:
1.) the ~~diff~~ 12 members are partitioned into 6 pair so that the difference between the members of each pair is 6 units. Members of each pair are then assigned to antipodal nodes (points) of the icosahedron, in whatever sequence one chooses. This way gives a spherical-clock-like effect* to the direction of the intervals. That is, antipodal intervals are reciprocals of each other, unless they are seen as parts of a single continuing line circulating around the surface ~~of the sphere~~ & about the center of a sphere.

*If you can imagine such a thing!

(16)a



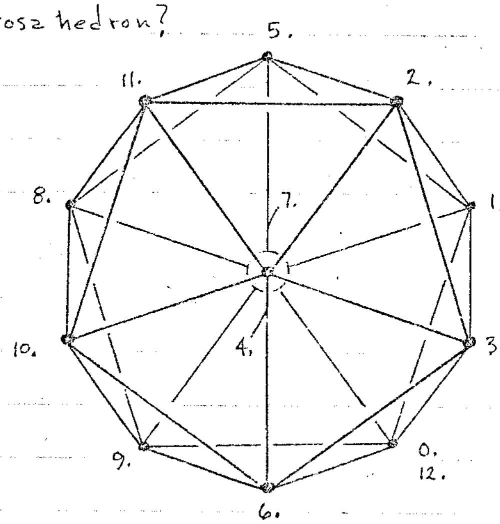
(16)b



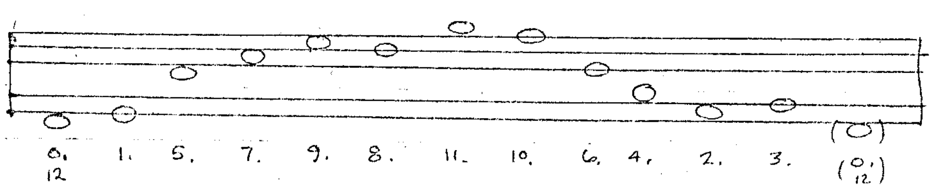
(cycle analogous to (12)) The melody repeats itself, after the first 6 tones, on the augmented Fourth, and returns to the Octave of the original tone, where it resumes the cycle. Several repetitions of such a cycle, strictly observed will take you right off one end, or another, of the keyboard. Frankly, the approach has very little appeal to my aesthetic sensibility. The 2nd method does to have considerable appeal.

2) The 12 members are partitioned into 6 pair so that the sums of the members of each pair ~~is~~ are identical, & equal to any same odd number. 11 is the most convenient sum to use, and yeilds this partition: 0,11 1,10 2,9 3,8 4,7 & 5,6. How many different ways may these pair be assigned across the antipodal nodes of icosahedron?

(17)a



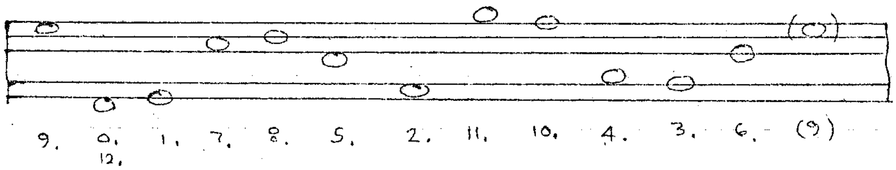
(17)b



(cycle (12) analog) The melody begins to to mirror itself after six tones. Depending on where we begin in the cycle, the interval of the tone beginning the mirror (or its reciprocal) may be any of the odd-numbered intervals to the original tone. ~~(or its reciprocal)~~. Cycles, strictly observed, return to point of origen.

Cycle (17)b, ~~is structurally~~ meandering about the surface of a sphere, is structurally equivalent to cycle (15), which moves about the circumference of a circle. I may, however, perform a manipulation of the cycle (12) analog about the surface of a sphere, icosahedron (17)a, which is utterly unthinkable about the circumference of a circle, dodecagon (15). Cycle (12) may be rotated in 20 distinct ways about the surface ~~of~~ of the icosahedron (17)a. This gives rise to a family of 20 related, but acoustically distinct melodies. I'll not enumerate these, but will give one example, where the image of the cycle (12) is rotated 72° clockwise (on paper):

(17)c



I am going to interrupt letter at page 51. It will continue on page 52.

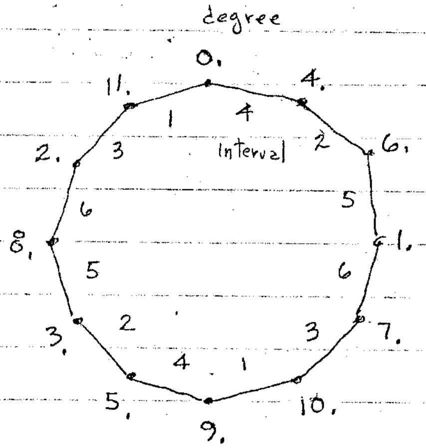
Erv Wilson
May 14, 1971

A dodekagon cycle which traverses each of the 6 intervals before repeating.

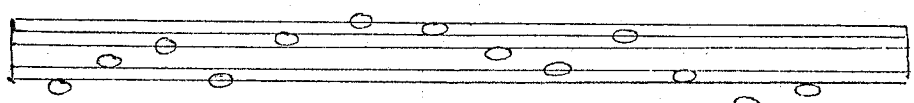
An interval is considered equivelent to its complement: 1 unit is equivalent to 11 units

2	"	10
3	"	9
4	"	8
5	"	7
6	"	6

(18)



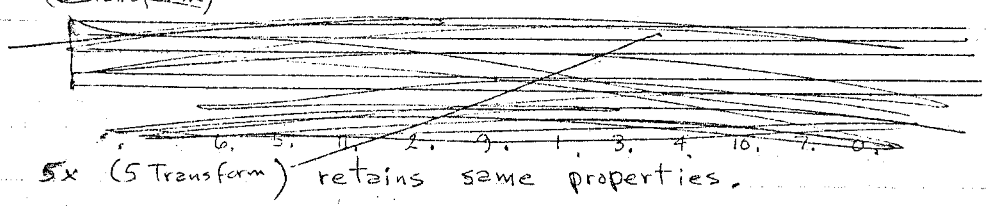
1x



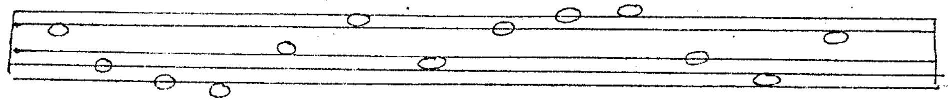
0. 4. 6. 1. 7. 10. 9. 5. 3. 8. 2. 11. 0.

4 2 5 6 3 1 4 2 5 6 3 1

~~(5 Transform)~~



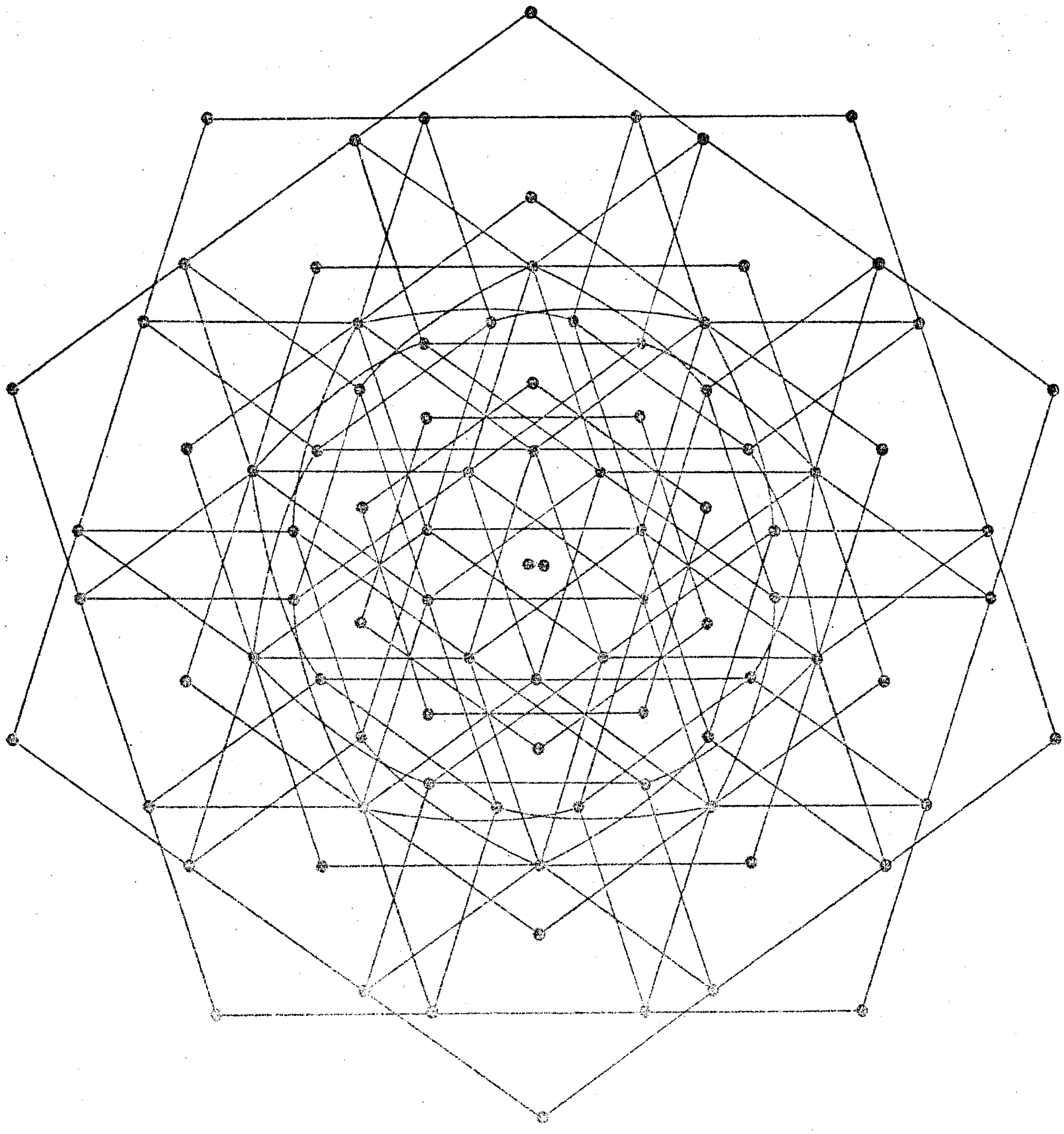
5x (5 Transform) retains same properties.



0. 8. 6. 5. 11. 2. 9. 1. 3. 4. 10. 7. 0.

4 2 1 6 3 5 4 2 1 6 3 5

How many such cycles can there be?



Erv Wilson 1967