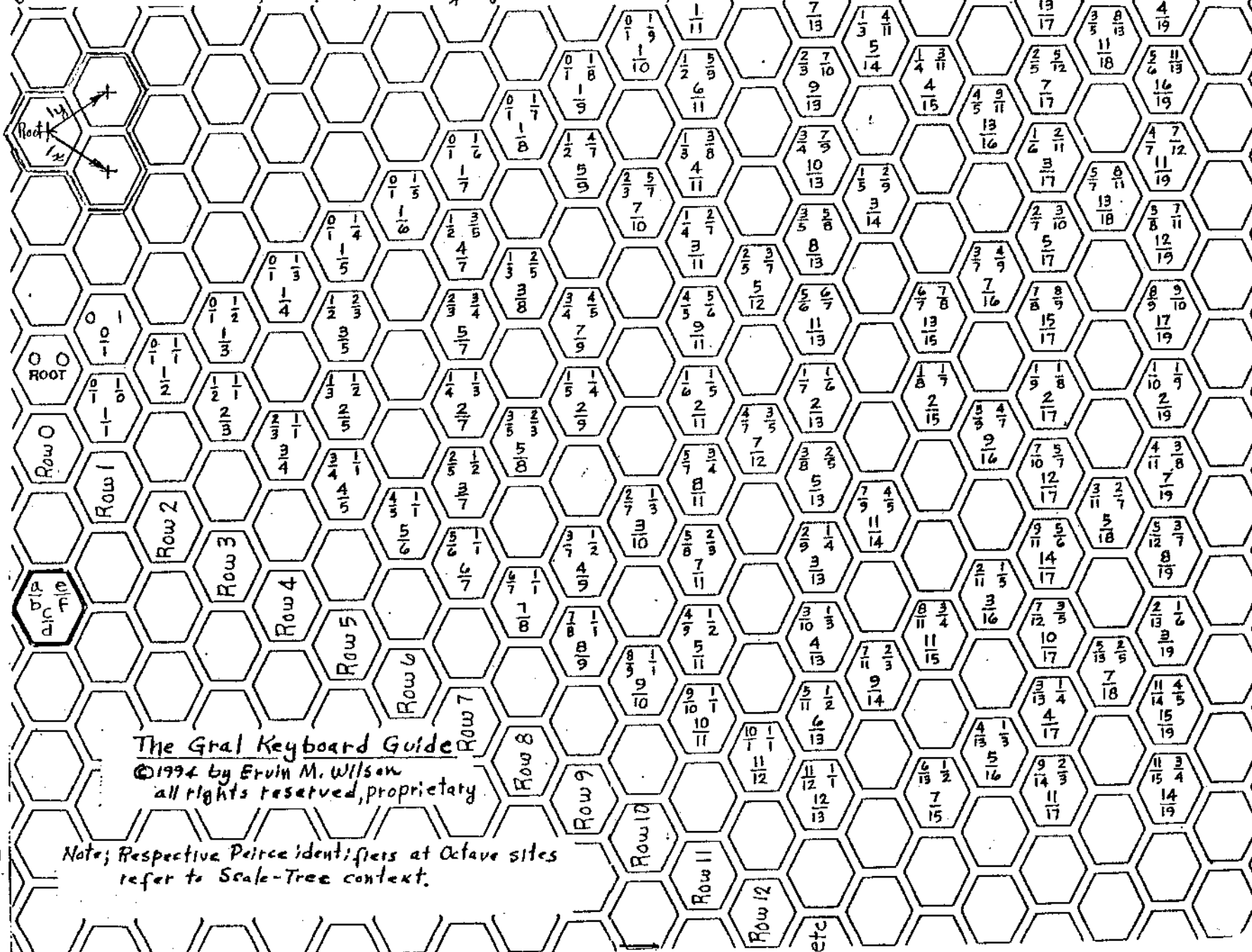


DIOPHANTINE TRIPLETS and x, y Coordinates, Applied to the GRAAL KEYBOARD, Co-Prime Format.

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To Craig Grady from Erv Wilson 13 April 2001

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The Graal Keyboard Guide
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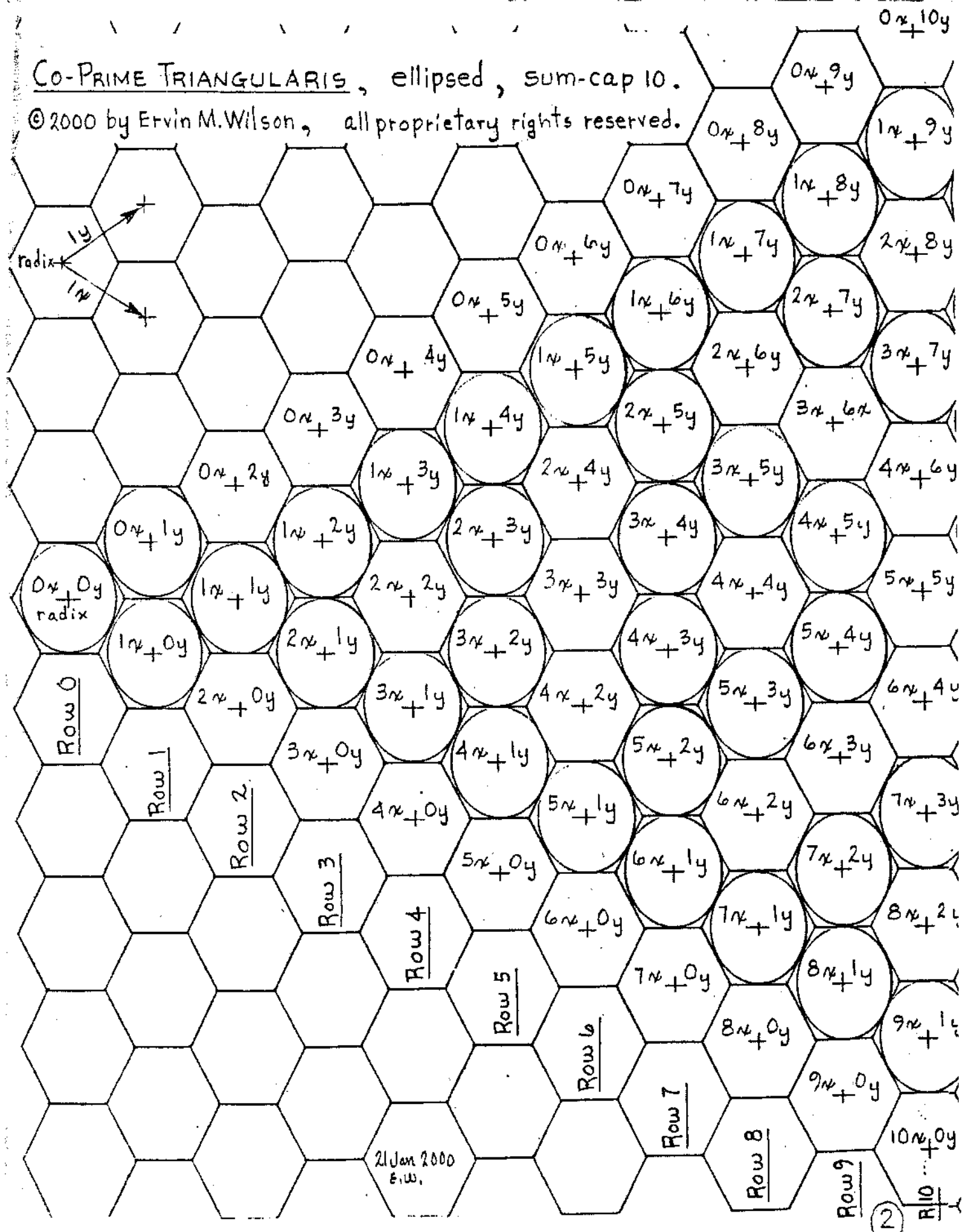
Note; Respective Peirce identifiers at Octave sites refer to Scale-Tree context.

Sheet 1
of 23 sheets

①

CO-PRIME TRIANGULARIS, ellipsed, sum-cap 10.

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21 Jan 2000
E.W.

Definition: $\frac{0}{1} \frac{1}{1} \frac{1}{0}$ is the most extensive Diophantine Triplet $\frac{a}{b} \frac{c}{d} \frac{e}{f}$; $be - af = 1$
 $bc - ad = 1$
 $de - cf = 1$

On The Application of Diophantine Equations To Musical Instrument Keyboard Format

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 (work in progress)

"Keyboard noun --- arrangement of the keys as of an organ, piano, etc."
 Funk & Wagnalls New College Standard Dictionary 1947

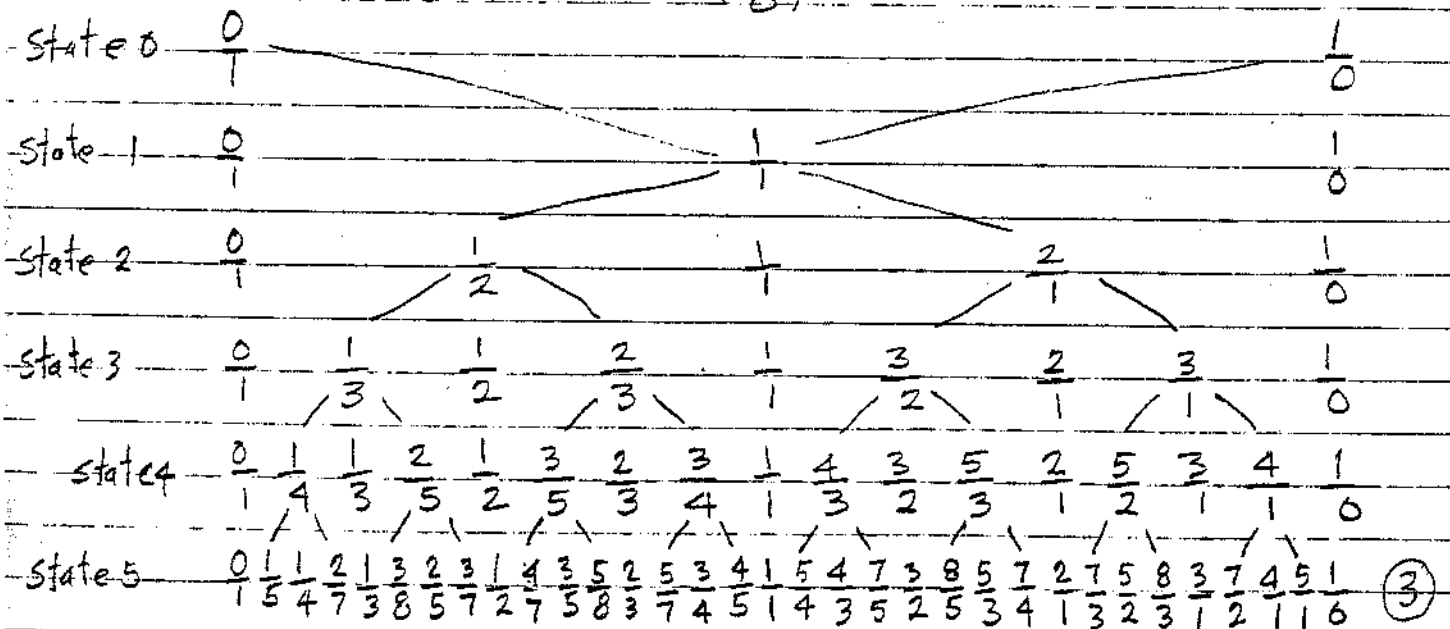
Diophantus of Alexandria was a 3rd century mathematician

His equation, $b \cdot c - a \cdot d = 1$, is applicable to

musical instrument keyboard format. Where $\frac{a}{b} \frac{c}{d}$, the most comprehensive form is $\frac{0}{1} \frac{1}{0}$. Charles Sanders Peirce, 19th century logician embodies this Diophantine Couplet in his series (Peirce Series), which I call the Scale Tree. This is how it progresses; add the top numbers (a+c) and the bottom numbers (b+d) to get the intermediate fraction, $\frac{0}{1} \frac{1}{1} \frac{1}{0}$, the Diophantine Triplet $\frac{a}{b} \frac{c}{d} \frac{e}{f}$.

Continue procedure to get $\frac{0}{1} \frac{1}{2} \frac{1}{1} \frac{2}{1} \frac{1}{0}$, and then $\frac{0}{1} \frac{1}{3} \frac{1}{2} \frac{2}{3} \frac{1}{1} \frac{3}{2} \frac{2}{1} \frac{3}{1} \frac{1}{0}$ and so on, endlessly, Thus;

($\frac{0}{0}$)



where $\begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$, then $\frac{c}{d} = \frac{a+e}{b+f}$, and (Diophantine equation)
 $b \cdot e - a \cdot f = 1$

Hence $b \cdot c - a \cdot d = 1$
 and $d \cdot e - c \cdot f = 1$

$\frac{0}{1} \frac{1}{1} \frac{1}{0}$.500
 →

$\frac{1}{2} \frac{2}{3} \frac{1}{1}$.666
 ←

$\frac{1}{2} \frac{3}{5} \frac{2}{3}$.600
 ←

achtung!
 application to
Keyboard Sites

(Foot)	Gen	8ve
○	○	○
a_x, o_y	a_x, e_y	b_x, f_y

Gen. info $\frac{1}{2} \frac{4}{7} \frac{3}{5}$.5714
 8ve. info →

$\frac{4}{7} \frac{7}{12} \frac{3}{5}$.5833
 ←

$\frac{4}{7} \frac{11}{19} \frac{7}{12}$.578947
 →

$\frac{11}{19} \frac{18}{31} \frac{7}{12}$.580645
 ←

$\frac{11}{19} \frac{29}{50} \frac{18}{31}$.586000
 →

$\frac{29}{50} \frac{47}{81} \frac{18}{31}$.580247
 →

$\frac{47}{81} \frac{65}{112} \frac{18}{31}$

- Notes 1. (x, y) coordinates cannot be casually slapped on the scale-tree.
 2. The complementary generators must be run to get all the keyboards.
 3. The Peirce triplet is ubiquitous in the Scale-Tree, the Lambda & the Triangle (imbué) substance
 The nuclear stuff

Six Items applied to Uath Keyboard 23JAN00.EW

- 1st Item: Every keyboard is a Boomsliter & Creel Work Station; each note is ear-tuneable.
- 2nd Item: Alter-Octaves are expressible in logarithms to the base of the respective alter-octave.
- 3rd Item: Notes assigned to the keyboard are optionally played in reciprocal; the same fingering will play the melody upside-down.
- 4th Item: Octaves are assignable to any key which is a co-prime move on the $x-y$ grid from the Root.
- 5th Item: The generator of a linear series is mutable live, and can be hand-controlled or set into continuum.
Example; With C_1^1 as root, and C_2^2 as the Octave, and $G_{\frac{3}{2}}$ as the generator of a chain-of-Fifths — the generator may be changed in size to give Pythagorean tuning or $\frac{1}{4}$ -comma meantone, or any linear tuning in between and beyond.
- 6th Item: The x, y grid is selectable on the keyboard, and may be rotated thru all its planes.

$$\sqrt[3]{10/3} = 1.49380158218\dots$$

$\frac{1}{3}$ -comma Meantone, Fifth

$$\rightarrow \text{Log}_2 .578988531383\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root	Generator	Octave
$0x, 0y$	ax, ey	bx, fy		$0x, 0y$	ax, ey	bx, fy
0/1	1/1	1/0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
←						
0/1	1/2	1/1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
→						
1/2	2/3	1/1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
←						
1/2	3/5	2/3	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
←						
1/2	4/7	3/5	.571429	$0x, 0y$	$1x, 3y$	<u>$2x, 5y$</u>
→						
4/7	7/12	3/5	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
←						
4/7	11/19	7/12	.578947	$0x, 0y$	$4x, 7y$	$7x, 12y$
→						
11/19	18/31	7/12	.580645	$0x, 0y$	$11x, 7y$	$19x, 12y$
←						
11/19	29/50	18/31	.580000	$0x, 0y$	$11x, 18y$	$19x, 31y$
←						
11/19	40/69	29/50	.579710	$0x, 0y$	$11x, 29y$	$19x, 50y$
←						
11/19	51/88	40/69	.579545	$0x, 0y$	$11x, 40y$	$19x, 69y$
←						
11/19	62/107	51/88	.579439	$0x, 0y$	$11x, 51y$	$19x, 88y$
←						
11/19	73/126	62/107	.579365	$0x, 0y$	$11x, 62y$	$19x, 107y$
←						
5x29	11/19	84/126	.579310	$0x, 0y$	$11x, 73y$	$19x, 126y$
←						
4x41	11/19	95/145	.579268	$0x, 0y$	$11x, 84y$	$19x, 145y$
←						
11/19	106/183	95/164	.579235	$0x, 0y$	$11x, 95y$	$19x, 164y$

see 744/1285 66 places ← etc

6

15APR00.EW

$$2/\sqrt[3]{10/3} = 1.33886590017\dots$$

1/3 Comma Meantone, Fourth

$$\text{Log}_2 .421011468617\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
b	d	f	$\frac{c}{d}$		$0n, 0y$	a_n, e_y	b_n, f_y
0	1	0	1.000000		$0n, 0y$	$0n, 1y$	$1n, 0y$
				←			
0	1	1	.500000		$0n, 0y$	$0n, 1y$	$1n, 1y$
				←			
0	1	2	.333333		$0n, 0y$	$0n, 1y$	$1n, 2y$
				→			
1	3	2	.400000		$0n, 0y$	$1n, 1y$	$3n, 2y$
				→			
2	5	2	.428571		$0n, 0y$	$2n, 1y$	$5n, 2y$
				←			
2	5	3	.416667		$0n, 0y$	$2n, 3y$	$5n, 7y$
				→			
5	12	3	.421053		$0n, 0y$	$5n, 3y$	$12n, 7y$
				←			
5	12	8	.419355		$0n, 0y$	$5n, 8y$	$12n, 19y$
				→			
13	31	8	.420000		$0n, 0y$	$13n, 8y$	$31n, 19y$
				→			
21	50	8	.420290		$0n, 0y$	$21n, 8y$	$50n, 19y$
				→			
29	69	8	.420455		$0n, 0y$	$29n, 8y$	$69n, 19y$
				→			
37	88	8	.420561		$0n, 0y$	$37n, 8y$	$88n, 19y$
				→			
45	107	8	.420635		$0n, 0y$	$45n, 8y$	$107n, 19y$
				→			
53	126	8	.420690		$0n, 0y$	$53n, 8y$	$126n, 19y$
				→			
61	145	8	.420732		$0n, 0y$	$61n, 8y$	$145n, 19y$
				→			
69	164	8	.420765		$0n, 0y$	$69n, 8y$	$164n, 19y$
				→ etc			

66 places

$$\sqrt[3]{\frac{5}{2}} = 1.35720880830\dots$$

Neo-Pèlog, Fourth

$$\log_2 .440642698299\dots$$

a	c	e	c	Root Generator	Octave
b	d	f	d dec.	$0x, 0y$	a_x, e_y
0	1	1	1.000000	$0x, 0y$	$0x, 1y$
1	1	0		$1x, 0y$	$1x, 0y$
←					
0	1	1	.500000	$0x, 0y$	$0x, 1y$
1	2	1		$1x, 0y$	$1x, 1y$
←					
0	1	1	.333333	$0x, 0y$	$0x, 1y$
1	3	2		$1x, 0y$	$1x, 2y$
→					
1	2	1	.400000	$0x, 0y$	$1x, 1y$
3	5	2		$3x, 0y$	$3x, 2y$
→					
2	3	1	.428571	$0x, 0y$	$2x, 1y$
5	7	2		$5x, 0y$	$5x, 2y$
→					
3	4	1	.444444	$0x, 0y$	$3x, 1y$
7	9	2		$7x, 0y$	$7x, 2y$
←					
3	7	4	.437500	$0x, 0y$	$3x, 4y$
7	16	9		$7x, 0y$	$7x, 9y$
→					
7	11	4	.440000	$0x, 0y$	$7x, 4y$
16	25	9		$16x, 0y$	$16x, 9y$
→					
11	15	4	.441176	$0x, 0y$	$11x, 4y$
25	34	9		$25x, 0y$	$25x, 9y$
←					
11	26	15	.440678	$0x, 0y$	$11x, 15y$
25	59	34		$25x, 0y$	$25x, 34y$
←					
11	37	26	.440476	$0x, 0y$	$11x, 26y$
25	84	59		$25x, 0y$	$25x, 59y$
→					
37	63	26	.440559	$0x, 0y$	$37x, 26y$
84	143	59		$84x, 0y$	$84x, 59y$
→					
63	89	26	.440594	$0x, 0y$	$63x, 26y$
143	202	59		$143x, 0y$	$143x, 59y$
→					
89	115	26	.440613	$0x, 0y$	$89x, 26y$
202	261	59		$202x, 0y$	$202x, 59y$
→					
115	141	26	.440625	$0x, 0y$	$115x, 26y$
261	320	59		$261x, 0y$	$261x, 59y$
→					
141	167	26	.440633	$0x, 0y$	$141x, 26y$
320	379	59		$320x, 0y$	$320x, 59y$
→					
167	193	26	.440639	$0x, 0y$	$167x, 26y$
379	438	59		$379x, 0y$	$379x, 59y$
→					
193	219	26	.440644	$0x, 0y$	$193x, 26y$
438	497	59		$438x, 0y$	$438x, 59y$
←					



$$\sqrt[3]{16/5} = 1.47361259945\dots$$

Neo-Pèlog, Fifth

$$\text{Log}_2 .559357301701\dots$$

a	c	e	c	dec.	Root Generator Octave
b	d	f	d		$0n, 0y$ a_n, e_y b_n, f_y
0	1	1	1.000000		$0n, 0y$ $0n, 1y$ $1n, 0y$
1	1	0		←	
0	1/2	1	.500000		$0n, 0y$ $0n, 1y$ $1n, 1y$
1	1	0		→	
1	2/3	1	.666667		$0n, 0y$ $1n, 1y$ $2n, 1y$
2	3	1		←	
1	3/5	2	.600000		$0n, 0y$ $1n, 2y$ $2n, 3y$
2	5	3		←	
1	4/7	3	.571429		$0n, 0y$ $1n, 3y$ $2n, 5y$ ✓
2	7	5		←	
1	5/9	4	.555556		$0n, 0y$ $1n, 4y$ $2n, 7y$
2	9	7		→	
5	9/16	4	.562500		$0n, 0y$ $5n, 4y$ $9n, 7y$
9	16	7		←	
5	14/25	9	.560000		$0n, 0y$ $5n, 9y$ $9n, 16y$
9	25	16		←	
5	19/34	14	.558824		$0n, 0y$ $5n, 14y$ $9n, 25y$
9	34	25		→	
19	33/59	14	.559322		$0n, 0y$ $19n, 14y$ $34n, 25y$
34	59	25		→	
33	47/84	14	.559524		$0n, 0y$ $33n, 14y$ $59n, 25y$
59	84	25		←	
33	80/143	47	.559441		$0n, 0y$ $33n, 47y$ $59n, 84y$
59	143	84		←	
33	113/202	80	.559406		$0n, 0y$ $33n, 80y$ $59n, 143y$ ✓
59	202	143		←	
33	146/261	113	.559387		$0n, 0y$ $33n, 113y$ $59n, 202y$
59	261	202		←	
33	179/320	146	.559375		$0n, 0y$ $33n, 146y$ $59n, 261y$
59	320	261		←	
33	212/379	179	.559367		$0n, 0y$ $33n, 179y$ $59n, 320y$
59	379	320		←	
33	245/438	212	.559361		$0n, 0y$ $33n, 212y$ $59n, 379y$
59	438	379		←	
33	278/497	245	.559356		$0n, 0y$ $33n, 245y$ $59n, 438y$
59	497	438		→	

$$\sqrt[6]{5/2} = 1.16499305075\dots$$

Neo-Pèlog, subminorThird

$$\text{Log}_2 .220321349146\dots$$

a	c	e	c	dec.	Root	Generator	Octave
b	d	f	d		$0x, 0y$	ax, ey	bx, fy
0	1	1	1.000000		$0x, 0y$	$0x, 1y$	$1x, 0y$
				←			
0	1/2	1	.500000		$0x, 0y$	$0x, 1y$	$1x, 1y$
				←			
0	1/3	2	.333333		$0x, 0y$	$0x, 1y$	$1x, 2y$
				←			
0	1/4	3	.250000		$0x, 0y$	$0x, 1y$	$1x, 3y$
				←			
0	1/5	4	.200000		$0x, 0y$	$0x, 1y$	$1x, 4y$
				→			
1	2/9	4	.222222		$0x, 0y$	$1x, 1y$	$5x, 4y$
				←			
1	3/14	2	.214286		$0x, 0y$	$1x, 2y$	$5x, 9y$
				→			
3	5/23	2	.217391		$0x, 0y$	$3x, 2y$	$14x, 9y$
				→			
5	7/32	2	.218750		$0x, 0y$	$5x, 2y$	$23x, 9y$
				→			
7	9/41	2	.219512		$0x, 0y$	$7x, 2y$	$32x, 9y$
				→			
9	11/50	2	.220000		$0x, 0y$	$9x, 2y$	$41x, 9y$
				→			
11	13/59	2	.220339		$0x, 0y$	$11x, 2y$	$50x, 9y$
				←			
11	24/109	13	.220183		$0x, 0y$	$11x, 13y$	$50x, 59y$
				→			
24	37/168	13	.220238		$0x, 0y$	$24x, 13y$	$109x, 59y$
				→			
37	50/227	13	.220264		$0x, 0y$	$37x, 13y$	$168x, 59y$
				→			
50	63/286	13	.220280		$0x, 0y$	$50x, 13y$	$227x, 59y$
				→			
15 places		etc					

$$2/\sqrt[6]{5/2} = 1.71674843787\dots \quad \text{Neo-Pèlog, supramajor Sixth}$$

$$\log_2 .779678650856\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
b	d	f			$0n, 0y$	a_n, e_y	b_n, f_y
0	1	0	1.000000		$0n, 0y$	$0n, 1y$	$1n, 0y$
				←			
0	1	1	.500000		$0n, 0y$	$0n, 1y$	$1n, 1y$
				→			
1	2	1	.666667		$0n, 0y$	$1n, 1y$	$2n, 1y$
				→			
2	3	1	.750000		$0n, 0y$	$2n, 1y$	$3n, 1y$
				→			
3	4	1	.800000		$0n, 0y$	$3n, 1y$	$4n, 1y$
				←			
3	7	4	.777778		$0n, 0y$	$3n, 4y$	$4n, 5y$
				→			
7	11	4	.785714		$0n, 0y$	$7n, 4y$	$9n, 5y$
				←			
7	18	11	.782609		$0n, 0y$	$7n, 11y$	$9n, 14y$
				←			
7	25	18	.781250		$0n, 0y$	$7n, 18y$	$9n, 23y$
				←			
7	32	25	.780488		$0n, 0y$	$7n, 25y$	$9n, 32y$
				←			
7	39	32	.780000		$0n, 0y$	$7n, 32y$	$9n, 41y$
				←			
7	46	39	.779661		$0n, 0y$	$7n, 39y$	$9n, 50y$
				→			
46	85	39	.779817		$0n, 0y$	$46n, 39y$	$59n, 50y$
				←			
46	131	85	.779762		$0n, 0y$	$46n, 85y$	$59n, 109y$
				←			
46	177	131	.779736		$0n, 0y$	$46n, 131y$	$59n, 168y$
				←			
46	223	177	.779720		$0n, 0y$	$46n, 177y$	$59n, 227y$
				←			

✓ new good

$$\sqrt[4]{5} = 1.49534878122\dots$$

1/4-comma Meantone, Fifth

$$\rightarrow \text{Log}_2 .580482023721\dots$$

5FEB00·EW

	a	c	e	$\frac{c}{d}$	Root $0x, 0y$	Generator a_n, e_y	Octave b_n, f_y
	b	d	f	$\frac{c}{d}$			
	0	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
	1	1	0	←			
	0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
	1	2	1	→			
	1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
	2	3	1	←			
1/4-comma 5th meantone	1	3	2	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
	2	5	3	←			
	1	4	3	.571429	$0x, 0y$	$1x, 3y$	$2x, 5y$
	2	7	5	→			
	4	7	3	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
	7	12	5	←			
	4	11	7	.578947	$0x, 0y$	$4x, 7y$	$7x, 12y$
	7	19	12	→			
	11	18	7	.580645	$0x, 0y$	$11x, 7y$	$19x, 12y$
	19	31	12	←			
	11	29	18	.580000	$0x, 0y$	$11x, 18y$	$19x, 31y$
	19	50	31	→			
	29	47	18	.580247	$0x, 0y$	$29x, 18y$	$50x, 31y$
	50	81	31	→			
	47	65	18	.580357	$0x, 0y$	$47x, 18y$	$81x, 31y$
	81	112	31	→			
→	65	83	18	.580420	$0x, 0y$	$65x, 18y$	$112x, 31y$
	112	143	31	→			
	83	101	18	.580460	$0x, 0y$	$83x, 18y$	$143x, 31y$
	143	174	31	→			
	101	119	18	.580488	$0x, 0y$	$101x, 18y$	$174x, 31y$
	174	205	31	←			
	101	220	119	.580475	$0x, 0y$	$101x, 119y$	$174x, 205y$
	174	379	205	→			
	220	339	119	.580479	$0x, 0y$	$220x, 119y$	$379x, 205y$
	379	584	205	→			

checked

(12)

$$2/\sqrt[4]{5} = 1.33748060995\dots$$

1/4-comma Meantone, Fourth

$$\rightarrow \text{Log}_2 .419517976279\dots$$

	a	c	e	c	Root	Generator	Octave
	b	d	f	d	$0x, 0y$	$a'x, e'y$	$b'x, f'y$
gen.	0	1	1		$0x, 0y$		
Oct.	1	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
				←			
	0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
				←			
	0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
				→			
	1	2	1	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
				→			
	2	3	1	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
				←			
	2	5	3	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
				→			
	5	8	3	.421053	$0x, 0y$	$5x, 3y$	$12x, 7y$
				←			
	5	13	8	.419355	$0x, 0y$	$5x, 8y$	$12x, 19y$
				→			
	13	21	8	.420000	$0x, 0y$	$13x, 8y$	$31x, 19y$
				←			
	13	34	21	.419753	$0x, 0y$	$13x, 21y$	$31x, 50y$
				←			
	13	47	34	.419643	$0x, 0y$	$13x, 34y$	$31x, 81y$
				←			
	13	60	47	.419580	$0x, 0y$	$13x, 47y$	$31x, 112y$
				←			
	13	73	60	.419540	$0x, 0y$	$13x, 60y$	$31x, 143y$
				←			
	13	86	73	.419512	$0x, 0y$	$13x, 73y$	$31x, 174y$
				→			
	86	159	73	.419525	$0x, 0y$	$86x, 73y$	$205x, 174y$
				←			
	86	245	159	.419521	$0x, 0y$	$86x, 159y$	$205x, 379y$
				←			

(13)

$$\sqrt[5]{\frac{15}{2}} = 1.49627786974\dots$$

1/5-Comma Meantone, Fifth

$$\rightarrow \log_2 .581378119122\dots$$

a	c	e	c	dec.	Root	Generator	Octave
b	d	f	d		$0x, 0y$	ax, ey	bx, fy
0	1	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0		←			
0	1	1	2	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1		→			
1	2	1	3	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
2	3	1		←			
1	3	2	5	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
2	5	3		←			
1	4	3	7	.571429	$0x, 0y$	$1x, 3y$	$2x, 5y$
2	7	5		→			
4	7	3	5	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
7	12	5		←			
4	11	7	12	.578947	$0x, 0y$	$4x, 7y$	$7x, 12y$
7	19	12		→			
11	18	7	12	.580645	$0x, 0y$	$11x, 7y$	$19x, 12y$
19	31	12		→			
18	25	7	12	.581395	$0x, 0y$	$18x, 7y$	$31x, 12y$
31	43	12		←			
18	43	25	43	.581081	$0x, 0y$	$18x, 25y$	$31x, 43y$
31	74	43		→			
43	68	25	43	.581197	$0x, 0y$	$43x, 25y$	$74x, 43y$
74	117	43		→			
68	93	25	43	.581250	$0x, 0y$	$68x, 25y$	$117x, 43y$
117	160	43		→			
93	118	25	43	.581281	$0x, 0y$	$93x, 25y$	$160x, 43y$
160	203	43		→			
118	143	25	43	.581301	$0x, 0y$	$118x, 25y$	$203x, 43y$
203	246	43		→			
143	168	25	43	.581315	$0x, 0y$	$143x, 25y$	$246x, 43y$
246	289	43		→			
168	193	25	43	.581325	$0x, 0y$	$168x, 25y$	$289x, 43y$
289	332	43		→			

21x21

17x17

$$2/\sqrt[5]{15/2} = 1.33665012392\dots$$

1/5-comma Meantone, Fourth

$$\rightarrow \text{Log}_2 .418621880078\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root Ox, Oy	Generator $a'x, e'y$	Octave $b'x, f'y$
0/1	1/1	1/0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0/1	1/2	1/1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0/1	1/3	1/2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			→			
1/3	2/5	1/2	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
			→			
2/5	3/7	1/2	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
			←			
2/5	5/12	3/7	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
			→			
5/12	8/19	3/7	.421053	$0x, 0y$	$5x, 3y$	$12x, 7y$
			←			
5/12	13/31	8/19	.419355	$0x, 0y$	$5x, 8y$	$12x, 19y$
			←			
5/12	18/43	13/31	.418605	$0x, 0y$	$5x, 13y$	$12x, 31y$
			→			
18/43	31/74	13/31	.418919	$0x, 0y$	$18x, 13y$	$43x, 31y$
			←			
18/43	49/117	31/74	.418803	$0x, 0y$	$18x, 31y$	$43x, 74y$
			←			
18/43	67/160	49/117	.418750	$0x, 0y$	$18x, 49y$	$43x, 117y$
			←			
18/43	85/203	67/160	.418719	$0x, 0y$	$18x, 67y$	$43x, 160y$
			←			
18/43	103/246	85/203	.418699	$0x, 0y$	$18x, 85y$	$43x, 203y$
			←			
18/43	121/289	103/246	.418685	$0x, 0y$	$18x, 103y$	$43x, 246y$
			←			
18/43	139/332	121/289		$0x, 0y$	$18x, 121y$	$43x, 289y$
			←			

25x5

(15)

$$4/3 = 1.3333333333333333 \dots$$

$$\rightarrow \log_2 .415037499279 \dots$$

Pythagorean Fourth

gen. Oct.	$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root $0x, 0y$	Generator a_n, e_y	Octave b_n, f_y
0	1	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
				←			
0	1	2	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
				←			
0	1	3	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
				→			
1	3	5	2	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
				→			
2	5	7	2	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
				←			
2	5	12	3	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
				←			
2	5	17	5	.411765	$0x, 0y$	<u>$2x, 5y$</u>	$5x, 12y$
				→			
7	17	29	5	.413793	$0x, 0y$	$7x, 5y$	$17x, 12y$
				→			
12	29	41	5	.414634	$0x, 0y$	$12x, 5y$	$29x, 12y$
				→			
17	41	53	5	.415094	$0x, 0y$	$17x, 5y$	$41x, 12y$
				←			
17	41	94	22	.414894	$0x, 0y$	$17x, 22y$	$41x, 53y$
				→			
39	94	147	22	.414966	$0x, 0y$	$39x, 22y$	$94x, 53y$
				→			
61	147	200	22	.415000	$0x, 0y$	$61x, 22y$	$147x, 53y$
				→			
83	200	253	22	.415020	$0x, 0y$	$83x, 22y$	$200x, 53y$
				→			
105	253	306	22	.415033	$0x, 0y$	$105x, 22y$	$253x, 53y$
				→			
127	306	359	22	.415042	$0x, 0y$	$127x, 22y$	$306x, 53y$
				←			
127	306	665	149	.4150376	$0x, 0y$	$127x, 149y$	$306x, 359y$

✓
✓

$3/2 = 1.5$
 $\rightarrow \text{Log}_2 .584962500721\dots$

Pythagorean Fifth

	a	c	e	$\frac{c}{d}$ dec.	Root $0x, 0y$	Generator a_n, e_n	Octave b_n, f_n
gen.	b	d	f				
0	1	1	1	1.000000	$0x, 0y$	$0n, 1y$	$1n, 0y$
Oct.	1	1	0				
				←			
0	1	2	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
1	2	1	1				
				→			
1	2	3	1	.666667	$0n, 0y$	$1n, 1y$	$2n, 1y$
2	3	1	1				
				←			
1	2	3	2	.600000	$0n, 0y$	$1n, 2y$	$2n, 3y$
2	3	2	3				
				←			
1	2	3	5	.571429	$0n, 0y$	$1n, 3y$	$2n, 5y$
2	3	5	7				
				→			
4	7	3	5	.583333	$0n, 0y$	$4n, 3y$	$7n, 5y$
7	12	3	5				
				→			
7	10	3	5	.588235	$0n, 0y$	$7n, 3y$	$12n, 5y$
12	17	3	5				
				←			
7	17	10	17	.586207	$0n, 0y$	$7n, 10y$	$12n, 17y$
12	29	10	17				
				←			
7	24	17	29	.585366	$0n, 0y$	$7n, 17y$	$12n, 29y$
12	41	17	29				
				←			
7	31	24	41	.584906	$0n, 0y$	$7n, 24y$	$12n, 41y$
12	53	24	41				
				→			
31	55	24	41	.585106	$0n, 0y$	$31n, 24y$	$53n, 41y$
53	94	24	41				
				←			
31	86	55	94	.585034	$0n, 0y$	$31n, 55y$	$53n, 94y$
53	147	55	94				
				←			
31	117	86	147	.585000	$0n, 0y$	$31n, 86y$	$53n, 147y$
53	200	86	147				
				←			
31	148	117	200	.584980	$0n, 0y$	$31n, 117y$	$53n, 200y$
53	253	117	200				
				←			
31	179	148	253	.584967	$0n, 0y$	$31n, 148y$	$53n, 253y$
53	306	148	253				
				←			
31	210	179	306	.584958	$0n, 0y$	$31n, 179y$	$53n, 306y$
53	359	179	306				
				→			
210	389	179	306	.5849624	$0n, 0y$	$210n, 179y$	$359n, 306y$
359	665	179	306				

✓
 ✓ remarkable
 ✓

$\sqrt[8]{10} = 1.33352143216 \dots$ Helmholtz $\frac{1}{8}$ -skhisma, Fourth

$\log_2 .415241011858 \dots$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root Generator Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	$0x, 0y$ a_n, e_y b_n, f_y
0/1	1/1	0/0	1.000000	$0x, 0y$ $0x, 1y$ $1x, 0y$
			←	
0/1	1/2	1/1	.500000	$0x, 0y$ $0x, 1y$ $1x, 1y$
			←	
0/1	1/3	2/2	.333333	$0x, 0y$ $0x, 1y$ $1x, 2y$
			→	
1/3	2/5	1/2	.400000	$0x, 0y$ $1x, 1y$ $3x, 2y$
			→	
2/5	3/7	1/2	.428571	$0x, 0y$ $2x, 1y$ $5x, 2y$
			←	
2/5	5/12	3/7	.416667	$0x, 0y$ $2x, 3y$ $5x, 7y$
			←	
2/5	7/17	5/12	.411765	$0x, 0y$ $2x, 5y$ $5x, 12y$ ✓
			→	
7/17	12/29	5/12	.413793	$0x, 0y$ $7x, 5y$ $17x, 12y$
			→	
12/29	17/41	5/12	.414634	$0x, 0y$ $12x, 5y$ $29x, 12y$
			→	
17/41	22/53	5/12	.415094	$0x, 0y$ $17x, 5y$ $41x, 12y$
			→	
22/53	27/65	5/12	.415385	$0x, 0y$ $22x, 5y$ $53x, 12y$
			←	
22/53	49/118	27/65	.415254	$0x, 0y$ $22x, 27y$ $53x, 65y$
			←	
22/53	71/171	49/118	.415205	$0x, 0y$ $22x, 49y$ $53x, 118y$ ✓
			→	
71/171	120/289	49/118	.415225	$0x, 0y$ $71x, 49y$ $171x, 118y$
			→	
120/289	169/407	49/118	.415233	$0x, 0y$ $120x, 49y$ $289x, 118y$
			→	
169/407	218/525	49/118	.415238	$0x, 0y$ $169x, 49y$ $407x, 118y$
			→	
218/525	267/643	49/118	.4152410575	$0x, 0y$ $218x, 49y$ $525x, 118y$
			←	

$$\sqrt[8]{128/5} = 1.49978841866 \dots \text{ Helmholtz } \frac{1}{8} \text{skhisma, Fifth}$$

$$\text{Log}_2 .584758988142 \dots$$

a	c	e	$\frac{c}{d}$	dec.	Root Generator Octave
b	d	f			$0x, 0y$ a_n, e_y b_n, f_y
0	1	1	1.000000		$0x, 0y$ $0x, 1y$ $1x, 0y$
			←		
0	1	1	.500000		$0x, 0y$ $0x, 1y$ $1x, 1y$
			→		
1	2	1	.666667		$0x, 0y$ $1x, 1y$ $2x, 1y$
			←		
1	3	2	.600000		$0x, 0y$ $1x, 2y$ $2x, 3y$
			←		
1	4	3	.571429		$0x, 0y$ $1x, 3y$ <u>$2x, 5y$</u>
			→		
4	7	3	.583333		$0x, 0y$ $4x, 3y$ $7x, 5y$
			→		
7	10	3	.588235		$0x, 0y$ $7x, 3y$ $12x, 5y$
			←		
7	17	10	.586207		$0x, 0y$ $7x, 10y$ $12x, 17y$
			←		
7	24	17	.585366		$0x, 0y$ $7x, 17y$ $12x, 29y$
			←		
7	31	24	.584906		$0x, 0y$ $7x, 24y$ $12x, 41y$
			←		
7	38	31	.584615		$0x, 0y$ $7x, 31y$ $12x, 53y$
			→		
38	69	31	.584746		$0x, 0y$ $38x, 31y$ $65x, 53y$
			→		
69	100	31	.584795		$0x, 0y$ $69x, 31y$ $118x, 53y$
			←		
69	169	100	.584775		$0x, 0y$ $69x, 100y$ $118x, 171y$
			←		
69	238	169	.584767		$0x, 0y$ $69x, 169y$ $118x, 289y$
			←		
69	307	238	.584762		$0x, 0y$ $69x, 238y$ $118x, 407y$
			←		
69	376	307	.584758942		$0x, 0y$ $69x, 307y$ $118x, 525y$
			←		
69	443	525			

and then 52 places → 1 pl.

20 APR 00. EW

19

$$\sqrt[3]{5} = 1.22284454499\dots$$

Neutral-Third, $\sqrt{\quad}$ Variation
on $1/4$ -comma meantone

$$\text{Log}_2 .290241011856\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	Root $0x, 0y$	Generator a, n, e, y	Octave b, n, f, y
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0		$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1		$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
1	3	2		$0x, 0y$	$0x, 1y$	$1x, 2y$
			←			
0	1	1	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
1	4	3		$0x, 0y$	$0x, 1y$	$1x, 3y$
			→			
1	2	1	.285714	$0x, 0y$	$1x, 1y$	$4x, 3y$
4	7	3		$0x, 0y$	$1x, 1y$	$4x, 3y$
			→			
2	3	1	.300000	$0x, 0y$	$2x, 1y$	$7x, 3y$
7	10	3		$0x, 0y$	$2x, 1y$	$7x, 3y$
			←			
2	5	3	.294118	$0x, 0y$	$2x, 3y$	$7x, 10y$
7	17	10		$0x, 0y$	$2x, 3y$	$7x, 10y$
			←			
2	7	5	.291667	$0x, 0y$	$2x, 5y$	$7x, 17y$
7	24	17		$0x, 0y$	$2x, 5y$	$7x, 17y$
			←			
2	9	7	.290323	$0x, 0y$	$2x, 7y$	$7x, 24y$
7	31	24		$0x, 0y$	$2x, 7y$	$7x, 24y$
			←			
2	11	9	.289474	$0x, 0y$	$2x, 9y$	$7x, 31y$
7	38	31		$0x, 0y$	$2x, 9y$	$7x, 31y$
			→			
11	20	9	.289855	$0x, 0y$	$11x, 9y$	$38x, 31y$
38	69	31		$0x, 0y$	$11x, 9y$	$38x, 31y$
			→			
20	29	9	.290000	$0x, 0y$	$20x, 9y$	$69x, 31y$
69	100	31		$0x, 0y$	$20x, 9y$	$69x, 31y$
			→			
29	38	9	.290076	$0x, 0y$	$29x, 9y$	$100x, 31y$
100	131	31		$0x, 0y$	$29x, 9y$	$100x, 31y$
			→			
38	47	9	.290123	$0x, 0y$	$38x, 9y$	$131x, 31y$
131	162	31		$0x, 0y$	$38x, 9y$	$131x, 31y$
			→			
47	56	9	.290155	$0x, 0y$	$47x, 9y$	$162x, 31y$
162	193	31		$0x, 0y$	$47x, 9y$	$162x, 31y$
			→			
56	65	9	.290179	$0x, 0y$	$56x, 9y$	$193x, 31y$
193	224	31		$0x, 0y$	$56x, 9y$	$193x, 31y$
			→			

$$2/\sqrt[3]{5} = 1.63553086792\dots$$

Neutral Sixth, $\sqrt{}$ variation
on 1/4-comma Meantone

$$\text{Log}_2 .709758988144\dots$$

a	c	e	c	Root	Generator	Octave
b	d	f	d	$0x, 0y$	a, x, ey	b, x, fy
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			→			
1	2	3	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
			→			
2	3	4	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
			←			
2	5	3	.714286	$0x, 0y$	$2x, 3y$	$3x, 4y$
			←			
2	7	5	.700000	$0x, 0y$	$2x, 5y$	$3x, 7y$
			→			
7	12	5	.705882	$0x, 0y$	$7x, 5y$	$10x, 7y$
			→			
12	17	5	.708333	$0x, 0y$	$12x, 5y$	$17x, 7y$
			→			
17	22	5	.709677	$0x, 0y$	$17x, 5y$	$24x, 7y$
			→			
22	27	5	.710526	$0x, 0y$	$22x, 5y$	$31x, 7y$
			←			
22	49	27	.710145	$0x, 0y$	$22x, 27y$	$31x, 38y$
			←			
22	71	49	.710000	$0x, 0y$	$22x, 49y$	$31x, 69y$
			←			
22	93	71	.709924	$0x, 0y$	$22x, 71y$	$31x, 100y$
			←			
22	115	93	.709877	$0x, 0y$	$22x, 93y$	$31x, 131y$
			←			
22	137	115	.709845	$0x, 0y$	$22x, 115y$	$31x, 162y$
			←			
22	159	137	.709821	$0x, 0y$	$22x, 137y$	$31x, 193y$
			←			

(21)

$$\sqrt{\frac{3}{2}} = 1.22474487139\dots$$

Neutral Third, $\sqrt{\text{variation}}$
on Pythagorean

$$\rightarrow \text{Log}_2 .292481250359\dots$$

a	c	e	$\frac{c}{d}$	Root $0x, 0y$	Generator $a'x, e'y$	Octave $b'x, f'y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
1	3	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			←			
0	1	3	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
1	4	3	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
			→			
1	2	1	.285714	$0x, 0y$	$1x, 1y$	$4x, 3y$
4	7	3	.285714	$0x, 0y$	$1x, 1y$	$4x, 3y$
			→			
2	3	1	.300000	$0x, 0y$	$2x, 1y$	$7x, 3y$
7	10	3	.300000	$0x, 0y$	$2x, 1y$	$7x, 3y$
			←			
2	5	3	.294118	$0x, 0y$	$2x, 3y$	$7x, 10y$
7	17	10	.294118	$0x, 0y$	$2x, 3y$	$7x, 10y$
			←			
2	7	5	.291667	$0x, 0y$	$2x, 5y$	$7x, 17y$
7	24	17	.291667	$0x, 0y$	$2x, 5y$	$7x, 17y$
			→			
7	12	5	.292683	$0x, 0y$	$7x, 5y$	$24x, 17y$
24	41	17	.292683	$0x, 0y$	$7x, 5y$	$24x, 17y$
			←			
7	19	12	.292308	$0x, 0y$	$7x, 12y$	$24x, 41y$
24	65	41	.292308	$0x, 0y$	$7x, 12y$	$24x, 41y$
			→			
19	31	12	.292453	$0x, 0y$	$19x, 12y$	$65x, 41y$
65	106	41	.292453	$0x, 0y$	$19x, 12y$	$65x, 41y$
			→			
31	43	12	.292517	$0x, 0y$	$31x, 12y$	$106x, 41y$
106	147	41	.292517	$0x, 0y$	$31x, 12y$	$106x, 41y$
			←			
31	74	43	.292490	$0x, 0y$	$31x, 43y$	$106x, 147y$
106	253	147	.292490	$0x, 0y$	$31x, 43y$	$106x, 147y$
			←			
31	105	74	.292479	$0x, 0y$	$31x, 74y$	$106x, 253y$
106	359	253	.292479	$0x, 0y$	$31x, 74y$	$106x, 253y$
			→			
105	179	74	.292484	$0x, 0y$	$105x, 74y$	$359x, 253y$
359	612	253	.292484	$0x, 0y$	$105x, 74y$	$359x, 253y$
			←			
105	284	179	.292482	$0x, 0y$	$105x, 179y$	$359x, 612y$
359	971	612	.292482	$0x, 0y$	$105x, 179y$	$359x, 612y$
			←			

$$2/\sqrt{\frac{3}{2}} = 1.63299316186\dots$$

Neutral Sixth, $\sqrt{\text{variation}}$
on Pythagorean

$$\rightarrow \text{Log}_2 .707518749641\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	Root $0x, 0y$	Generator ax, ey	Octave bx, fy
0/1	1/1	1/0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0/1	1/2	1/1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			→			
1/2	2/3	1/1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
			→			
2/3	3/4	1/1	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
			←			
2/3	5/7	3/4	.714286	$0x, 0y$	$2x, 3y$	$3x, 4y$
			←			
2/3	7/10	5/7	.700000	$0x, 0y$	<u>$2x, 5y$</u>	$3x, 7y$
			→			
7/10	12/17	5/7	.705882	$0x, 0y$	$7x, 5y$	$10x, 7y$
			→			
12/17	17/24	5/7	.708333	$0x, 0y$	$12x, 5y$	$17x, 7y$
			←			
12/17	29/41	17/24	.707317	$0x, 0y$	$12x, 17y$	$17x, 24y$
			→			
29/41	46/65	17/24	.707692	$0x, 0y$	$29x, 17y$	$41x, 24y$
			←			
29/41	75/106	46/65	.707547	$0x, 0y$	$29x, 46y$	$41x, 65y$
			←			
29/41	104/147	75/106	.707483	$0x, 0y$	$29x, 75y$	$41x, 106y$
			→			
104/147	179/253	75/106	.707510	$0x, 0y$	$104x, 75y$	$147x, 106y$
			→			
179/253	254/359	75/106	.707521	$0x, 0y$	$179x, 75y$	$253x, 106y$
			←			
179/253	433/612	254/359	.707516	$0x, 0y$	$179x, 254y$	$253x, 359y$
			→			
433/612	687/971	254/359	.707518	$0x, 0y$	$433x, 254y$	$612x, 359y$
			→			

$$\sqrt[6]{3} = 1.20093695518\dots$$

$$\rightarrow \text{Log}_2 .264160416792\dots$$

Tanaka/Hanson
Kleismatic Temperament
Thirds

a	c	e	c	Root	Generator	Octave
b	d	f	d	dec.	$\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$	$\frac{b}{d}, \frac{c}{f}$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
			←			
0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
			←			
0	1	1	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
			←			
0	1	1	.250000	$0n, 0y$	$0n, 1y$	$1n, 3y$
			→			
1	2	1	.285714	$0n, 0y$	$1n, 1y$	$4n, 3y$
			←			
1	3	2	.272727	$0n, 0y$	$1n, 2y$	$4n, 7y$
			←			
1	4	3	.266667	$0n, 0y$	$1n, 3y$	$4n, 11y$
			←			
1	5	4	.263158	$0n, 0y$	$1n, 4y$	$4n, 15y$
			→			
5	9	4	.264706	$0n, 0y$	$5n, 4y$	$19n, 15y$
			←			
5	14	9	.264151	$0n, 0y$	$5n, 9y$	$19n, 34y$
			→			
14	23	9	.264368	$0n, 0y$	$14n, 9y$	$53n, 34y$
			←			
14	37	23	.264286	$0n, 0y$	$14n, 23y$	$53n, 87y$
			←			
14	51	37	.264249	$0n, 0y$	$14n, 37y$	$53n, 140y$
			←			
14	65	51	.264228	$0n, 0y$	$14n, 51y$	$53n, 193y$
6x41						
			←			
14	79	65	.264214	$0n, 0y$	$14n, 65y$	$53n, 246y$
18x23						
			←			
14	93	79	.264205	$0n, 0y$	$14n, 79y$	$53n, 299y$
38x11						
			←			
36 Places			etc.			

$$2/\sqrt[6]{3} = 1.66536635531\dots$$

$$\rightarrow \text{Log}_2 \underline{.735839583208\dots}$$

Tanaka/Hanson
Kleismatic Temperament
Sixths

a	c	e	c	dec.	Root	Generator	Octave	
b	d	f	d		$0x, 0y$	a_x, e_y	b_x, f_y	
0	1	0	1.000000		$0x, 0y$	$0x, 1y$	$1x, 0y$	
←								
0	1	1	.500000		$0x, 0y$	$0x, 1y$	$1x, 1y$	
→								
1	2	1	.666667		$0x, 0y$	$1x, 1y$	$2x, 1y$	
→								
2	3	1	.750000		$0x, 0y$	$2x, 1y$	$3x, 1y$	
←								
2	3	5	.714286		$0x, 0y$	$2x, 3y$	$3x, 4y$	✓
→								
5	7	8	.727273		$0x, 0y$	$5x, 3y$	$7x, 4y$	
→								
8	11	11	.733333		$0x, 0y$	$8x, 3y$	$11x, 4y$	
→								
11	15	14	.736842		$0x, 0y$	$11x, 3y$	$15x, 4y$	
←								
11	15	25	.735294		$0x, 0y$	$11x, 14y$	$15x, 19y$	✓
→								
25	34	39	.735849		$0x, 0y$	$25x, 14y$	$34x, 19y$	
←								
25	34	64	.735632		$0x, 0y$	$25x, 39y$	$34x, 53y$	
→								
64	87	103	.735714		$0x, 0y$	$64x, 39y$	$87x, 53y$	
→								
103	140	142	.735751		$0x, 0y$	$103x, 39y$	$140x, 53y$	
→								
142	193	181	.735772		$0x, 0y$	$142x, 39y$	$193x, 53y$	
→								
181	246	220	.735786		$0x, 0y$	$181x, 39y$	$246x, 53y$	
→								
220	299	259	.735795		$0x, 0y$	$220x, 39y$	$299x, 53y$	25
→								

$$\sqrt[3]{3} = 1.16993081276\dots$$

Subminor-Third

$$\rightarrow \text{Log}_2 .226423214390\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	Root $0x, 0y$	Generator $a'x, e'y$	Octave $b'x, f'y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			←			
0	1	3	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
			←			
0	1	4	.200000	$0x, 0y$	$0x, 1y$	$1x, 4y$
			→			
1	2	1	.222222	$0x, 0y$	$1x, 1y$	$5x, 4y$
			→			
2	3	1	.230769	$0x, 0y$	$2x, 1y$	$9x, 4y$
			←			
2	5	3	.227273	$0x, 0y$	$2x, 3y$	$9x, 13y$
			←			
2	7	5	.225806	$0x, 0y$	<u>$2x, 5y$</u>	$9x, 22y$
			→			
7	12	5	.226415	$0x, 0y$	$7x, 5y$	$31x, 22y$
			→			
12	17	5	.226667	$0x, 0y$	$12x, 5y$	$53x, 22y$
			←			
12	29	17	.226563	$0x, 0y$	$12x, 17y$	$53x, 75y$
			←			
12	41	29	.226519	$0x, 0y$	$12x, 29y$	$53x, 128y$
			←			
12	53	41	.226496	$0x, 0y$	$12x, 41y$	$53x, 181y$
			←			
12	65	53	.226481	$0x, 0y$	$12x, 53y$	$53x, 234y$
			←			
12	77	65	.226471	$0x, 0y$	$12x, 65y$	$53x, 287y$
			←			

$$2/\sqrt{3} = 1.70950279981\dots$$

Supramajor - Sixth

$$\rightarrow \text{Log}_2 .773576785610\dots$$

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a\#x, e\#y$	$b\#x, f\#y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0				
			←			
0	1/2	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	1/2	1				
			→			
1	2/3	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
2	2/3	1				
			→			
2	3/4	1	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
3	3/4	1				
			→			
3	4/5	1	.800000	$0x, 0y$	$3x, 1y$	$4x, 1y$
4	4/5	1				
			←			
3	7/9	4	.777778	$0x, 0y$	$3x, 4y$	$4x, 5y$
4	7/9	4				
			←			
3	10/13	7	.769231	$0x, 0y$	$3x, 7y$	$4x, 9y$
4	10/13	7				
			→			
10	17/22	7	.772727	$0x, 0y$	$10x, 7y$	$13x, 9y$
13	17/22	7				
			→			
17	24/31	7	.774194	$0x, 0y$	$17x, 7y$	$22x, 9y$
22	24/31	7				
			←			
17	41/53	24	.773585	$0x, 0y$	$17x, 24y$	$22x, 31y$
22	41/53	24				
			←			
17	58/75	41	.773333	$0x, 0y$	$17x, 41y$	$22x, 53y$
22	58/75	41				
			→			
58	99/128	41	.773438	$0x, 0y$	$58x, 41y$	$75x, 53y$
75	99/128	41				
			→			
99	140/181	41	.773481	$0x, 0y$	$99x, 41y$	$128x, 53y$
128	140/181	41				
			→			
140	181/234	41	.773504	$0x, 0y$	$140x, 41y$	$181x, 53y$
181	181/234	41				
			→			
181	222/287	41	.773519	$0x, 0y$	$181x, 41y$	$234x, 53y$
234	222/287	41				
41x7			→			
222	263/340	41	.773529	$0x, 0y$	$222x, 41y$	$287x, 53y$
287	263/340	41				
			→			

$$\sqrt[2]{40} = 1.50663019029\dots$$

$$\text{Log}_2 = .591325343871\dots$$

$$\sqrt[9]{20,480}$$

$$\underline{19,683}$$

Fifth

a	c	e	c	Root Generator	Octave	
b	d	f	d dec.	$0x, 0y$	a, ey	b, fy
0	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			→			
1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
			←			
1	2	2	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
			←			
1	2	3	.571429	$0x, 0y$	$1x, 3y$	$2x, 5y$
			→			
4	7	3	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
			→			
7	12	5	.588235	$0x, 0y$	$7x, 3y$	$12x, 5y$
			→			
10	17	5	.590909	$0x, 0y$	$10x, 3y$	$17x, 5y$
			→			
13	22	5	.592593	$0x, 0y$	$13x, 3y$	$22x, 5y$
			←			
13	22	16	.591837	$0x, 0y$	$13x, 16y$	$22x, 27y$
			←			
13	22	29	.591549	$0x, 0y$	$13x, 29y$	$22x, 49y$
			←			
13	22	42	.591398	$0x, 0y$	$13x, 42y$	$22x, 71y$
			←			
13	22	55	.591304	$0x, 0y$	$13x, 55y$	$22x, 93y$
			→			
68	115	55	.591346	$0x, 0y$	$68x, 55y$	$115x, 93y$
			←			
68	115	123	.591331	$0x, 0y$	$68x, 123y$	$115x, 208y$
			←			
68	115	191	.591324	$0x, 0y$	$68x, 191y$	$115x, 323y$
			→			

$$\sqrt[9]{64/5} = 1.32746576624\dots$$

$$\text{Log}_2 = \underline{.408674656129\dots}$$

$$\sqrt[9]{20,480} = 19,683$$

Fourth

a	c	e	c	Root Generator	Octave
b	d	f	d dec.	$0x, 0y$	a_x, e_y
				$0x, 0y$	b_x, f_y
0	1	1	1.000000	$0x, 0y$	$1x, 0y$
			←		
0	1/2	1	.500000	$0x, 0y$	$1x, 1y$
			←		
0	1/3	2	.333333	$0x, 0y$	$1x, 2y$
			→		
1	2/5	2	.400000	$0x, 0y$	$3x, 2y$
			→		
2	3/7	2	.428571	$0x, 0y$	$5x, 2y$
			←		
2	5/12	3	.416667	$0x, 0y$	$5x, 7y$
			←		
2	7/17	5	.411765	$0x, 0y$	$5x, 12y$
			←		
2	9/22	7	.409091	$0x, 0y$	$5x, 17y$
			←		
2	11/27	9	.407407	$0x, 0y$	$5x, 22y$
			→		
11	20/49	9	.408163	$0x, 0y$	$27x, 22y$
			→		
20	29/71	9	.408451	$0x, 0y$	$49x, 22y$
			→		
29	38/93	9	.408602	$0x, 0y$	$71x, 22y$
			→		
38	47/115	9	.408696	$0x, 0y$	$93x, 22y$
			←		
38	85/208	47	.408654	$0x, 0y$	$93x, 115y$
			→		
85	132/323	47	.408669	$0x, 0y$	$208x, 115y$
			→		
132	179/438	47	.408676	$0x, 0y$	$323x, 115y$
			←		

$$15 + (22 \times 25) = 665$$

(29)

28MAY00.EW

$$\sqrt[2]{\frac{40}{3}} = 1.33350053098\dots$$

$$\text{Log}_2 = .415218399348\dots$$

1/9 Skhisma
Fourth

a	c	e	c	Root Generator	Octave
b	d	f	d dec.	$0n, 0y$	a_n, e_y
0	1	1	1.000000	$0n, 0y$	$1n, 0y$
			←		
0	1	1	.500000	$0n, 0y$	$1n, 1y$
			←		
0	1	1	.333333	$0n, 0y$	$1n, 2y$
			→		
1	2	1	.400000	$0n, 0y$	$3n, 2y$
			→		
2	3	1	.428471	$0n, 0y$	$5n, 2y$
			←		
2	5	3	.416667	$0n, 0y$	$5n, 7y$
			←		
2	7	5	.411765	$0n, 0y$	$5n, 12y$
			→		
7	12	5	.413793	$0n, 0y$	$17n, 12y$
			→		
12	17	5	.414634	$0n, 0y$	$29n, 12y$
			→		
17	22	5	.415094	$0n, 0y$	$41n, 12y$
			→		
22	27	5	.415385	$0n, 0y$	$53n, 12y$
			←		
22	49	27	.415254	$0n, 0y$	$53n, 65y$
			←		
22	71	49	.415205	$0n, 0y$	$53n, 118y$
			→		
71	120	49	.415225	$0n, 0y$	$171n, 118y$
			←		
71	191	120	.415217	$0n, 0y$	$171n, 289y$
			→		
191	311	120	.415220	$0n, 0y$	$460n, 289y$
			←		

$$\sqrt[9]{\frac{192}{5}} = 1.49981192623 \dots$$

$$\log_2 = .584781600652 \dots$$

1/9 skhisma
Fifth

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root $0n, 0y$	Generator a_n, e_y	Octave b_n, f_y
0/1	1/1	1/0	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
			←			
0/1	1/2	1/1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
			→			
1/2	2/3	1/1	.666667	$0n, 0y$	$1n, 1y$	$2n, 1y$
			←			
1/2	3/5	2/3	.600000	$0n, 0y$	$1n, 2y$	$2n, 3y$
			←			
1/2	4/7	3/5	.571429	$0n, 0y$	$1n, 3y$	$2n, 5y$
			→			
4/7	7/12	3/5	.583333	$0n, 0y$	$4n, 3y$	$7n, 5y$
			→			
7/12	10/17	3/5	.588235	$0n, 0y$	$7n, 3y$	$12n, 5y$
			←			
7/12	17/29	10/17	.586207	$0n, 0y$	$7n, 10y$	$12n, 17y$
			←			
7/12	24/41	17/29	.585366	$0n, 0y$	$7n, 17y$	$12n, 29y$
			←			
7/12	31/53	24/41	.584906	$0n, 0y$	$7n, 24y$	$12n, 41y$
			←			
7/12	38/65	31/53	.584615	$0n, 0y$	$7n, 31y$	$12n, 53y$
			→			
38/65	69/118	31/53	.584746	$0n, 0y$	$38n, 31y$	$65n, 53y$
			→			
69/118	100/171	31/53	.584795	$0n, 0y$	$69n, 31y$	$118n, 53y$
			←			
69/118	169/289	100/171	.584775	$0n, 0y$	$69n, 100y$	$118n, 171y$
			→			
169/289	269/460	100/171	.584783	$0n, 0y$	$169n, 100y$	$289n, 171y$
			←			
169/289	438/749	269/460	.584780	$0n, 0y$	$169n, 269y$	$289n, 460y$
			→			

(31)

28MAY00.EW

Zig-Zag Series (1←, 1→), Limit Kornerup's Golden Fifth $\frac{c}{d}$

$\log_2 1.580178728295$

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a	c	e	$\frac{c}{d}$ dec
b	d	f	
1	5	2	.600000
2	5	3	←
2	4	3	.571429
4	7	5	→
7	12	5	.583333
			←
4	11	7	.578947
7	19	12	→
11	18	7	.580645
19	31	12	←
11	29	18	.580000
19	50	31	→
29	47	18	.580247
50	81	31	←
29	76	47	.580153
50	131	81	→
76	123	47	.580189
131	212	81	←
76	199	123	.580175
131	343	212	→
199	322	123	.580180
343	555	212	←
199	521	322	.5801782
343	898	555	→
521	843	322	.5801789
898	1453	555	←
521	1,364	843	.5801786
898	2,351	1453	→
1,364	2,207	843	.5801788
2,351	3,804	1453	←
1,364	3,571	2,207	.5801787
2,351	6,155	3,804	

Ref: Acoustic Methods of Work, Thorvald Kornerup, 1934
 A Theory of Evolving Tonality, Joseph Yasser, 1932
 Diophantine Triplets and N, Y Coordinates, Ervin M. Wilson, 2000

Zig-Zag Series (1→, 1←), Limit Kornerup's Golden Fourth $\frac{c}{d}$

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Sheet 2.

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a	c	e	$\frac{c}{d}$ dec
1	2	1	.400000
3	5	2	→
2	3	1	.428571
5	7	2	←
2	5	3	.416667
5	12	7	→
5	8	3	.421053
12	19	7	←
5	13	8	.419355
12	31	19	→
13	21	8	.420000
31	50	19	←
13	34	21	.419753
31	81	50	→
34	55	21	.419847
81	131	50	←
34	89	55	.419811
81	212	131	→
89	144	55	.419825
212	343	131	←
89	233	144	.419820
212	555	343	→
233	377	144	.4198218
555	898	343	←
233	610	377	.4198211
555	1453	898	→
610	987	377	.4198214
1453	2351	898	←
610	1597	987	.41982124
1453	3804	2351	→
1597	2584	987	.41982128
3804	6155	2351	