

ACOUSTIC METHODS OF WORK
IN RELATION TO
SYSTEMATIC COMPARATIVE MUSICOLOGY

INCLUDING SOME ACOUSTIC TABLES

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COPENHAGEN F.

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CHAPTER IV

THE AUTHORITATIVE TONE-SYSTEM, THE GOLDEN SYSTEM AS AN OBJECTIVE MEANS OF VALUATION.

However, in order to pass judgement on the various historical tone-systems, the chief defect of which is the displacement of **Commas** between for example »d and d+«, »des— and des« etc., it is necessary first to construct an authoritative tone-system **without** these Comma-displacements, which claims that many tones with plus would be made much lower, and many tones with minus much higher.

In August 1930 I found the new Fifth in two ways which gave the same result.

$$2 \times 66.7 + 7 \times 23.5$$

from above down-
to the uppermost in
me 7 according to

Group A. Arithmetic Series I—VI, retrograde and direct.

The sum of 2 golden adjacent Nos. forms the following No., and the sum of the cents of two golden adjacent tones forms the following cents in the same golden Series.

1. The astronomer J. Kepler (1571—1630) combined tones with the sum of the numerators and the sum of the denominators respectively of 5 tone-fractions:

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{8}{5}$	$\frac{13}{8}$ Partial-tone 840 5 cents
C	c	g	a	as	gisis

The golden gisis..... 843.2 »

In the 19-toned and the 31-toned golden section respectively we transform the tone-sequence in this manner (see **Scheme 10**):

Fourth- Series I retrograde:	{ 19 toned 31 — 50 —	1 cis	2 des	3 d	5 es	8 f	13 as
		1 deses	2 cis	3 des	5 d	8 es	13 f	21 as
		1 bisis	2 deses	3 cis	5 des	8 d	13 es	21 f	34 as

Double super division

Factor → 819 = 3 × 3 × 3 × 31
 819 = 3 × 273
 273 = 3 × 91
 91 = 7 × 13

Ludwig Sonnenberg, principal teacher in Bonn (1820-88) called the sequence 1, 2, 3, 5, 8, 13: «Kepler's Series» (Notes 12-13), our golden **Fourth-Series I** retrograde, in 1844 to No. 17 by Gabriel Lamé, in 1929 up to No. 40 by L. Kaiser.

2. Dr. J. Kaiser, the mathematician, found by pure mathematics (Note 14) (without reference to tones) a similar sequence; we call the latter «Kaiser's Series», our **Fifth-Series II**, which we use in the following manner:

Fifth-Series II retrograde	Double super division.				
	19-toned	4 dis	7 fes	11 g	18 ces
	31- —	7 dis	11 fes	18 g	29 ces

3. In May 1930 Andreas Kornerup, engineer, directed my attention to the fact that the Fifth in these systems has tone-Nos. which form similar sequences of Nos. for

→ example $\frac{\text{Fifth}}{\text{Octave}} = \frac{7}{12}, \frac{11}{19}, \frac{18}{31}, \frac{29}{50}, \frac{47}{81}$ etc.

We name the sequence 12, 19, 31, 50, 81 the **Octave-Series III**, Andreas Kornerup's Series, the denominators of the fractions.

a) It occurred to me to calculate these fractions in cents, which I did, August 15th 1930. The value moved like a **pendulum** quickly approaching the point of balance, where the seventh decimal would be stable at the Fifth $\frac{1200 \times 3371}{6155} = 696.2145$ cents (Acoustic statics).
Fourth 1200 × 2584 / 6155 = 503.7855 error 3571

The temperaments, the octaves of which are the denominators of these fractions, I called the **organic** temperaments, and, at the same time, it occurred to me to construct an **authoritative** »tone-system of the Fifth« with this Fifth which, however, **also** generally appears by means of the super-division (golden cut) of the octave, **Scheme 10**, Series III. I calculated the Fifth on August 17th 1930, as shown below:

b) The fraction of super-division	Cents	Series III:
∞	0,618.03398 × 1200 gives 741.64078 direct ases
1 - ∞	0,381.96602 × 1200 458.35922 retrograde eis
Total 1,0 1200 cents c'

Further { ases is 11 Fourths from c and 4 octaves back then f = 503.7855
 eis is 11 Fifths — — — — 6 — — — — g = 696.2145
 Total = 1200 cents

In 19-toned and 31-toned golden sections respectively we use »Andreas Kornerup's Series« in the following manner:

Octave-Series III retrograde	Double super division				
	19-toned	O c	7 eis	12 ases	19 c'
	31-toned	O c	12 eis	19 ases	31 c'

The difference between golden eis = 458.4
 and syntonic — = 457.0
 Schemes 14 and 15: nearly $k = \frac{1}{15}$ Comma = 1.4

4. Out of the calc instance 31-toned see S

Great Third-Series
Tritonos —
Great Sixth-

Group B.

The golden tones at **Scheme 8**, which indic

1) above, part of a parts by means of 2 c (the Fourth f), this in

in {

2) below: a square into a »small double :

corresp

3) further: **within**

in

Scheme 9 shows the same chord C...

Fourth-Series

Further: $\frac{C...(Es)}{C..E} =$

Scheme 10 shows of a square are divid

{ from c upwards in — c' downwards

Factor → 819 = 729 / 895
 8 / 352 / 4095

38
 3 = 729 / 2250 / 729 / 656 / 14096 = 2

4. Out of the calculated golden system I have formed some other Series for instance 31-toned see **Scheme 20**:

		Double super division.		
Great Third-Series IV:	4 cisis 6 eses	10 e	16 ges	26 bes
Tritonos — V:	9 feses	15 fis	24 beses
Great Sixth- — VI:	9 disis	14 geses	23 a

Group B. Geometrical Constructions of golden Tones.

The golden tones and intervals can also be constructed geometrically as shown in **Scheme 8**, which indicates:

1) above, part of a regular **Pentagon**, the angle of which = 108° is divided into 3 parts by means of 2 chords; if the chord be equal to a Tetrachord — the interval C-F, (the Fourth f), this interval will be super-divided in »great double super-division«

$$\text{in } \left\{ \begin{array}{l} C \text{ — D Es — F = Series I} \\ 192.43 + 118.93 + 192.43 = 503.79 \text{ cents} \end{array} \right\}$$

2) below: a **square** placed inside a semicircle, whereby the diameter is super-divided into a »small double super division«

$$\text{in cents } \left\{ \begin{array}{l} \text{Cis D — E F} \\ \text{D Es — F Ges} \\ \text{B C — D Es} \\ 118.93 + 192.43 + 118.93 = 430.29 \end{array} \right.$$

$$\text{corresponding with: } \left\{ \begin{array}{l} C \text{ D — F G} \\ 192.4 + 311.4 + 192.4 = 696.2 \end{array} \right.$$

3) further: **within** the square we can form a »great double super division«

$$\text{in } \left\{ \begin{array}{l} D \text{ — Dis Es — E} \\ 73.50 + 45.43 + 73.50 = 192.43 \text{ cents} \end{array} \right\}$$

Scheme 9 shows the golden system constructed by means of the **Pentagram** with the same chord C.. F, so that the side is = C.. Es in the Pentagon.

Fourth-Series I: C, Deses, Cis, Des, D, Es, F and further As, Des'.

Further: $\frac{C..(Es)}{C..E} = \frac{es}{e} = \text{Cosinus } 36^\circ$, the relation between small and great Third.

Scheme 10 shows »double super-division« in 4 Series; the upper and lower edges of a square are divided:

- { from c upwards into ases, direct,
- c' downwards into eis, retrograde.

the sequence 1, 2, I retrograde, in y L. Kaiser. matics (Note 14) •Kaiser's Series»,

to the fact that ences of Nos. for

ideas Kornerup's

I did, August 15th point of balance,

$\frac{1}{5} = 696.2145 \text{ cents}$
 error 3571

of these fractions, ed to me to con- which, however, ut) of the octave, is shown below:

eries III: rect ases grade eis c'

then f = 503.7855

— g = 696.2145

Total = 1200 cents

ideas Kornerup's

If we continue a single super-division,

1) direct, upwards, we will get **decreasing** intervals upwards,

retrograde, downwards, we will get decreasing intervals downwards. We use

oblique lines { from the corner upwards on the right } forming 8 double super-divisions, which give the result:

	upwards, retrograde: Cents	downwards, direct, Scheme 10:
Series I	c—es.. f —as 815	e—g .. a —c'
— IV	c—e .. ges —bes 1008	cis—f .. gis—c'
— II	c—fes .. g —ces 1126	d—fis .. as—c'
— III	c—eis —c' 1200	c —ases—c'

We continue double super-division only in Series I:

Series I:	c—des .. d —es 311	a —bes .. b —c'
	c—cis .. des—d 192	bes—b .. ces—c'
	c—deses .. cis —des 119	b —ces .. bis—c'

The tones are in pairs supplement-tones (making an octave together) for instance:

Series I	retrograde	deses	cis	des	d	es	f	as	c'
	direct	bis	ces	b	bes	a	g	e	c

Scheme 20 shows golden tones in 6 Series formed geometrically by means of parallel lines in a Pentagon, for instance:

The tones :	Eis	F	Fis	Ges	G	As	A	C'	Des'
Series I-III:	III.	I.	II.	I.	III.	I.	
— IV-VI:	V.	IV.	VI.		

Scheme 21 shows cents (to 4 decimals) for an organic 19-toned section of the infinite golden tone-system with two Molecules:

{ t = 73.501 cis, which is the smallest interval in this section,
v = 45.426 deses, the greatest interval in the next **organic** section on 31 golden tones.

	v.	t.	cents
The Molecules in 12-toned golden section	5 × cis	+ 7 × des	= 1200
19- — — —	7 × deses	+ 12 × cis	= 1200
31- — — —	12 × bisis	+ 19 × deses	= 1200

v. outside the section.

Group

Example: $d = \frac{10}{9}$
merges into the gold
of the string between

162
d
Particles 648
cents 182.4

so that the golden t
The tertiary half

The structure of
(x = great Third, y =

Triple chord:

Major 2nd form
Minor 3rd —

For teachers of
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119 or 192 cents, »c
intervals in cents:

From	{ o
	{ c
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in cents	
to	{ c
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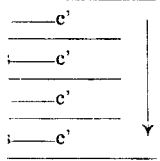
downwards. We use
 ming 8 double super-

Group C. All Comma-displacements disappear.

Example: $d = \frac{10}{9}$ and $d+ = \frac{9}{8}$

merges into the golden tone $d = 192.43$ cents, approximating to the **secondary** halving of the string between d and $d+$, which in the Formula X gives:

ect, Scheme 10:



	162	161	160	} or $\frac{180}{161} = 193.12$ cents, the difference is inaudible,
	d	d	d+	
Particles	648	644.3	640	
cents	182.4	193.1	203.9	

Why not $\frac{19}{77} = 192.56$ cents? 4th 8/24/78

so that the golden tone can easily be found approximately on the string.

The **tertiary** halving gives a somewhat higher tone (Formula Y):

.. b — c'
 .. ces — c'
 .. bis — c'

$\frac{160}{144} = \frac{161}{144}$ or $\frac{162}{144} = 193.20$ cents.

together) for instance:

The structure of the triple chord is often the golden cut directly put to use (x = great Third, y = small Third, (2) and (3) formations):

f | as | c'
 g | e | c

Triple chord:	Symbol	Intervals	Small Sixth with golden cut:
Major 2nd form	xy (2)	e g c	retrograde: $311 + 504 =$
Minor 3rd —	yx (3)	g c es	direct: $504 + 311 =$ } 815 cents

etrically by means of

A C' Des'
 III. I.
 . VI.

-toned section of the

For teachers of harmony the golden system can thus also be of use through its clear and logical construction; thus xyy (1) the Dominant quadruple chord $c e g bes$ will, on the 1st step, be **resolved** into xy (3) = the tonic triple chord on the 3rd step: $(c) f a c'$ (on the Tonic f) by which means the tones e, g and bes **glide** either 119 or 192 cents, »c des« and »c d« respectively, systematically up or down, with the intervals in cents:

ion on 31 golden tones.

t. cents
 × des = 1200
 × cis = 1200
 × deses = 1200

From	{	o	385	696	1007	1200.	Symbols
		c	e	g	bes	c'	xyy (1)
Gliding in cents	}		→		←	→	
			119	←	→	119	
				192	192		192
to	}	c	f	a	c'		xy (3)
		o	504	888	1200		

Since all Comma displacements disappear in the golden system, one can therefore propose any scale whatever on any golden tone as Tonic (key-note) with the **best possible number** of tones; the authoritative Golden tone-system represents in this field the formula for the principle of **the smallest activity**, Nature's **economic minimum** principle. It is the system with the smallest possible number of tones (acoustic Okology), an outcome of Nature's wonderful **power of adaptation**, — the supporting principle of all life in Nature.

Andreas Kornerup has called the fraction of the golden cut 0,618.034 Omega, which is recommended as an international expression in the formula $\frac{1}{\omega} = \frac{\omega}{1-\omega}$ or: $\omega^2 + \omega = 1$ or: $eis + ases = c'$, Series I, or $458 + 742 = 1200$ cents, which we here designate, once and for all, as the authoritative principle of relativity in the field of acoustics: The essential formula for **the universal sense of harmony**.

Even if the composers do not know of this formula, the Danish Physicist H. C. Ørsted (1771—1851) is indeed right in saying: 'The work of the composer is based on mathematics although in a **deeper** measure than has ever dawned upon us.' (Note 15). It will be the task of the future student of the theory of music to get to the bottom of this deeper-lying **law** of Nature, the golden cut, the super division — the basis of the future renaissance of harmonics.

The Octave-Serie **organic** temperaments, namely the sequence-toned temperament

How many tones for practical measurement unworkable. This is

•Ceterum censeo circumstances choose requirements the St:

All the inorganic **Scheme 11** show is carried through 1 for two neighbour t the 24-tone system,

Further: Dr. P. : 1915 proved, by me better than the 12-t 19-toned. To have t how much the singl nes, and by summi and #, b, 2# and 2b

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CHAPTER V

TEMPERAMENTS.

Tone-systems with 1 Molecule only.

The Octave-Series III, Andreas Kornerup's Series, marks the boundary of the **organic** temperaments, i. e. the only **rational**, the only temperaments fit for use, namely the sequence: 12, 19, 31, 50, 81 — with 19-toned as the **practical** and with 31-toned temperament as the **Standard**-temperament, see **Schemes 12** and **18**.

How many tones will be required for pianos, organs etc. is indeed a question; but, for practical measures only the organic **19** and **31** are efficient; all the others are unworkable. This is the authoritative judgement passed on the matter in question.

•Ceterum censeo: If we wish to abandon the 12-toned pianoforte we can in no circumstances choos any other than the practical 19-tonic temperament, and for finer requirements the Standard 31- (or the 50-) toned temperament.

All the inorganic temperaments ought to be excluded.

Scheme 11 shows, by way of example, how the Fourth-Series I, Kepler's Series, is carried through logically, in 19- or 31-toned temperament, where the sum of cents for two neighbour tones gives the next tone in the Series, but is split in, for instance, the 24-tone system, which is therefore authoritatively considered to be unworkable.

Further: Dr. P. S. Wedell and N. P. J. Bertelsen, actuary, Copenhagen, in January 1915 proved, by means of 'the method of the smallest squares', that the 19-toned is better than the 12-toned temperament, and that the 31-toned, again, is better than the 19-toned. To have this judgement expressed in numbers it can easily be calculated how much the single tone in the different temperaments deviate from the golden tones, and by summing-up such deviations for 35 tones (the tones of the 7 white keys and $\sharp, \flat, 2\sharp$ and $2\flat$) the result is found to be as follows see **Scheme 12**:

	12-toned Temperament 1174 cents estimated at 100 %			
Deviation	19- —	—	458 —	thus 39 —
for	31- —	—	174 —	— 15 —
	50- —	—	67 —	— 6 —

In comparison it may be quoted that in the pure consistent Pythagorean system, the collective deviation for the same 35 tones is 1780 cents or 52 % **greater** than for the 12-toned temperament.

Among the numerous observations as to the change of the piano-temperaments we shall quote the following few selections:

Director Gotfred Skjerne says (1909): »We are indebted to the tempered tuning for our musical progress but the ear is, as a matter of fact, **coarsened**.«

Dr. José Würschmidt showed 1920—28 »that a division of the octave into 18, 24 or 36 parts can represent **no** natural extension of our tone system, but that we have before us such an extension in a division of **19** steps. (Note 17).

Professor Joseph Yasser, New York, recommends the 19-toned temperament with the purpose: »to **enrich** our musical language, particularly when one takes into account the **growing** significance of the independent twelve-tone foundation in modern music«. (Note 18).

Professor Louis Kelterborn, Neuchâtel, (Note 19) proposes, further, on practical grounds, that simultaneously with the introduction of the 19-toned temperament, the pitch of »the tone a« should be made **a little lower**, so that **c** may keep the same pitch as it has now.

Scheme 13 shows, geometrically, an equilateral hyperbole through the tones No. 5 in various organic temperaments forming the tones in the Golden Fourth-Series I, retrograde, (with approximate pitches of tone).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

CHANGE OF T

a) In **Schemes 1** Fifths (the Partial-t lines with an angle in 1890.

The axis through difference between t most nil, just as is th and golden gisis (eq

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4 golden
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All the tones in tones by adding or s in all the lines whi regular rise in numb number-designs, or 1 Nicolai Thiele (1838-

Scheme 14 is th formed on about 9

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CHAPTER VI

CHANGE OF THE SYNTONIC SYSTEM INTO GOLDEN TONES.

a) In **Schemes 14 and 15** we have the syntonic system, built on Pythagorean Fifths (the Partial-tone = $3/2$) in horizontal lines and the Thirds = $5/4$ in diagonal lines with an angle of 60° , according to the proposal of the Japanese Shohé Tanaka in 1890.

The axis through »fes, c or gisis« in the schemes is called **Zero-Axis**, because the difference between the syntonic and golden fes (equalling 15 golden Fourths) is almost nil, just as is the difference between the corresponding supplement-tones, syntonic and golden gisis (equal to 15 golden Fifths), which is seen in the following table.

{	4 syntonic Thirds.....	give 1545.255 cents		
	minus 1 — Fifth, the Partial-tone $3/2 = g$	701.955	—	
		gives syntonic gisis	843.300	—
	15 golden Fifths minus 8 Octaves, golden gisis , is	843.217	—	
		inaudible difference	0.083	—
	Further: syntonic Third, the tone $5/4 = e$		386.314	—
	4 golden Fifths minus 2 Octaves: golden e.....	384.858	—	
		Slight deviation	1.456	—
	1/15 of a syntonic Comma 21.5062 cents is $k =$	1.434	—	
		inaudible deviation	0.022	—

All the tones in **Scheme 14** can thus, in a practical way, be changed to golden tones by adding or subtracting a number of »k«. These numbers are regularly grouped in all the lines which can be enclosed through the tones in the Scheme, i. e. with regular rise in numbers of k, so that the Scheme will be reminiscent of mathematical number-designs, or **number-figures**, constructed by the Danish astronomer Thorvald Nicolai Thiele (1838—1910) in 1872 (Note 20).

Scheme 14 is thus divided into 9 symmetrical **figures**, each comprising 15 tones, formed on about 9 central points of, respectively + 15 or 0 or - 15k:

TH. KORNERUP:

	+	Zero	-
	gisis — + 15	gisis 0 0	gisis + - 15
	c — + 15	c 0	c + - 15
	fesese — + 15	fesese 0	fesese + - 15
resp.	from + 22 to + 8	from + 7 to - 7	from - 8 to - 22

Thus, outside the Zero-Axis the tones are here changed by »addition on the left side«, »subtraction on the right side« of fifteenth parts of a Comma, increasing evenly in all directions.

Axis:	Direction:				Result: Central-points
1) Pythagorean Fifth-Axis, horizontal ... then one Small Third-step	to the right downward	o. c	- 4. g	- 8. d +	- 12. a + - 3. - 15. c +
2) Pythagorean Fifth-Axis, horizontal ... then one Great Sixth-step	to the left upward	o. c	- 4. f	+ 8. bes-	+ 12. es - + 3. + 15. c -
3) Great Third, Axis, 60° upward then one Great Sixth-step	to the right - - left	o. c	- 1. e	- 2. gis	- 3 bis + 3 0 gisis
4) Small Sixth-Axis 60° downward then one Small-Third-step	to the left - - right	o. c	+ 1. as	+ 2. fes	+ 3 deses - 3 0 fesese
5) Small Third-Axis through the figure .	from »+ 15 gisis -« (through »o. c«) to »- 15 fesese +«.				

If all the Third-lines, inclining 60°, be elongated they will cut the Zero-Axis in nothing but Zero-points, — ad infinitum.

Two examples of the division of 1 Comma will finally be quoted:

d + sunk	8 k = 11.4700
d raised	7 k = 10.0362
Total 15 k = 21.5062	

es sunk	3 k = 4.3 cents
es - raised	12 k = 17.2 —
5 × 3 k = Total 15 k = 21.5 — = 1 Comma.	

The golden d = 192.43 cents is only a trifle below the secondary halving of the distance on the string between d and d +, as above mentioned (Chapter IV, Group C).

Analogously the syntonic es = $\frac{6}{5}$ is s

with the p

An extreme exam

Pythagorean 6

Scheme 14: p

b) Scheme 15 sh
(15 tones) and gisis.

representing »the 2 bu

1) + 1.373 cents eis	2
- 1.456 plus e	
- 0.083 gisis	

c) Rationa lexpla
ment-pairs) gives »re

d) Similar method
famous systems of c

(X unknown

The Bohemia

The Italian, (

Analogously the distance on the string between Pythagorean $es = \frac{32}{27}$ and the syntonic $es = \frac{6}{5}$ is secondarily 5-parted according to the Formula X:

	480		
	405	401	400
	es —	golden	es
with the particles	607,5	601,5	600

An extreme example:

Pythagorean 6q = ges 2 — =	588.3 cents	Syntonic fis + =	590.2 cents
Scheme 14: plus 24 k = + 34.4 —		Minus 9 k = — 12.9 —	
golden ges = 622.7 —		golden fis = 577.3 —	
Scheme 21:		plus Molecule v = + 45.4 —	
		ges = 622.7 —	

addition on the left
1a, increasing evenly

	Result: Central-points
3. d +	— 12. a +
.....	— 3.
	— 15. c +
.bes—	+ 12. es —
.....	+ 3.
	+ 15. c —
.gis	— 3 bis
.....	+ 3
	0 gisis
.fes	+ 3 deses
.....	— 3
	0 fesese
»o. ce«	

b) **Scheme 15** shows the exact change (to three decimals) in the middle figure (15 tones) and gisis. The number of cents diminish or increase evenly with

$$\left\{ \begin{array}{l} 5.741 \text{ cents in all horizontal Fifth-Axis} \\ 1.456 \text{ — — — oblique Third —} \end{array} \right\}$$

representing »the 2 building materials« in the syntonic mixing-system: Fifths and Thirds:

1) + 1.373 cents eis	2) — 2.912 gis	3) — 2.912 gis	4) 5 syntonic e = 378.2 fesese +
— 1.456 plus e	+ 2.829 plus ‡	— 2.829 minus ‡	5 golden e = 356.8 fesese
— 0.083 gisis	— 0.083 gisis	— 5.741 g	distance = 21.4 = 1 Comma

c) **Rationa lexplanation:** Super division of the octaves e ... e' and as ... as' (Supplement-pairs) gives »retrograde gisis« and »direct fesese« respectively.

d) Similar methods (but of inferior structure) were up to 1558 used in the three famous systems of chance by: (see **Scheme 16**):

ut the Zero-Axis in

ed:

s

= 1 Comma.

lary halving of the
apter IV, Group C).

	Zero-Axis	The unit	
		increases	decreases
(X unknown author) before 1511.....	a c es	+ 1/3 as	— 1/3 e
The Bohemian, Arnold Schlick 1511 ..	as c e	+ 1/4 a	— 1/4 es
The Italian, Gioseffo Zarlino 1558	ces c cis	+ 1/7 { as a	— 1/7 e — 1/7 es
here, Scheme 14 1930	fesese c gisis	+ 1/15 as	— 1/15 e

Scheme 16 shows the 3 Fifths oscillating about the golden g, »the truth«:

	X	Zarlino	Golden system	Schlick
the result g:	694.8	695.8	696.2	696.6
by means of:	$\frac{5}{15}$	$\frac{4}{14}$	$\frac{4}{15}$	$\frac{4}{16}$
in decimals:	= 0,333	= 0,286	= 0,267	= 0,250
Distance from the ideal:	+ 0,066	+ 0,019	Ideal	- 0,017

Addendum.

Scheme 17 shows diagrams of 7 Minors and 6 Majors, some of them in pairs symmetrically, see pages 10, 15 and 20.

Glareanus's erroneous nomenclature of the medieval scale-type should once for all be obliterated from the litterature of music, and should be replaced by that recommended in Scheme 17, compare Helmholtz's heartfelt cry: »Übrigens werde ich Glareans Namen nicht brauchen es wäre überhaupt besser, wenn man sie vergessen möchte,« repeated by Ellis: »But I shall not use Glareanus's names It would be better to forget them altogether.« (Note 21).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

Scheme 19: tertiary distances as arcs in a circle, see pages 7 and 16.

Scheme 20: construction of golden tones by means of parallel lines, see page 24.

Scheme 21: Golden tones in cents with 4 decimals, see page 24.

Scheme 22: examples of logarithms as training in the use of my Constant K = 0.600.5714, see page 7.

In Scheme 23 is set forth a proposal for a system of notation on music-paper, staff, in all other tone-systems than the 12-toned temperament, with the suggestion, offered by K. Steensen (Note 22) for a rational arrangement of the \sharp and \flat .

Scheme 24 shows how the cents can easily be calculated by taking the difference between the logarithms for Partial-tones, the numbers of which are respectively the numerator and the denominator in a tone-fraction, according to the table constructed by the organist Kai Kroman, Copenhagen, for example:

$$\text{oriental } d = \frac{10}{9} \left\{ \begin{array}{l} \text{Logarithm for Partial tone No. 10} = 3.986.3 \\ \text{--- --- --- --- 9} = 3.803.9 \\ \hline \text{difference ... 182.4 cents} \end{array} \right.$$

The word »Partial-tone« is preferred to »over-tone« in order to avoid the confusion, which Ellis characterises as »great confusion, Tyndall's erroneous translation« (Note 21) between

the German	ober	and the English	upper: adjective
—	über	—	—	over: preposition, adverb.

Page	Note	
4	1	Professor E. schungsmi Additamen
5	2	Ibid: »Musik p. 438.
—	3	Prof. E. v. H Bharata ur schaft«, Be
—	4	Professor E. steme«, Ch tische Para p. 10.
7	5	Professor Jo p. 21, Note
10	6	Professor A. Eastern Et
11	7	Prof. HORNB VICTOR MA servatoire
15	8	HELMUT RIT schr. f. ve Blac-quinte Chp. 9, p.
17	9	Administrato published Prof. v. H
19	10	Dr. phil. ALI 1898, p. 12
20	11	Director Gor music, wit »Handb. d der Musik
22	12	LUDW. SONN: Bonn, 1881

the truth:

chlick

696.6

 $\frac{4}{10}$

= 0,250

- 0,017

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5	2	Ibid: »Musikalische Tonsysteme« in »Handb. der Physik«, Bd. VIII, Chp. 9, p. 438.
—	3	Prof. E. v. HORNBOSEL and ROBERT LACHMANN: »Das indische Tonsystem bei Bharata und sein Ursprung«, in »Zeitschrift für vergleichende Musikwissenschaft«, Berlin, 1933, p. 89, Note 1.
—	4	Professor E. v. HORNBOSEL, Berlin, above mentioned: »Musikalische Tonsysteme«, Chp. 9, p. 429 and Prof. E. v. HORNBOSEL and R. LACHMANN: »Asiatische Parallelen zur Berbermusik«, in »Zeitschrift f. vergl. Musikw.«, 1933, p. 10.
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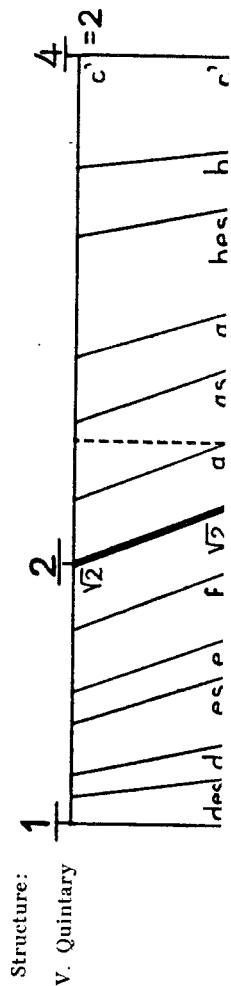
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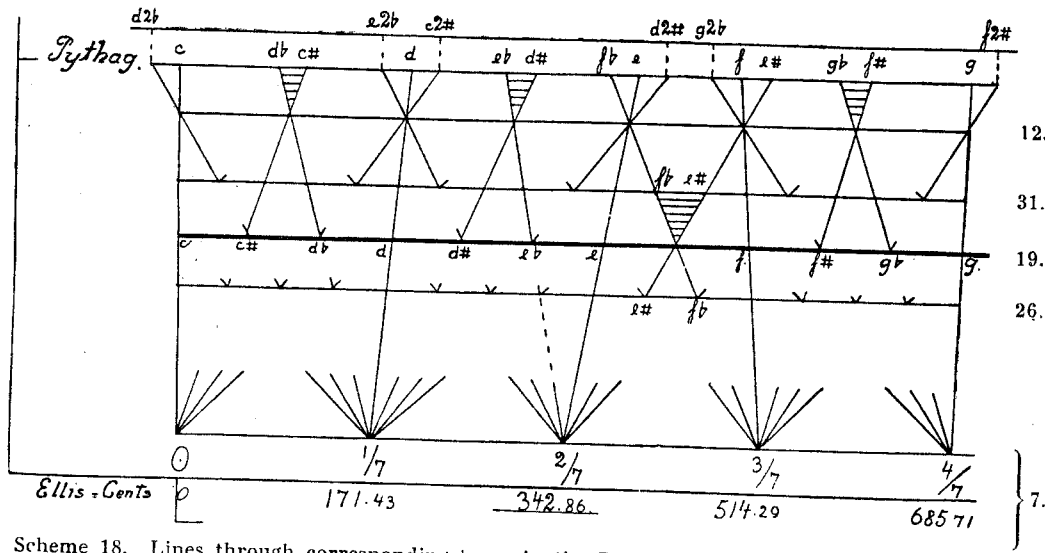
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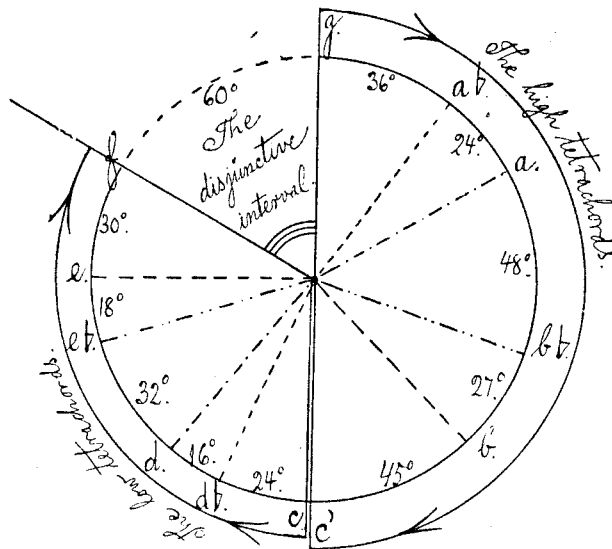
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B. Qualitati



Scheme 18. Lines through corresponding tones in the Pythag. system and 5 temperaments.



Scheme 19. Two Tetrachords tertiarily graduated as a semi-circle and a sector.

Scheme 20. (of many gold p

No.	Cents
26	16 bes 1007
21	13 as 815
16	10 ges 622
11	7 fes 430
8	5 es 311
3	2 des 118
29	18 ces 1126

Scheme 21.