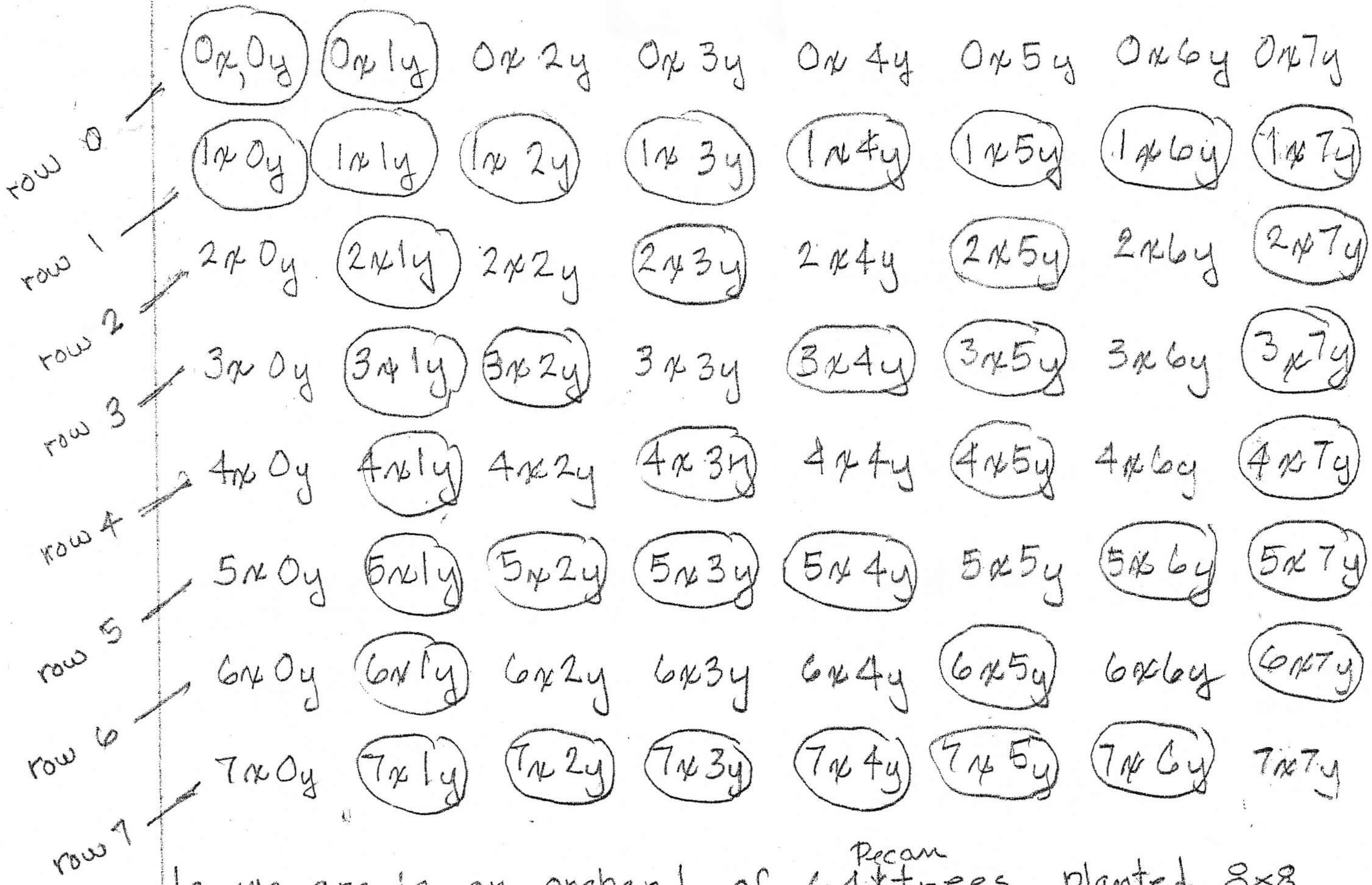
Pecan-Tree Patterns, In a Nut-Shell

Fig. 1 Biaxial Co-prime Pattern



row 0	0x,0y	0x 1y	0x 2y	0x 3y	0x 4y	0x 5y	0x 6y	0x 7y
row 1	1x 0y	1x 1y	1x 2y	1x 3y	1x 4y	1x 5y	1x 6y	1x 7y
row 2	2x 0y	2x 1y	2x 2y	2x 3y	2x 4y	2x 5y	2x 6y	2x 7y
row 3	3x 0y	3x 1y	3x 2y	3x 3y	3x 4y	3x 5y	3x 6y	3x 7y
row 4	4x 0y	4x 1y	4x 2y	4x 3y	4x 4y	4x 5y	4x 6y	4x 7y
row 5	5x 0y	5x 1y	5x 2y	5x 3y	5x 4y	5x 5y	5x 6y	5x 7y
row 6	6x 0y	6x 1y	6x 2y	6x 3y	6x 4y	6x 5y	6x 6y	6x 7y
row 7	7x 0y	7x 1y	7x 2y	7x 3y	7x 4y	7x 5y	7x 6y	7x 7y

If we are in an orchard of 64 Pecan trees planted 8×8 as shown and we stand at Tree "0x 0y" and sight thru the orchard, the trees we can <u>actually</u> <u>see</u> are shown in the circles. They form the <u>co-prime</u> <u>pattern</u>, which extended endlessly never exactly repeats itself, but is nonetheless precisely determined. This pattern is described in the Scale Tree/Peirce Series/Stern-Brocot Series, as it is likewise found in the Lambdoma/Farey Series. Variations on this pattern are found throut nature, the arts, the sciences, and in many surprisingly unexpected places. Interesting and diverse applications are found in <u>musical</u> <u>scales</u> and their associated <u>keyboards</u>.

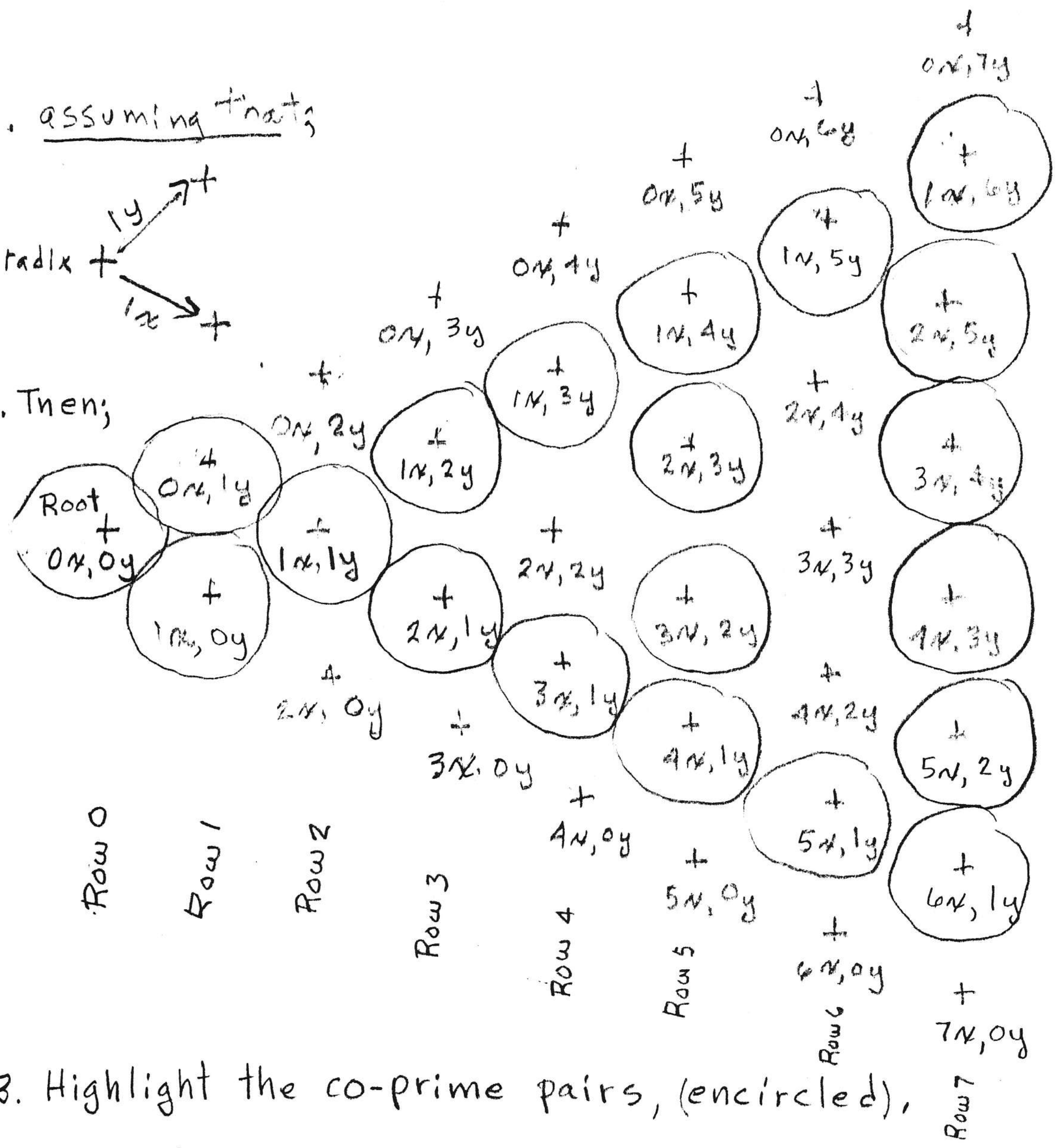
22 June 2000 EW

How to Construct a Co-Prime Grid

22Nov99.EW

fig 1. assuming that;

Fig 2. Then;



3. Highlight the co-prime pairs, (encircled),

4. Neglect or suppress the reducible pairs, (not encircled)

Comments; The above fig 2 shows, from the Root (0x, 0y), all generalized Octave sites and their respective Generator sites, out to row 7. The grid extends endlessly.

Co-prime Moves, on a 5+7 Cross-Grid

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0	5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~	~~35~~	~~40~~	~~45~~
7	12	17	22	27	32	37	42	47	52
~~14~~	19	~~24~~	29	~~34~~	39	~~44~~	49	~~54~~	59
~~21~~	26	31	~~36~~	41	46	~~51~~	56	61	~~66~~
~~28~~	33	~~38~~	43	~~48~~	53	~~58~~	63	(68)	73
~~35~~	40	45	50	55	~~60~~	65	70	75	80
~~42~~	47	~~52~~	~~57~~	~~62~~	67	~~72~~	77	~~82~~	~~87~~
~~49~~	54	59	64	69	74	79	~~84~~	89	94
~~56~~	61	~~66~~	71	~~76~~	81	~~86~~	91	~~96~~	101
~~63~~	68	73	~~78~~	83	88	~~93~~	98	103	~~108~~

I was doing something like this at BYU circa 1950, after reading Joseph Yasser's A Theory of Evolving Tonality. At the time I did not see the co-prime moves pattern from "0".

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Co-prime Moves, on a 5+7 Cross-Grid

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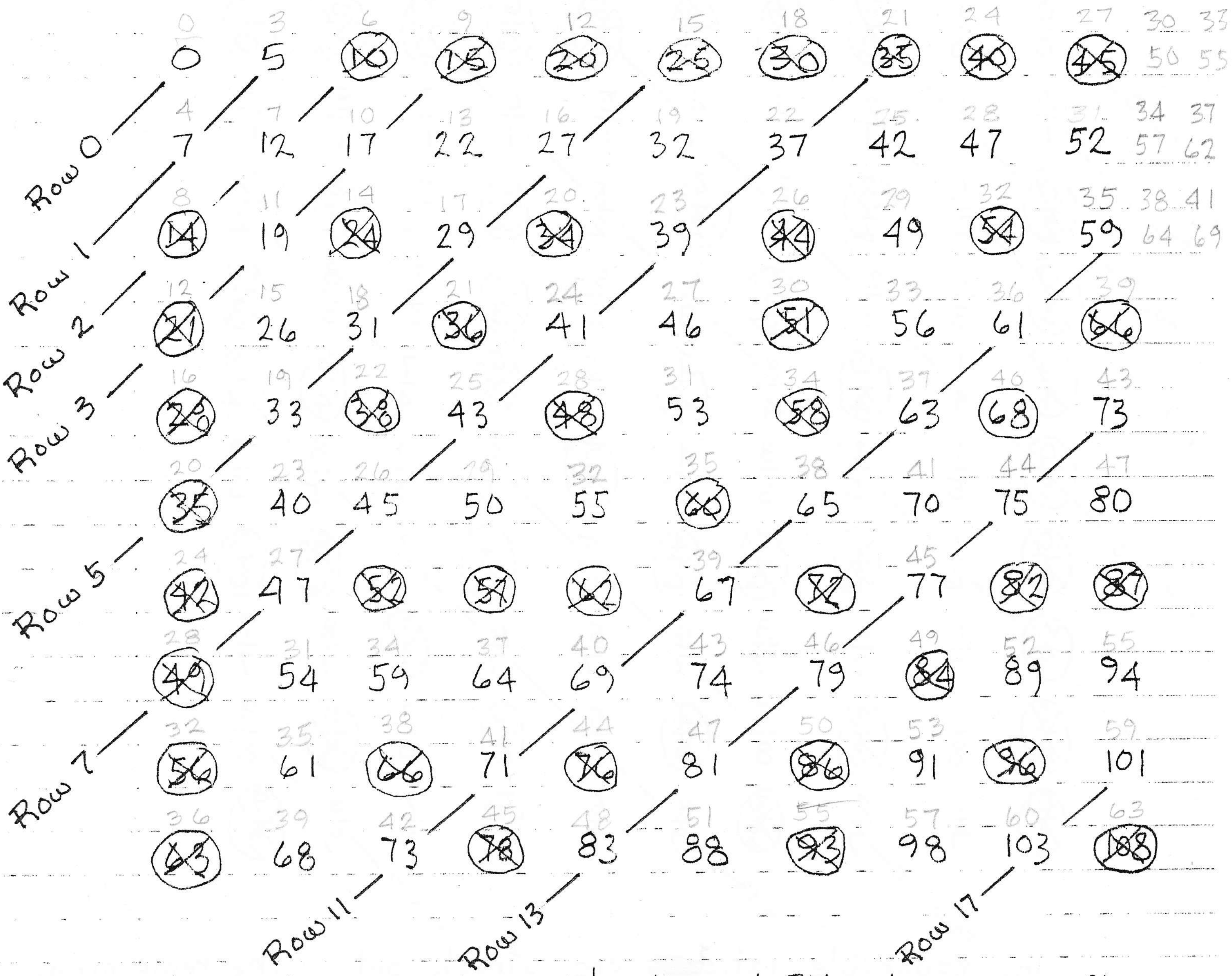

0 0	3 5	6 ~~10~~	9 ~~15~~	12 ~~20~~	15 ~~25~~	18 ~~30~~	21 ~~35~~	24 ~~40~~	27 ~~45~~	30 50	33 55
4 7	7 12	10 17	13 22	16 27	19 32	22 37	25 42	28 47	31 52	34 57	37 62
8 ~~14~~	11 19	14 ~~24~~	17 29	20 ~~34~~	23 39	26 ~~44~~	29 49	32 ~~54~~	35 59	38 64	41 69
12 ~~21~~	15 26	18 31	21 ~~36~~	24 41	27 46	30 ~~51~~	33 56	36 61	39 ~~66~~		
16 ~~28~~	19 33	22 ~~38~~	25 43	28 ~~48~~	31 53	34 ~~58~~	37 63	46 68	43 73		
20 ~~35~~	23 40	26 45	29 50	32 55	35 ~~60~~	38 65	41 70	44 75	47 80		
24 ~~42~~	27 47	~~52~~	~~57~~	~~62~~	39 67	~~72~~	45 77	~~82~~	~~87~~		
28 ~~49~~	31 54	34 59	37 64	40 69	43 74	46 79	49 ~~84~~	52 89	55 94		
32 ~~56~~	35 61	38 ~~66~~	41 71	44 ~~76~~	47 81	50 ~~86~~	53 91	~~96~~	59 101		
36 ~~63~~	39 68	42 73	45 ~~78~~	48 83	51 88	55 ~~93~~	57 98	60 103	63 ~~108~~		

I was doing something like this at BYU circa 1950, after reading Joseph Yasser's A Theory of Evolving Tonality, 1932. At the time I did not see the co-prime moves pattern from "0".

Annotated Dec 10, 2000 – Had I added the numerators as shown, and struck out the reducible fractions – I would have seen a small slice $\{\frac{3}{5}, \frac{4}{7}\}$? of the Farey Series, and a co-prime grid; and bridged the gap between the Yasser Series 5, 7, 12, 19, 31, 50, 81 ⋯ and the Farey Series. – The prime diagonals also shown; 0, 1, 2, 3, 5, 7, 11, 13, 17 ⋯

Ref: "Jumping Champions", Ian Stewart, Scientific American Dec 20 pp 106-107.

The Farey Series of Order 1025, E. H. Neville, 1950, University Press Cambridge.

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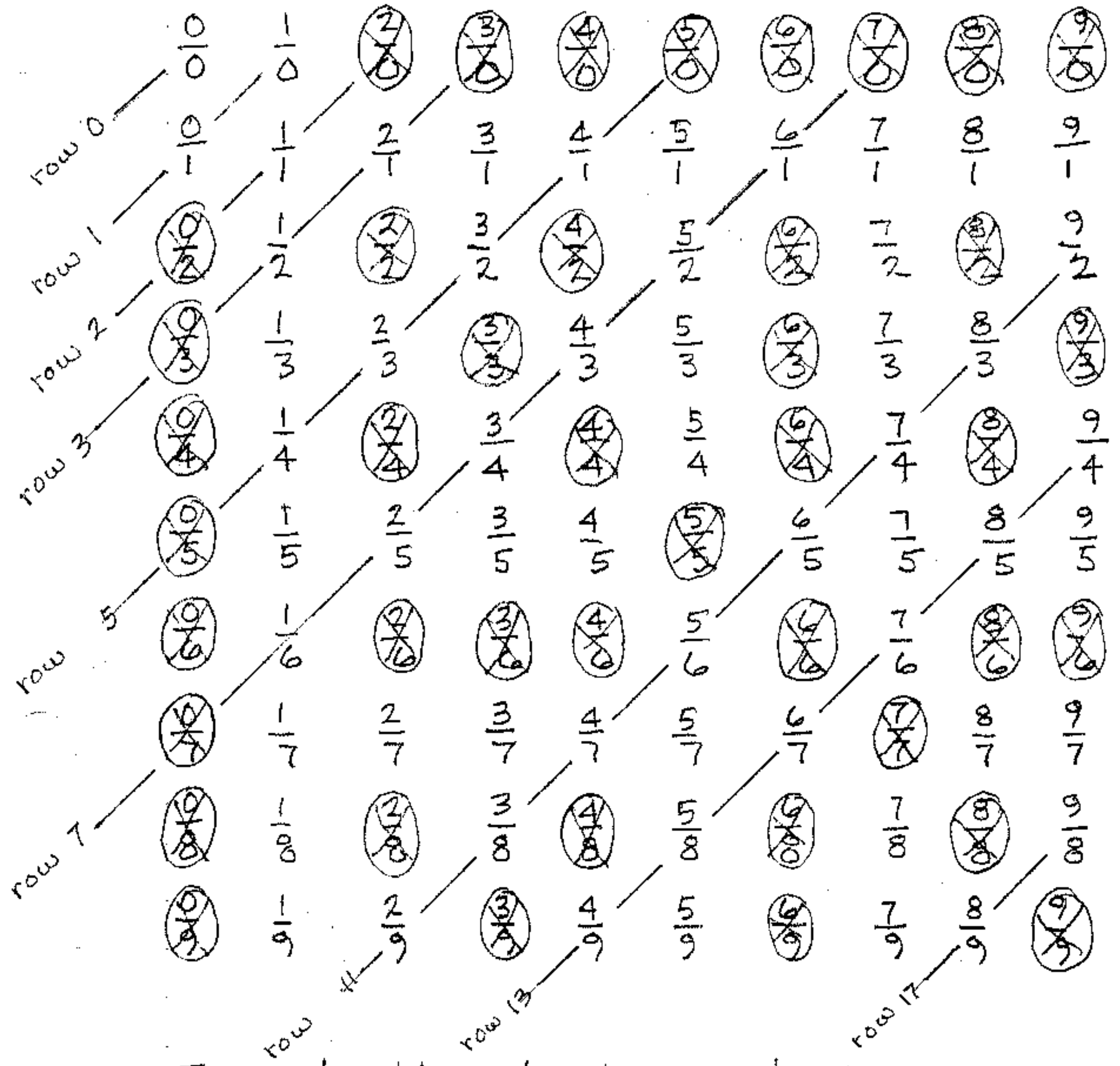
Co-Prime Moves, on the cap 9 Lambdoma



$\frac{0}{0}$	$\frac{1}{0}$	$\cancel{\frac{2}{0}}$	$\cancel{\frac{3}{0}}$	$\cancel{\frac{4}{0}}$	$\cancel{\frac{5}{0}}$	$\cancel{\frac{6}{0}}$	$\cancel{\frac{7}{0}}$	$\cancel{\frac{8}{0}}$	$\cancel{\frac{9}{0}}$
$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$	$\frac{9}{1}$
$\cancel{\frac{0}{2}}$	$\frac{1}{2}$	$\cancel{\frac{2}{2}}$	$\frac{3}{2}$	$\cancel{\frac{4}{2}}$	$\frac{5}{2}$	$\cancel{\frac{6}{2}}$	$\frac{7}{2}$	$\cancel{\frac{8}{2}}$	$\frac{9}{2}$
$\cancel{\frac{0}{3}}$	$\frac{1}{3}$	$\frac{2}{3}$	$\cancel{\frac{3}{3}}$	$\frac{4}{3}$	$\frac{5}{3}$	$\cancel{\frac{6}{3}}$	$\frac{7}{3}$	$\frac{8}{3}$	$\cancel{\frac{9}{3}}$
$\cancel{\frac{0}{4}}$	$\frac{1}{4}$	$\cancel{\frac{2}{4}}$	$\frac{3}{4}$	$\cancel{\frac{4}{4}}$	$\frac{5}{4}$	$\cancel{\frac{6}{4}}$	$\frac{7}{4}$	$\cancel{\frac{8}{4}}$	$\frac{9}{4}$
$\cancel{\frac{0}{5}}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\cancel{\frac{5}{5}}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$
$\cancel{\frac{0}{6}}$	$\frac{1}{6}$	$\cancel{\frac{2}{6}}$	$\cancel{\frac{3}{6}}$	$\cancel{\frac{4}{6}}$	$\frac{5}{6}$	$\cancel{\frac{6}{6}}$	$\frac{7}{6}$	$\cancel{\frac{8}{6}}$	$\cancel{\frac{9}{6}}$
$\cancel{\frac{0}{7}}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\cancel{\frac{7}{7}}$	$\frac{8}{7}$	$\frac{9}{7}$
$\cancel{\frac{0}{8}}$	$\frac{1}{8}$	$\cancel{\frac{2}{8}}$	$\frac{3}{8}$	$\cancel{\frac{4}{8}}$	$\frac{5}{8}$	$\cancel{\frac{6}{8}}$	$\frac{7}{8}$	$\cancel{\frac{8}{8}}$	$\frac{9}{8}$
$\cancel{\frac{0}{9}}$	$\frac{1}{9}$	$\frac{2}{9}$	$\cancel{\frac{3}{9}}$	$\frac{4}{9}$	$\frac{5}{9}$	$\cancel{\frac{6}{9}}$	$\frac{7}{9}$	$\frac{8}{9}$	$\cancel{\frac{9}{9}}$

The reducible rationals are struck out. The remaining, irreducible rationals form a co-prime moves pattern from "$\frac{0}{0}$". Compare this with <u>Yasserian Keyboard Guide</u>, by Erv Wilson 1994.

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Co-prime Moves, on the cap 9 Lambdoma

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$\frac{0}{0}$	$\frac{1}{0}$	$\cancel{\frac{2}{0}}$	$\cancel{\frac{3}{0}}$	$\cancel{\frac{4}{0}}$	$\cancel{\frac{5}{0}}$	$\cancel{\frac{6}{0}}$	$\cancel{\frac{7}{0}}$	$\cancel{\frac{8}{0}}$	$\cancel{\frac{9}{0}}$
$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$	$\frac{9}{1}$
$\cancel{\frac{0}{2}}$	$\frac{1}{2}$	$\cancel{\frac{2}{2}}$	$\frac{3}{2}$	$\cancel{\frac{4}{2}}$	$\frac{5}{2}$	$\cancel{\frac{6}{2}}$	$\frac{7}{2}$	$\cancel{\frac{8}{2}}$	$\frac{9}{2}$
$\cancel{\frac{0}{3}}$	$\frac{1}{3}$	$\frac{2}{3}$	$\cancel{\frac{3}{3}}$	$\frac{4}{3}$	$\frac{5}{3}$	$\cancel{\frac{6}{3}}$	$\frac{7}{3}$	$\frac{8}{3}$	$\cancel{\frac{9}{3}}$
$\cancel{\frac{0}{4}}$	$\frac{1}{4}$	$\cancel{\frac{2}{4}}$	$\frac{3}{4}$	$\cancel{\frac{4}{4}}$	$\frac{5}{4}$	$\cancel{\frac{6}{4}}$	$\frac{7}{4}$	$\cancel{\frac{8}{4}}$	$\frac{9}{4}$
$\cancel{\frac{0}{5}}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\cancel{\frac{5}{5}}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$
$\cancel{\frac{0}{6}}$	$\frac{1}{6}$	$\cancel{\frac{2}{6}}$	$\cancel{\frac{3}{6}}$	$\cancel{\frac{4}{6}}$	$\frac{5}{6}$	$\cancel{\frac{6}{6}}$	$\frac{7}{6}$	$\cancel{\frac{8}{6}}$	$\cancel{\frac{9}{6}}$
$\cancel{\frac{0}{7}}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\cancel{\frac{7}{7}}$	$\frac{8}{7}$	$\frac{9}{7}$
$\cancel{\frac{0}{8}}$	$\frac{1}{8}$	$\cancel{\frac{2}{8}}$	$\frac{3}{8}$	$\cancel{\frac{4}{8}}$	$\frac{5}{8}$	$\cancel{\frac{6}{8}}$	$\frac{7}{8}$	$\cancel{\frac{8}{8}}$	$\frac{9}{8}$
$\cancel{\frac{0}{9}}$	$\frac{1}{9}$	$\frac{2}{9}$	$\cancel{\frac{3}{9}}$	$\frac{4}{9}$	$\frac{5}{9}$	$\cancel{\frac{6}{9}}$	$\frac{7}{9}$	$\frac{8}{9}$	$\cancel{\frac{9}{9}}$

The reducible rationals are struck out. The remaining, irreducible rationals form a co-prime moves pattern from "$\frac{0}{0}$". Compare this with Yasserian Keyboard Guide, by Erv Wilson 1994.

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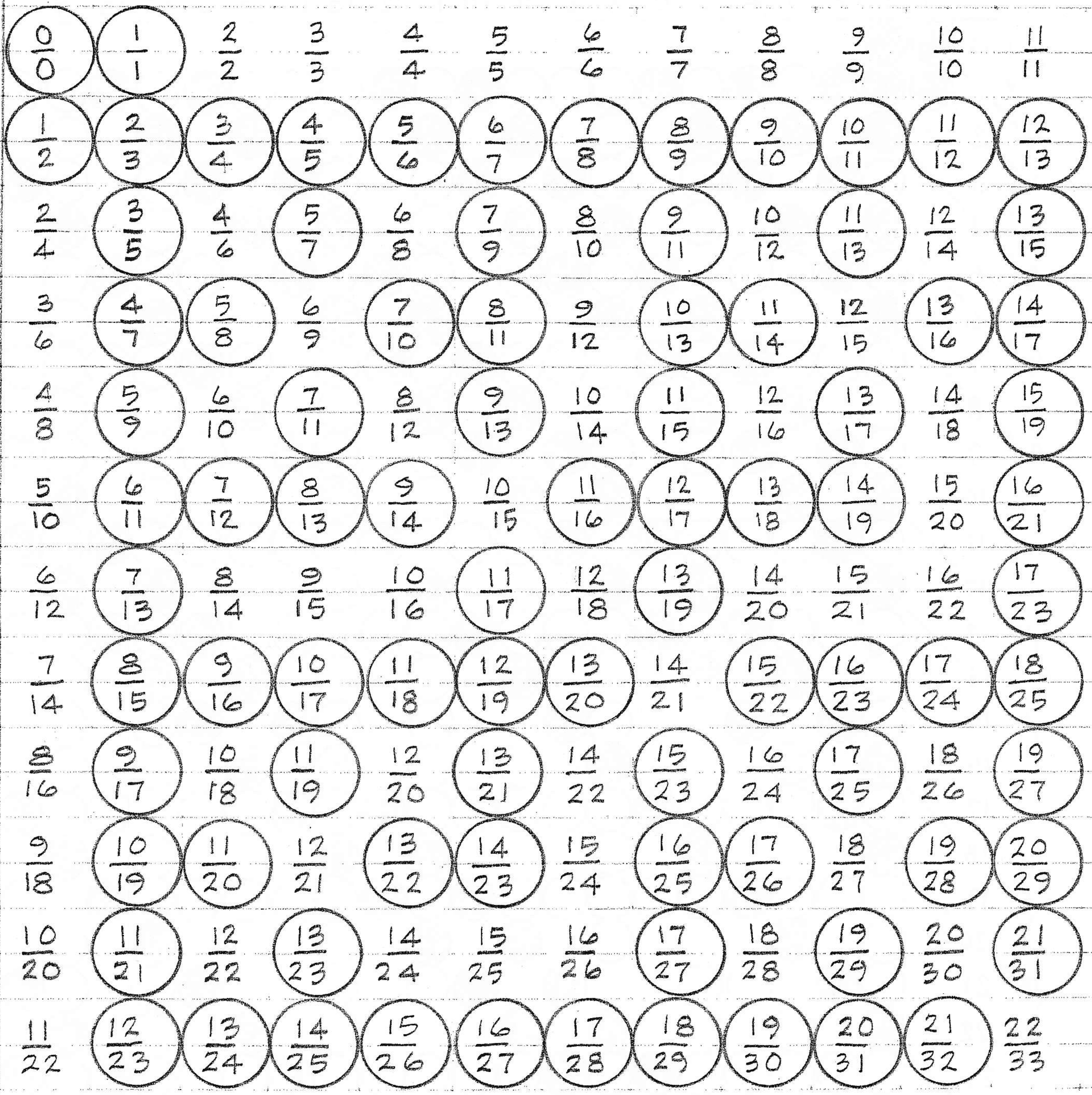
Lambdoma of Diophantine Couplet $\frac{1}{2}$ $\frac{1}{1}$, $(\frac{a}{b}\ \frac{c}{d})$ $b \cdot c - a \cdot d = 1$



$\frac{0}{0}$	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$	$\frac{11}{11}$
$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{10}{11}$	$\frac{11}{12}$	$\frac{12}{13}$
$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$	$\frac{5}{7}$	$\frac{6}{8}$	$\frac{7}{9}$	$\frac{8}{10}$	$\frac{9}{11}$	$\frac{10}{12}$	$\frac{11}{13}$	$\frac{12}{14}$	$\frac{13}{15}$
$\frac{3}{6}$	$\frac{4}{7}$	$\frac{5}{8}$	$\frac{6}{9}$	$\frac{7}{10}$	$\frac{8}{11}$	$\frac{9}{12}$	$\frac{10}{13}$	$\frac{11}{14}$	$\frac{12}{15}$	$\frac{13}{16}$	$\frac{14}{17}$
$\frac{4}{8}$	$\frac{5}{9}$	$\frac{6}{10}$	$\frac{7}{11}$	$\frac{8}{12}$	$\frac{9}{13}$	$\frac{10}{14}$	$\frac{11}{15}$	$\frac{12}{16}$	$\frac{13}{17}$	$\frac{14}{18}$	$\frac{15}{19}$
$\frac{5}{10}$	$\frac{6}{11}$	$\frac{7}{12}$	$\frac{8}{13}$	$\frac{9}{14}$	$\frac{10}{15}$	$\frac{11}{16}$	$\frac{12}{17}$	$\frac{13}{18}$	$\frac{14}{19}$	$\frac{15}{20}$	$\frac{16}{21}$
$\frac{6}{12}$	$\frac{7}{13}$	$\frac{8}{14}$	$\frac{9}{15}$	$\frac{10}{16}$	$\frac{11}{17}$	$\frac{12}{18}$	$\frac{13}{19}$	$\frac{14}{20}$	$\frac{15}{21}$	$\frac{16}{22}$	$\frac{17}{23}$
$\frac{7}{14}$	$\frac{8}{15}$	$\frac{9}{16}$	$\frac{10}{17}$	$\frac{11}{18}$	$\frac{12}{19}$	$\frac{13}{20}$	$\frac{14}{21}$	$\frac{15}{22}$	$\frac{16}{23}$	$\frac{17}{24}$	$\frac{18}{25}$
$\frac{8}{16}$	$\frac{9}{17}$	$\frac{10}{18}$	$\frac{11}{19}$	$\frac{12}{20}$	$\frac{13}{21}$	$\frac{14}{22}$	$\frac{15}{23}$	$\frac{16}{24}$	$\frac{17}{25}$	$\frac{18}{26}$	$\frac{19}{27}$
$\frac{9}{18}$	$\frac{10}{19}$	$\frac{11}{20}$	$\frac{12}{21}$	$\frac{13}{22}$	$\frac{14}{23}$	$\frac{15}{24}$	$\frac{16}{25}$	$\frac{17}{26}$	$\frac{18}{27}$	$\frac{19}{28}$	$\frac{20}{29}$
$\frac{10}{20}$	$\frac{11}{21}$	$\frac{12}{22}$	$\frac{13}{23}$	$\frac{14}{24}$	$\frac{15}{25}$	$\frac{16}{26}$	$\frac{17}{27}$	$\frac{18}{28}$	$\frac{19}{29}$	$\frac{20}{30}$	$\frac{21}{31}$
$\frac{11}{22}$	$\frac{12}{23}$	$\frac{13}{24}$	$\frac{14}{25}$	$\frac{15}{26}$	$\frac{16}{27}$	$\frac{17}{28}$	$\frac{18}{29}$	$\frac{19}{30}$	$\frac{20}{31}$	$\frac{21}{32}$	$\frac{22}{33}$

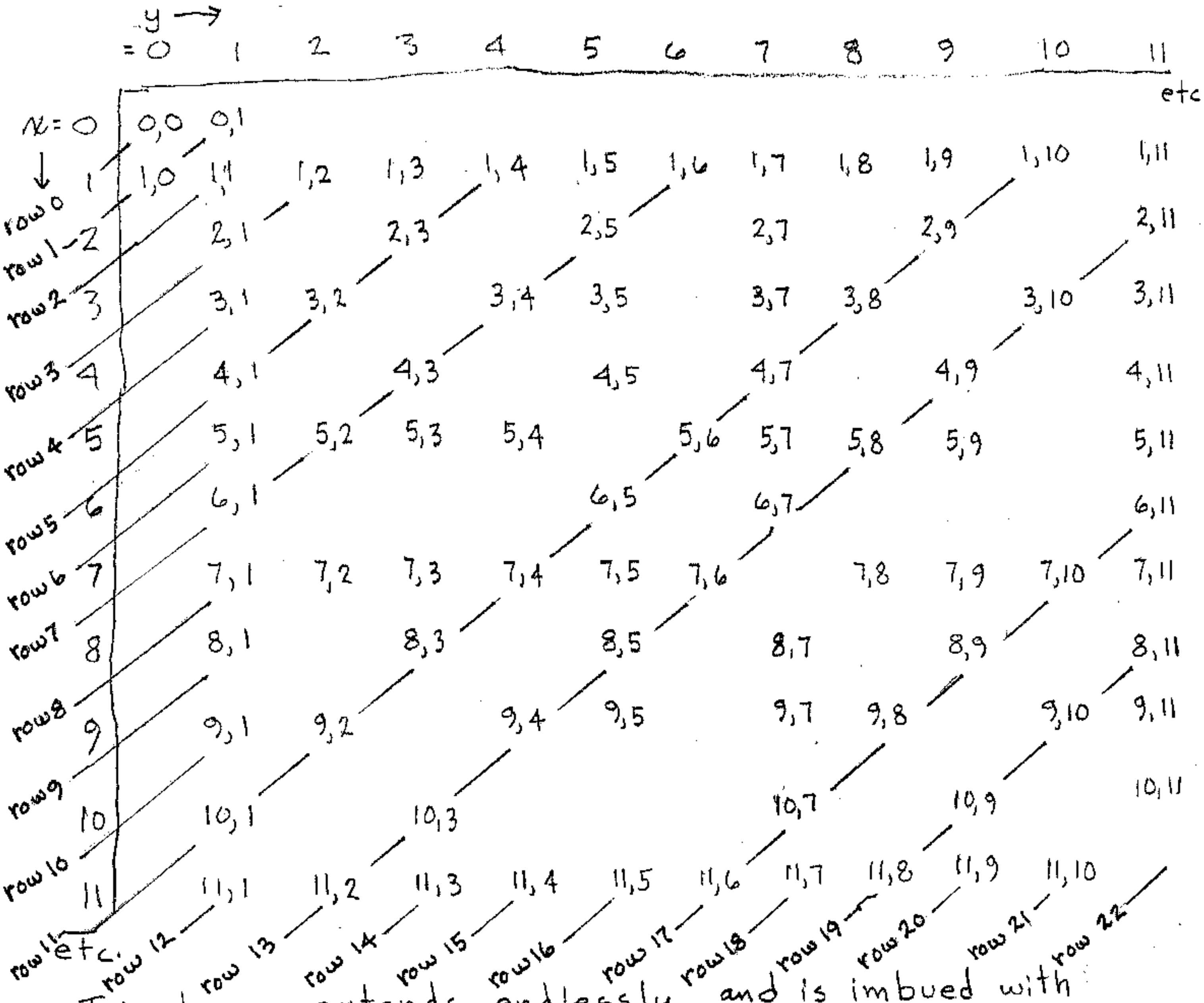
The Top-Lambdoma Is generated from the Diophantine Couplet $\frac{0}{1}$ $\frac{1}{0}$; $\frac{a}{b}$ $\frac{c}{d}$ are adjacent and $bc-ad=1$. Each subsequent couplet in the series can generate a Lambdoma sub-species, like shown above. Mediants and Epimoria hold, as does the Co-prime Pattern.

Ref: A Brief History of the Lambdoma, Barbara 1994, XH 16

So-Called Farey Series, extended $0/1$ to $1/0$ (Full Set of Gear Ratios), and Lamboma by Ervin M. Wilson 1992

11DEC99.EW

Co-Prime Logic Diagram, (to cap 11) 23 Nov 99 EW



y →	=0	1	2	3	4	5	6	7	8	9	10	11
x = 0 ↓	0,0	0,1										
1	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11
2		2,1		2,3		2,5		2,7		2,9		2,11
3		3,1	3,2		3,4	3,5		3,7	3,8		3,10	3,11
4		4,1		4,3		4,5		4,7		4,9		4,11
5		5,1	5,2	5,3	5,4		5,6	5,7	5,8	5,9		5,11
6		6,1				6,5		6,7				6,11
7		7,1	7,2	7,3	7,4	7,5	7,6		7,8	7,9	7,10	7,11
8		8,1		8,3		8,5		8,7		8,9		8,11
9		9,1	9,2		9,4	9,5		9,7	9,8		9,10	9,11
10		10,1		10,3				10,7		10,9		10,11
11		11,1	11,2	11,3	11,4	11,5	11,6	11,7	11,8	11,9	11,10	

This diagram extends endlessly, and is imbued with properties applicable to musical scales and generalized musical keyboards. It contains the essential elements of the Peirce Series (Scale-Tree), the Farey Series (Lambdoma) and the gamut of Fibonacci Series. It gives the x,y co-ordinates of the Co-Prime Grid.

Co-Prime Logic Diagram, (to cap 11) 23 Nov 99 EW

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etc

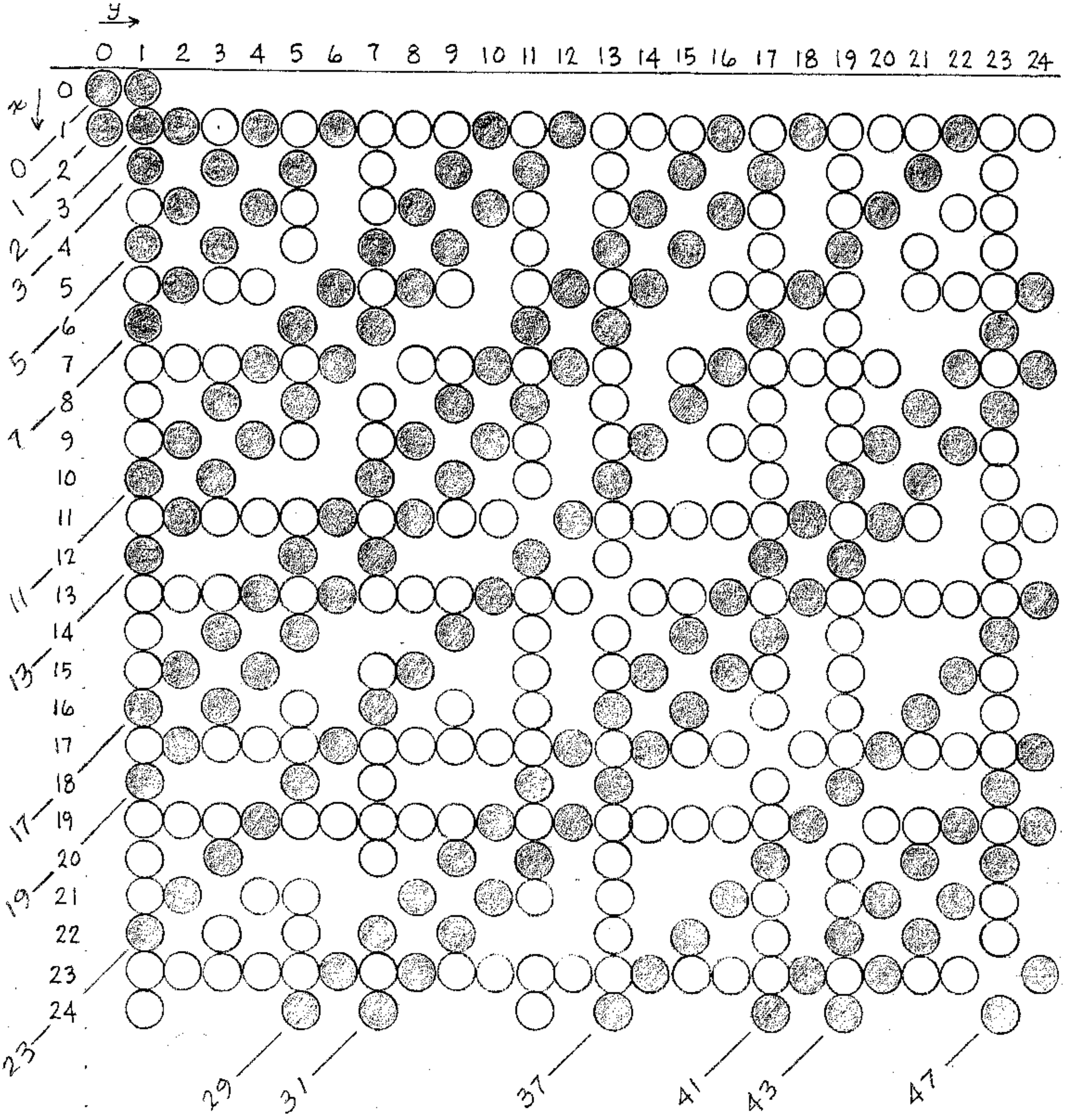
	y → =0	1	2	3	4	5	6	7	8	9	10	11
x=0	0,0	0,1										
↓ 1	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11
2		2,1		2,3		2,5		2,7		2,9		2,11
3		3,1	3,2		3,4	3,5		3,7	3,8		3,10	3,11
4		4,1		4,3		4,5		4,7		4,9		4,11
5		5,1	5,2	5,3	5,4		5,6	5,7	5,8	5,9		5,11
6		6,1				6,5		6,7				6,11
7		7,1	7,2	7,3	7,4	7,5	7,6		7,8	7,9	7,10	7,11
8		8,1		8,3		8,5		8,7		8,9		8,11
9		9,1	9,2		9,4	9,5		9,7	9,8		9,10	9,11
10		10,1		10,3				10,7		10,9		10,11
11		11,1	11,2	11,3	11,4	11,5	11,6	11,7	11,8	11,9	11,10	

etc.

This diagram extends endlessly, and is imbued with properties applicable to musical scales and generalized musical keyboards. It contains the essential elements of the Peirce Series (Scale-Tree), the Farey Series (Lambdoma), and the gamut of Fibonacci Series. It gives the x,y coordinates of the Co-Prime Grid.

Co-PRIME LOGIC Diagram, to Cap 24

23 NOV 99. EW



11/7

coprime 362
25 × 25 625
reducible 263

263/362 = .726519337

Co-PRIME LOGIC Diagram, to Cap 24

23 NOV 99. EW

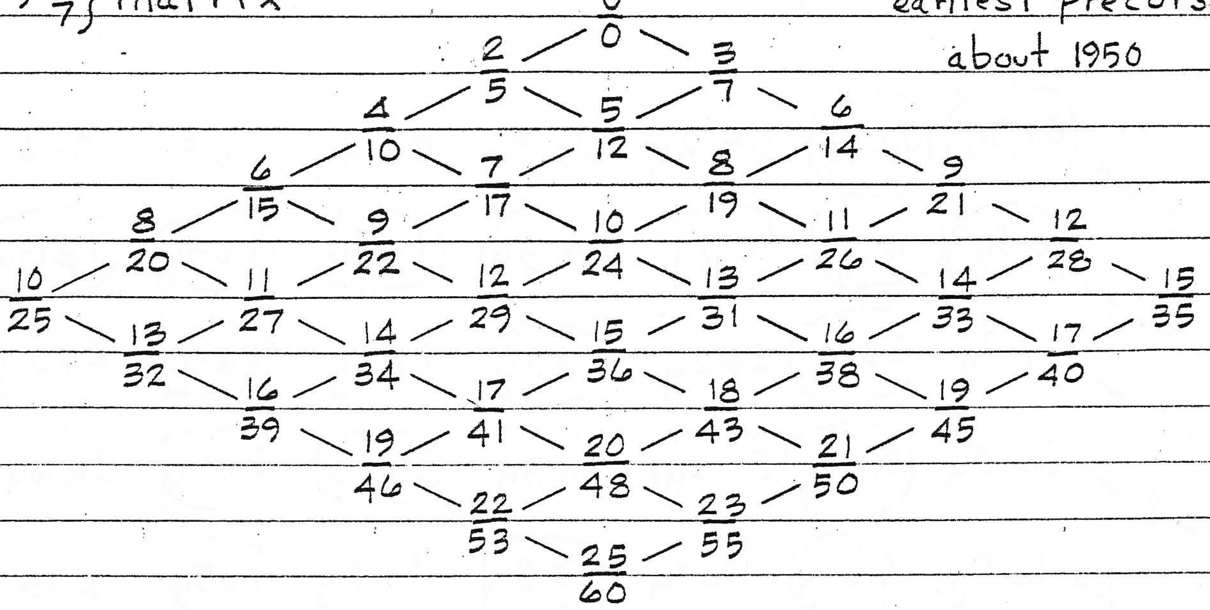
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	○	○																							
1	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
2		○		○		○		○		○		○		○		○		○		○		○		○	
3		○	○		○	○		○	○		○	○		○	○		○	○		○	○		○	○	
4		○		○		○		○		○		○		○		○		○		○		○		○	
5		○	○	○	○		○	○	○	○		○	○	○	○		○	○	○	○		○	○	○	○
6		○				○		○				○		○				○		○				○	
7		○	○	○	○	○	○		○	○	○	○	○	○		○	○	○	○	○	○		○	○	○
8		○		○		○		○		○		○		○		○		○		○		○		○	
9		○	○		○	○		○	○		○	○		○	○		○	○		○	○		○	○	
10		○		○				○		○		○		○				○		○		○		○	
11		○	○	○	○	○	○	○	○	○	○		○	○	○	○	○	○	○	○	○	○		○	
12		○				○		○				○		○				○		○				○	
13		○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○	○	○	○	○	○	○	○
14		○		○		○				○		○		○		○		○		○				○	
15		○	○		○			○	○			○		○	○		○	○		○			○	○	
16		○		○		○		○		○		○		○		○		○		○		○		○	
17		○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○	○	○	○
18		○				○		○				○		○				○		○				○	
19		○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○	
20		○		○				○		○		○		○				○		○		○		○	
21		○	○		○	○			○		○	○		○			○	○		○	○		○	○	
22		○		○		○		○		○				○		○		○		○		○		○	
23		○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		
24		○				○		○				○		○				○		○				○	

Notes on two precursors to the Scale-Tree

© 1996 by Erv Wilson

$\left\{\frac{2}{5}, \frac{3}{7}\right\}$ matrix

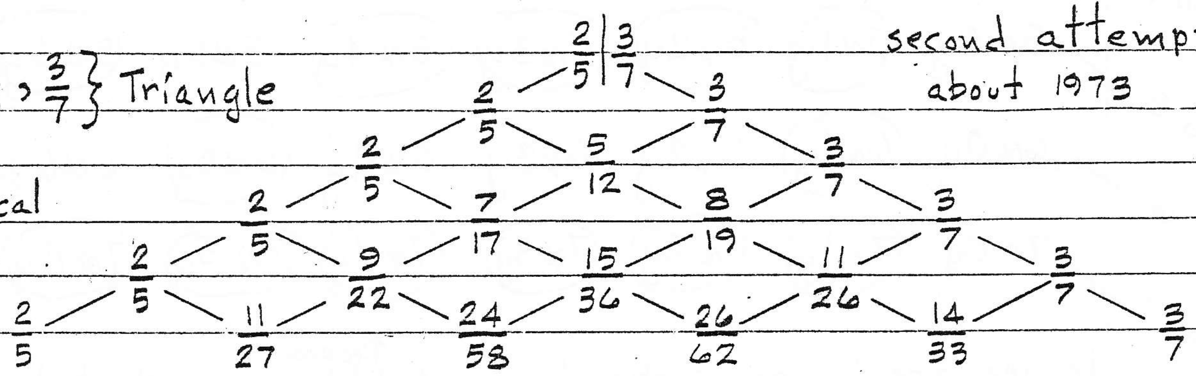
earliest precursor
about 1950



$\left\{\frac{2}{5}, \frac{3}{7}\right\}$ Triangle

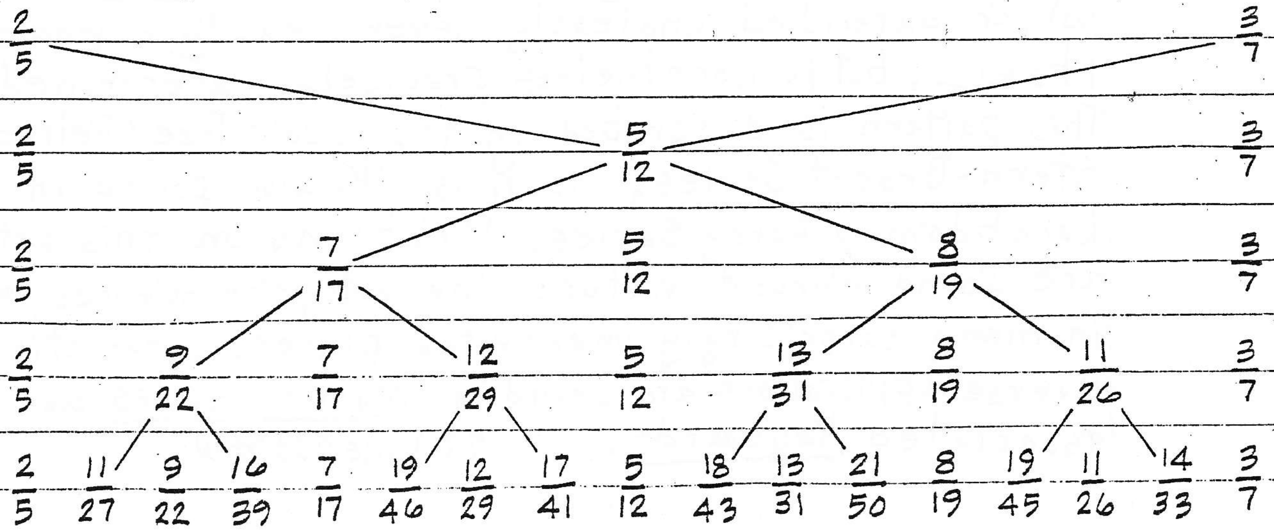
second attempt
about 1973

ref. Pascal



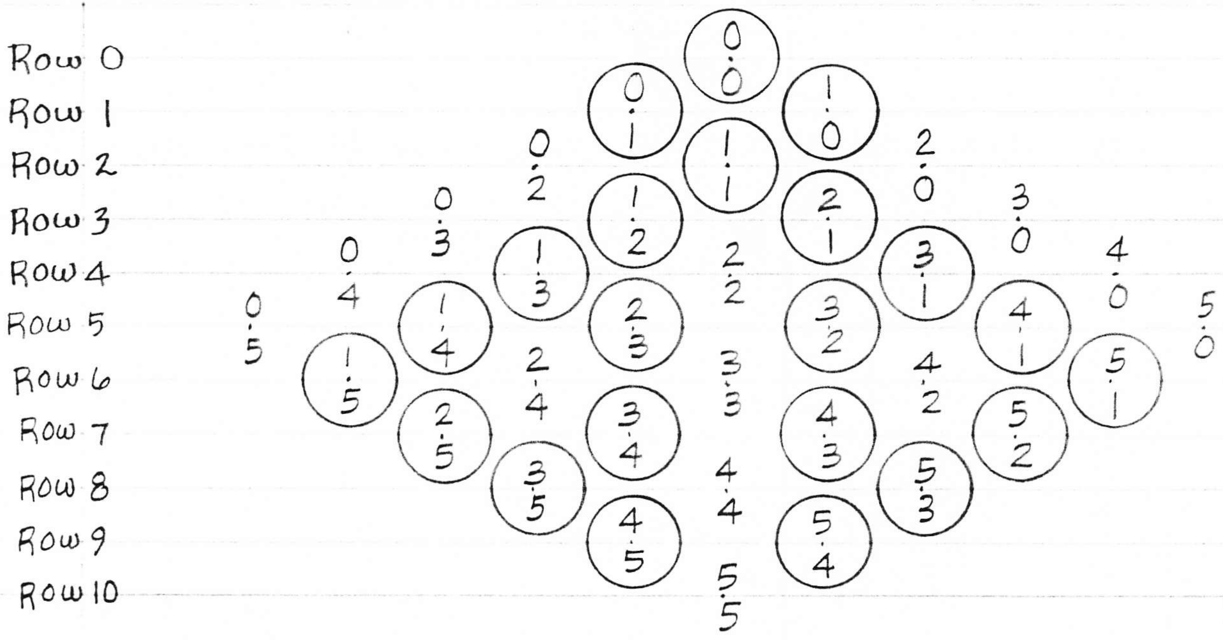
$\left\{\frac{2}{5}, \frac{3}{7}\right\}$ Peirce sequence, Scale-Tree outline

Peirce sequence
as observed by Wilso
1974



REVERSE TRUNCATION OF LAMDOMA

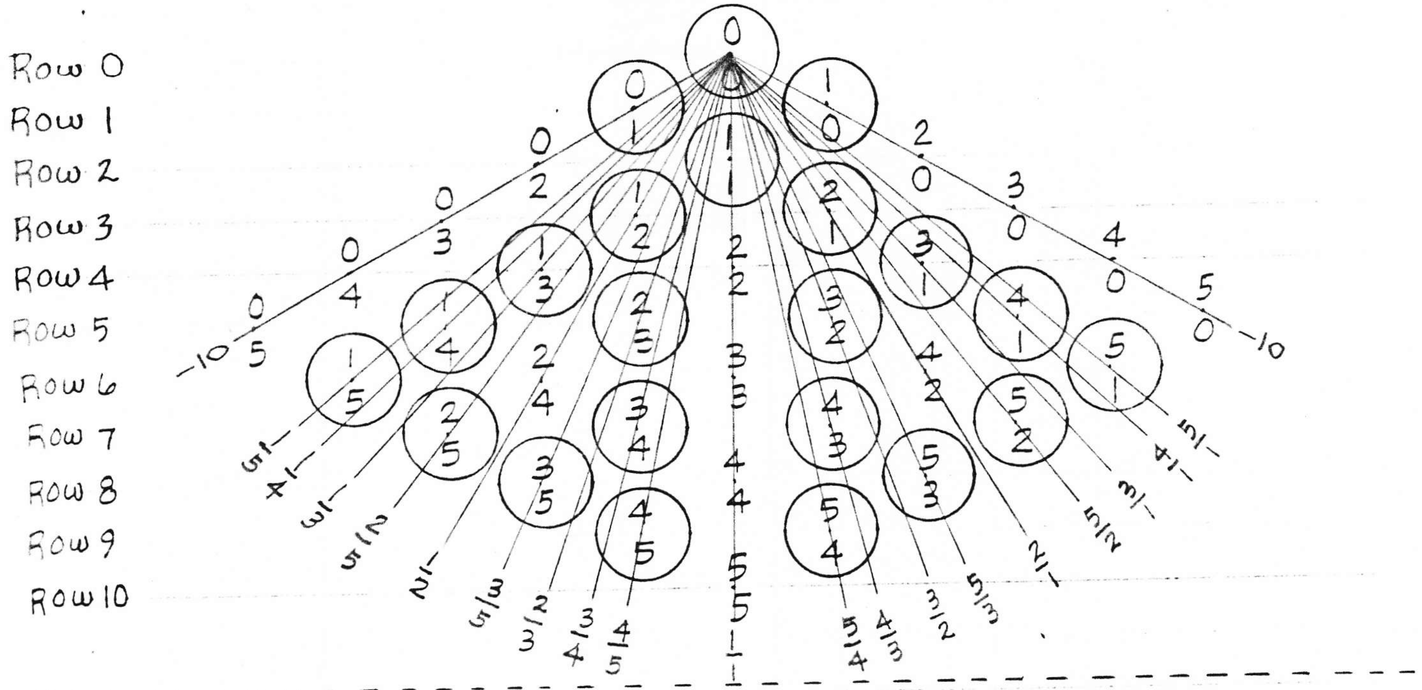
22 MAR 00. EW



R0											0/0										
R1	0/1																				1/0
R2	0/1										1/1										1/0
R3	0/1					1/2					1/1					2/1					1/0
R4	0/1			1/3		1/2					1/1					2/1		3/1			1/0
R5	0/1		1/4	1/3		1/2		2/3			1/1			3/2		2/1		3/1	4/1		1/0
R6	0/1	1/5	1/4	1/3		1/2		2/3			1/1			3/2		2/1		3/1	4/1	5/1	1/0
R7	0/1	1/5	1/4	1/3	2/5	1/2		2/3	3/4		1/1		4/3	3/2		2/1	5/2	3/1	4/1	5/1	1/0
R8	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4		1/1		4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0
R9	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4	4/5	1/1	5/4	4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0
R10	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4	4/5	1/1	5/4	4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0

REVERSE TRUNCATION OF LAMDOMA

22 MAR 00. EW



R0											0/0										
R1	0/1																				1/0
R2	0/1										1/1										1/0
R3	0/1					1/2					1/1					2/1					1/0
R4	0/1			1/3		1/2					1/1					2/1		3/1			1/0
R5	0/1		1/4	1/3		1/2		2/3			1/1			3/2		2/1		3/1	4/1		1/0
R6	0/1	1/5	1/4	1/3		1/2		2/3			1/1			3/2		2/1		3/1	4/1	5/1	1/0
R7	0/1	1/5	1/4	1/3	2/5	1/2		2/3	3/4		1/1		4/3	3/2		2/1	5/2	3/1	4/1	5/1	1/0
R8	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4		1/1		4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0
R9	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4	4/5	1/1	5/4	4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0
R10	0/1	1/5	1/4	1/3	2/5	1/2	3/5	2/3	3/4	4/5	1/1	5/4	4/3	3/2	5/3	2/1	5/2	3/1	4/1	5/1	1/0

Note; Solutions to the Diophantine equation b.c - a.d hold. Mediants hold.

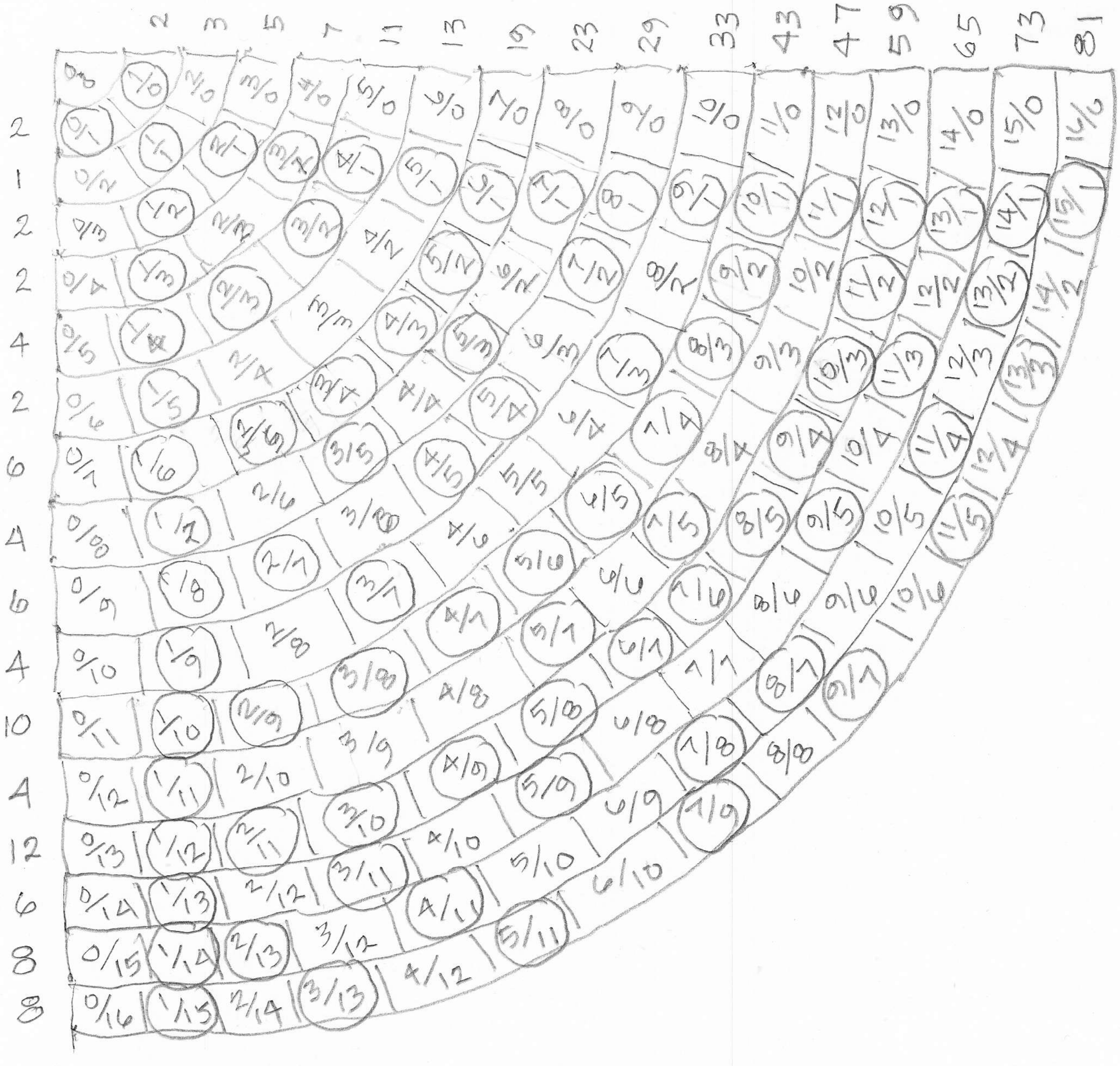
Ref: *A Brief History of the Lambdoma*, 1994, Barbara Hero
So-Called Farey Series, extended 0/1 To 1/0, 1996 Ervin M. Wilson

Zero is an integer and therefore it is part of the integer sequence. The application of the co-prime pattern to Keyboard design is the intellectual property of Erv Wilson, There are infinitely many co-prime patterns imbedded radially within the top set. This property is uniquely useful in the application to musical scale and associated musical instrument generalized Keyboard design.

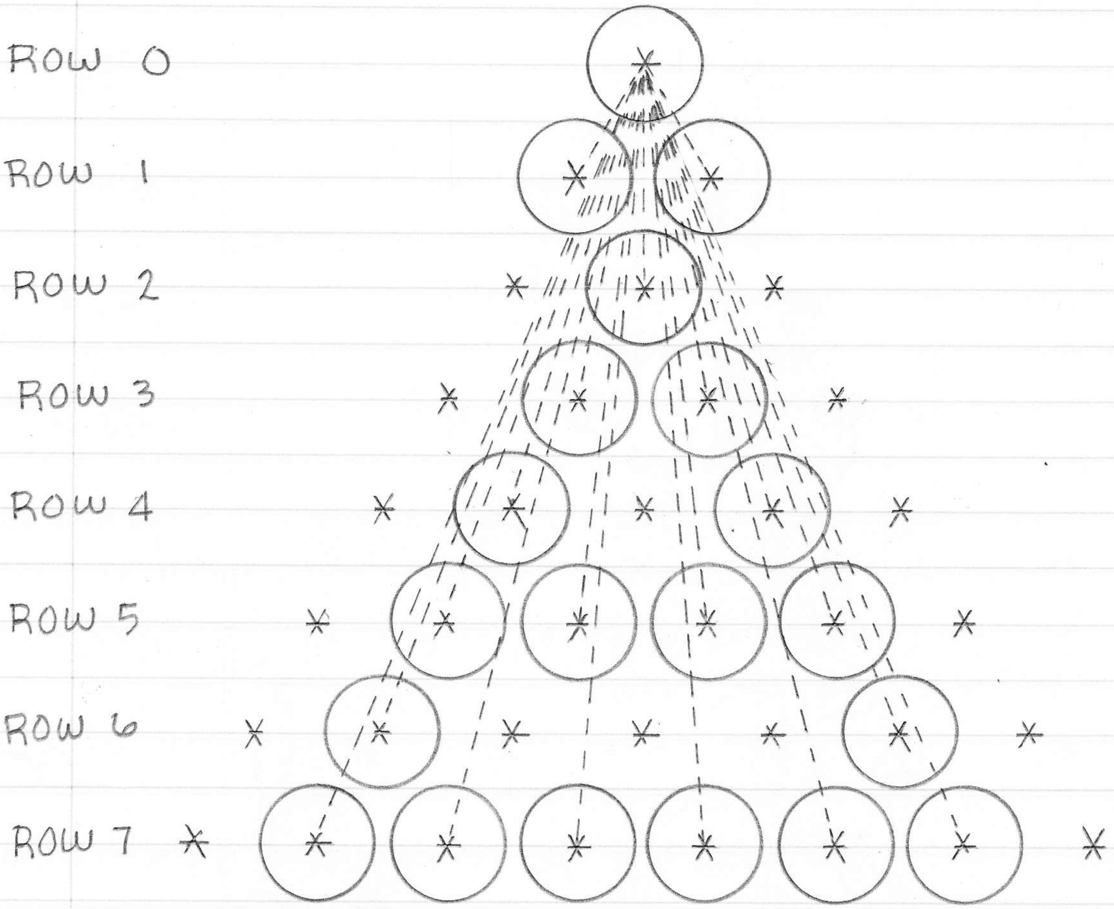
"An Hyperdimensional co-prime pattern fills the paradisal infinitude."

23Sep2003·EW . This lead of courge to the continuum.

- -



25Sep2003·EW

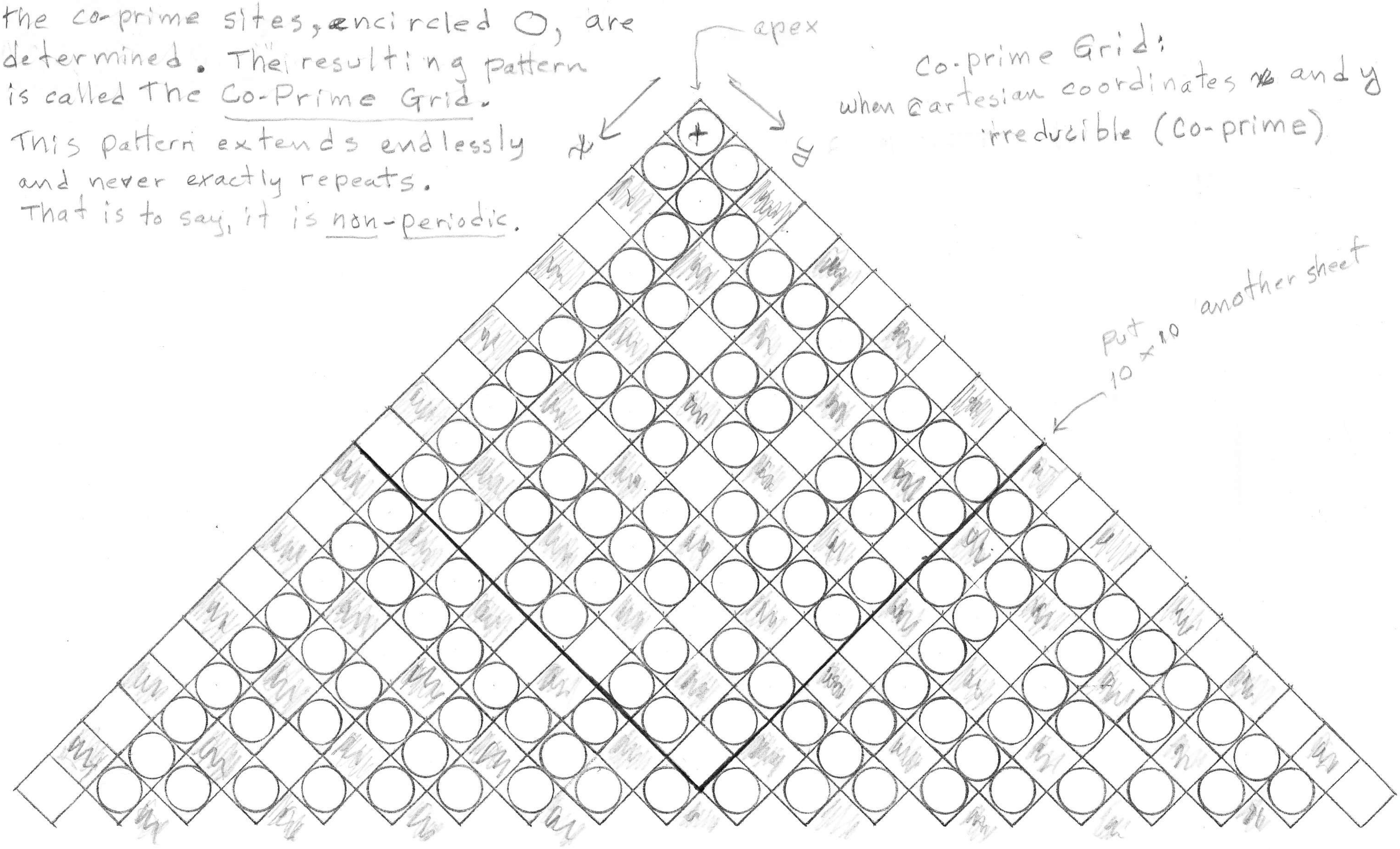

ROW 0
ROW 1
ROW 2
ROW 3
ROW 4
ROW 5
ROW 6
ROW 7

when (from the reference point at ~~the~~ Apex ⊕, x and y are irreducible (Co-prime) the co-prime sites, encircled ○, are determined. The resulting pattern is called The Co-Prime Grid.

This pattern extends endlessly and never exactly repeats. That is to say, it is non-periodic.

Scratched

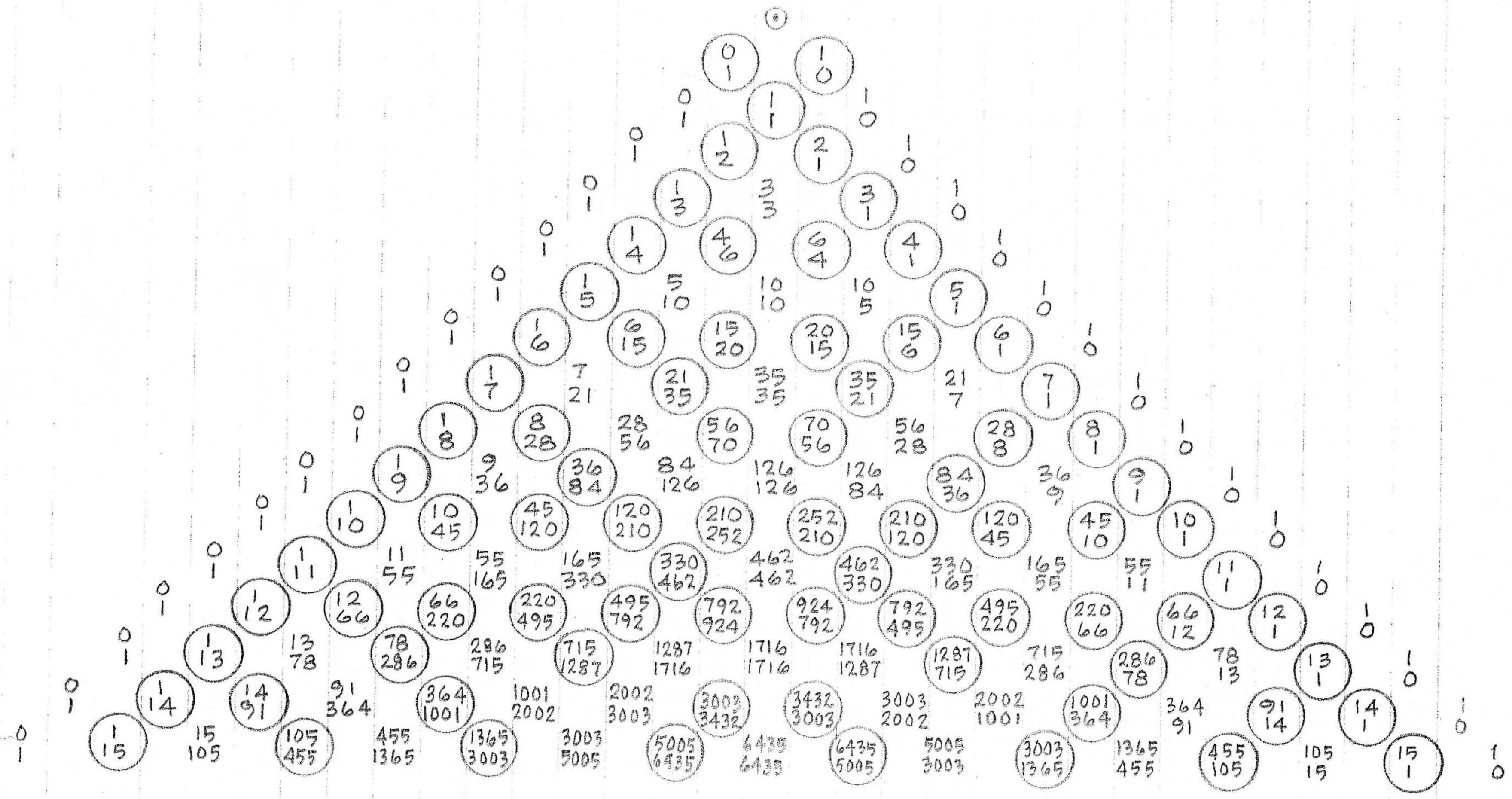
Co-prime Grid: when Cartesian coordinates x and y irreducible (Co-prime)



30 Jun 06 · EW

Archtypal Triangle $\left\{\frac{0}{1}, \frac{1}{0}\right\}$

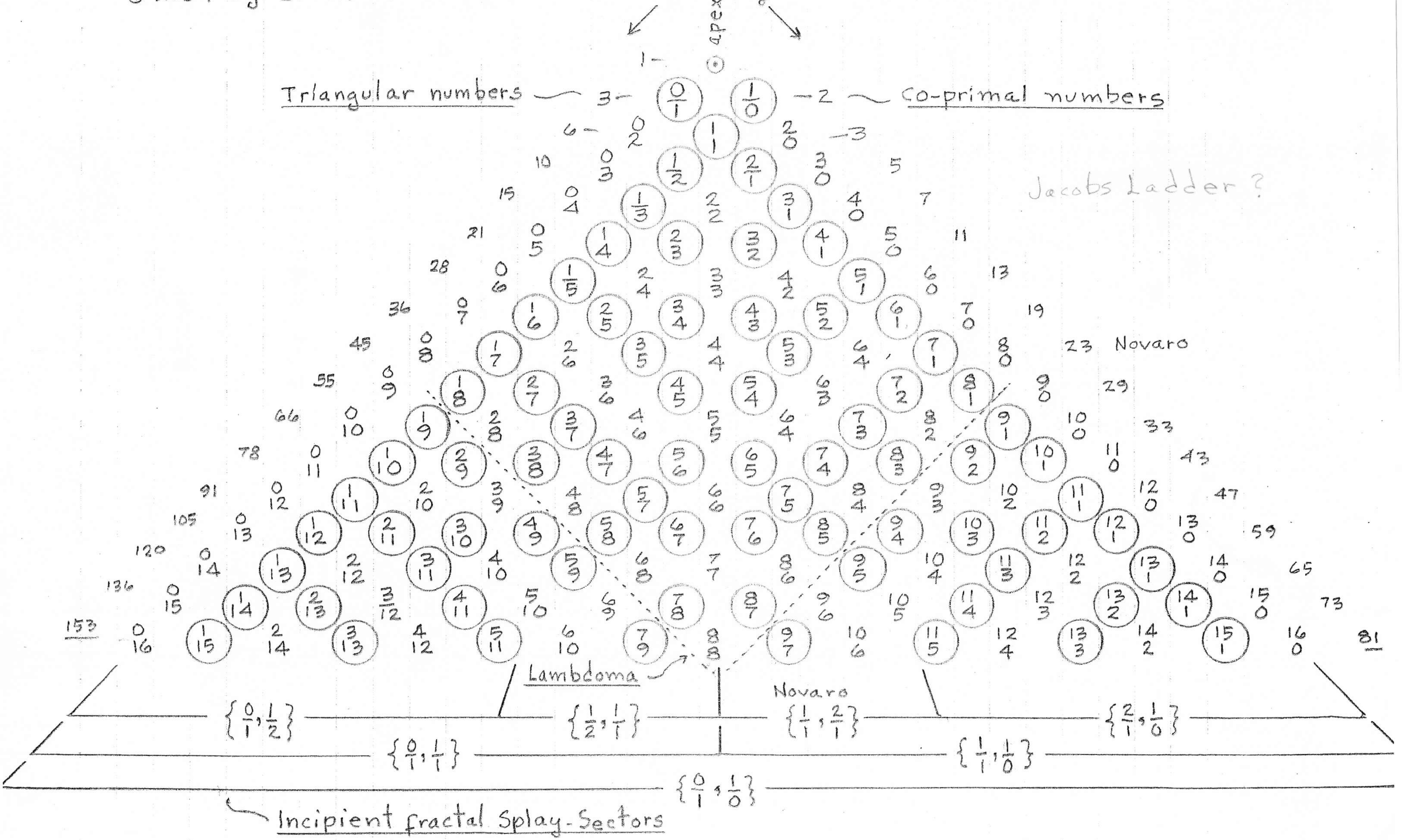
© 2004 by Ervin M. Wilson



Erv's Copy

Lambda $\left\{\frac{0}{1}, \frac{1}{0}\right\}$, shewing Co-Prime Grid ○

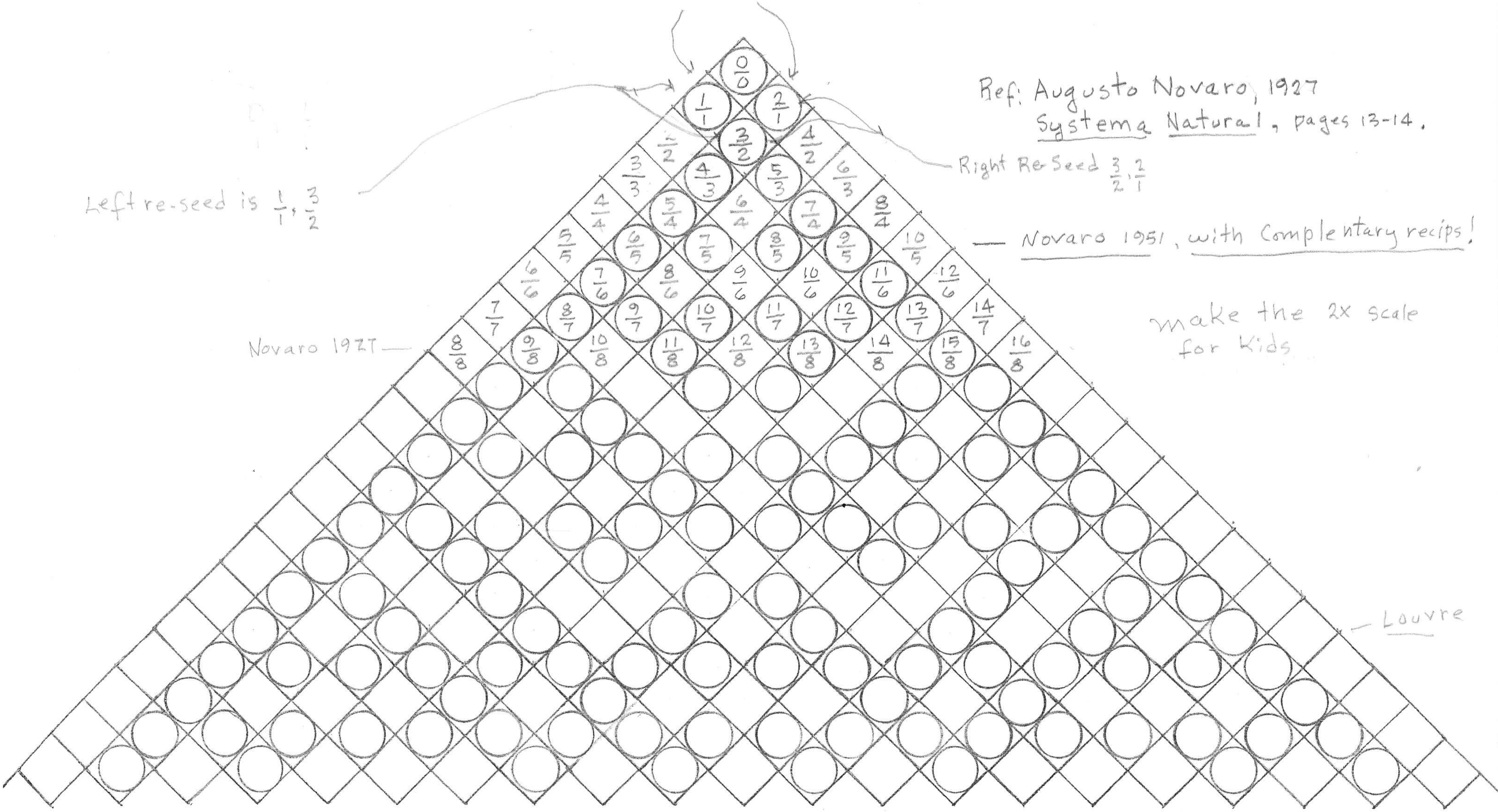
This defines the co-prime grid on a Cartesian x, y coordinate system



Triangularis, Delta

seed $\left(\frac{1}{1}, \frac{2}{1}\right)$

Ref: Augusto Novaro, 1927
<u>Systema</u> <u>Natural</u>, pages 13-14.



Left re-seed is $\frac{1}{1}$, $\frac{3}{2}$

Right Re-Seed $\frac{3}{2}$, $\frac{2}{1}$

— <u>Novaro 1951</u>, <u>with Complentary recips!</u>

make the 2x scale for kids

Novaro 1927—

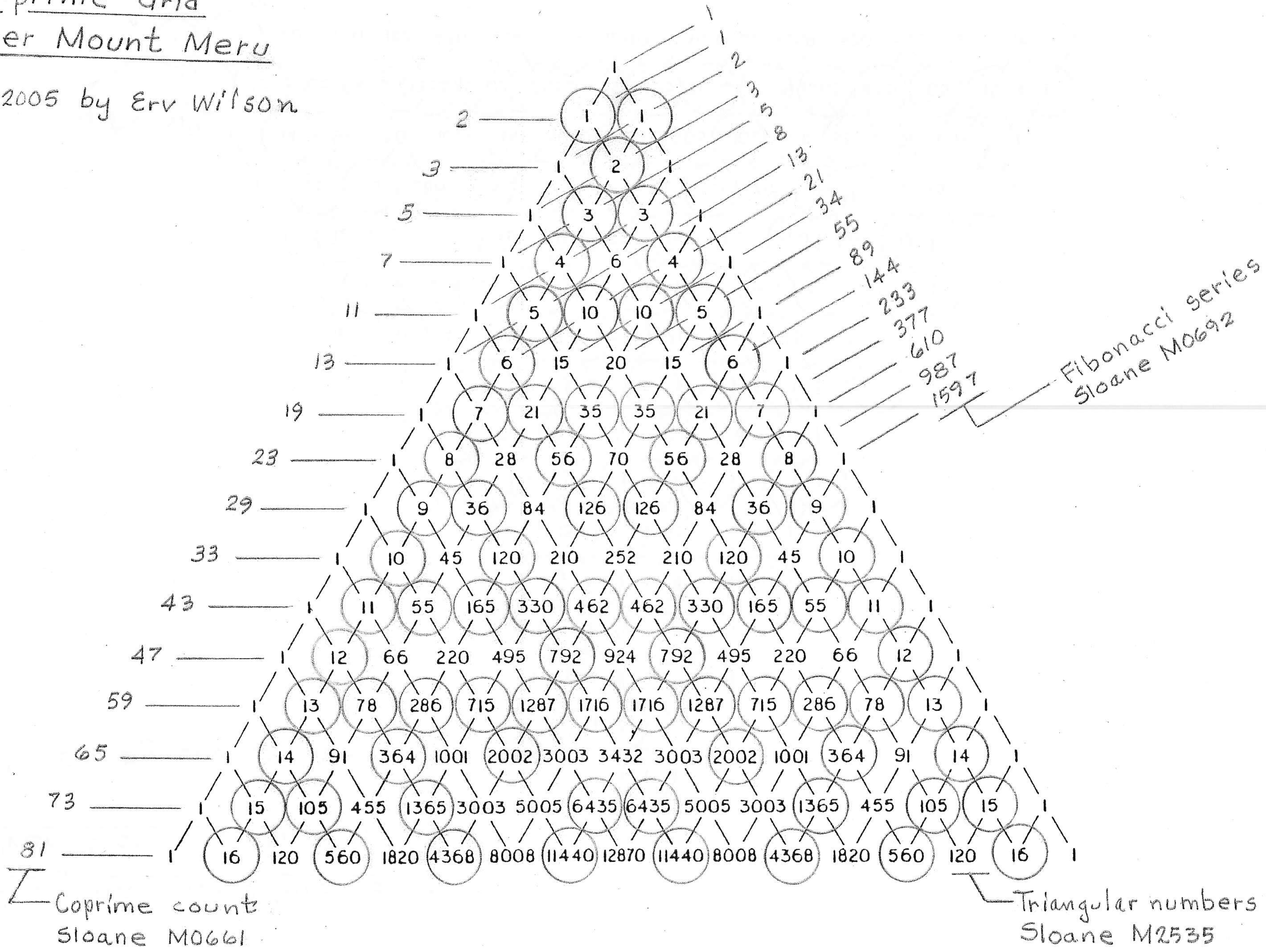
— <u>Louvre</u>

<u>Augusto Novaro's Triangle</u>, (as interpreted by Ervin M. Wilson)

© 20 Sep 06. EW

30 Jun 06·EW

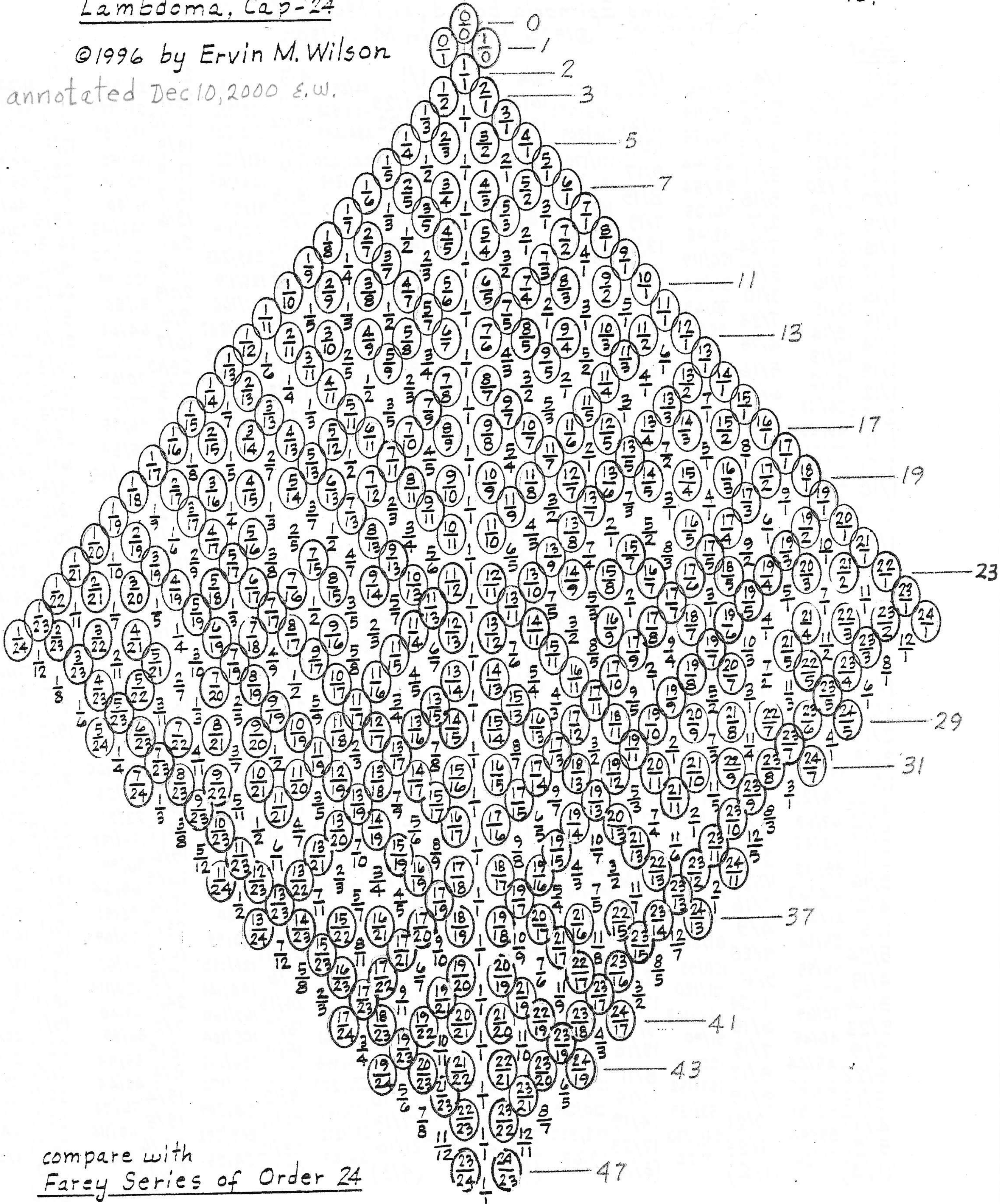
Co-prime Grid Over Mount Meru



Wilson

Lambdoma, Cap-24

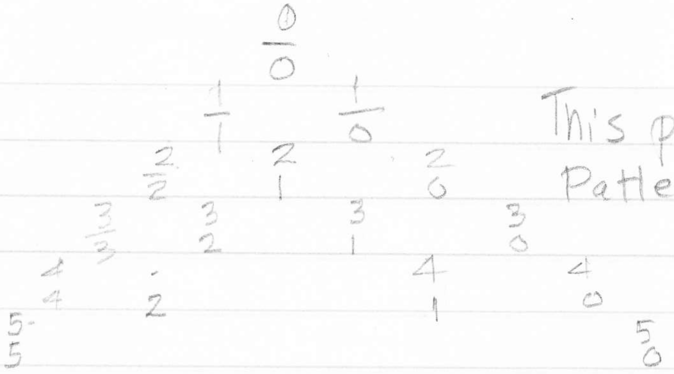
annotated Dec 10, 2000 E.W.



compare with
Farey Series of Order 24

Excerpt from So-Called Farey Series, extended 0/1 to 1/0 (Full Set of Gear Ratios), and Lambdoma, by Erv Wilson, 1996

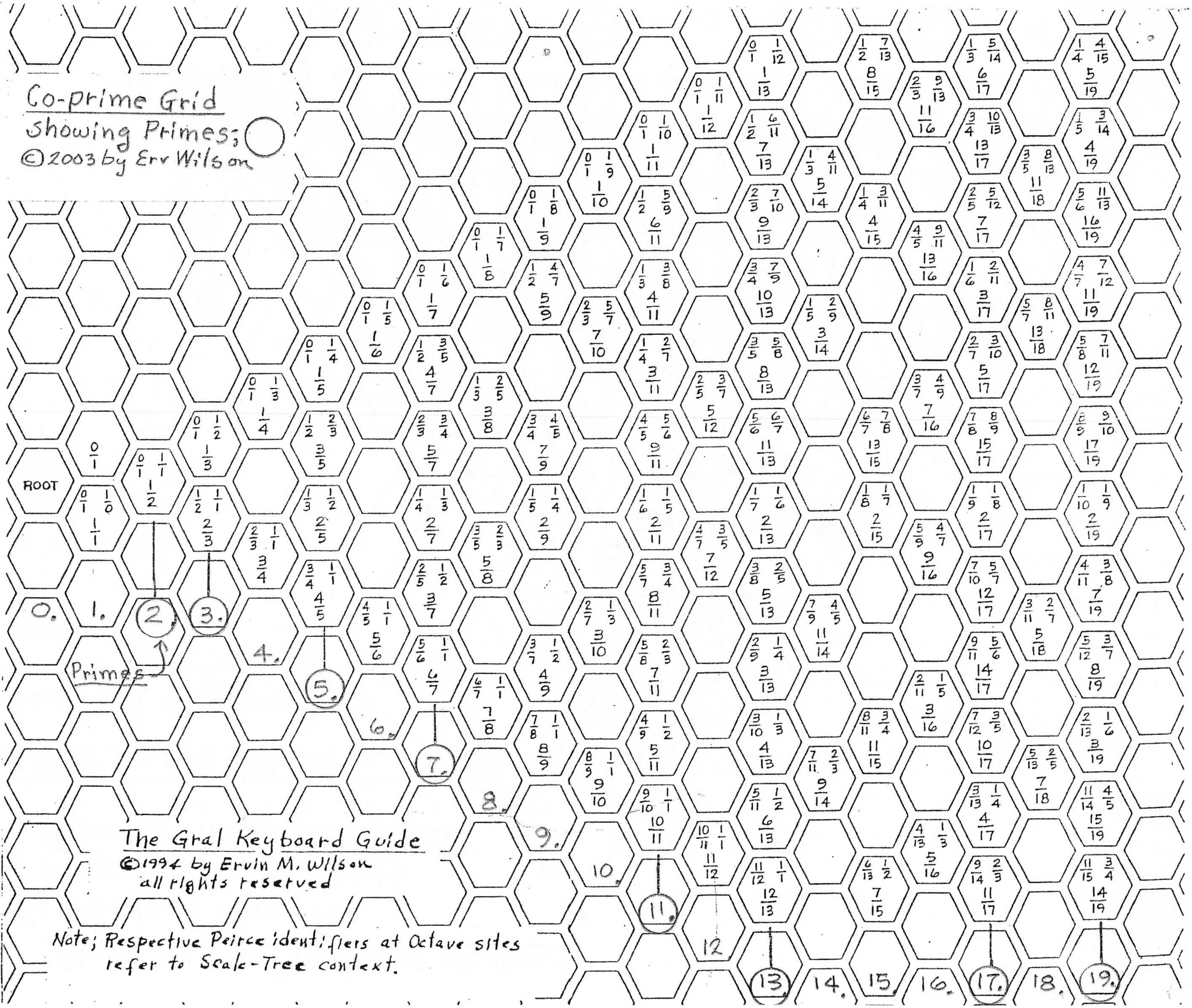
13DEC00



This proves it is a fractal Pattern

$\boxed{\frac{0}{0}}$	$\boxed{\frac{1}{0}}$	$\frac{2}{0}$	$\frac{3}{0}$	$\frac{4}{0}$	$\frac{5}{0}$	$\frac{6}{0}$	$\frac{7}{0}$
$\boxed{\frac{1}{1}}$	$\boxed{\frac{2}{1}}$	$\boxed{\frac{3}{1}}$	$\boxed{\frac{4}{1}}$	$\boxed{\frac{5}{1}}$	$\boxed{\frac{6}{1}}$	$\boxed{\frac{7}{1}}$	$\boxed{\frac{8}{1}}$
$\frac{2}{2}$	$\boxed{\frac{3}{2}}$	$\frac{4}{2}$	$\boxed{\frac{5}{2}}$	$\frac{6}{2}$	$\boxed{\frac{7}{2}}$	$\frac{8}{2}$	$\boxed{\frac{9}{2}}$
$\frac{3}{3}$	$\boxed{\frac{4}{3}}$	$\boxed{\frac{5}{3}}$	$\frac{6}{3}$	$\boxed{\frac{7}{3}}$	$\boxed{\frac{8}{3}}$	$\frac{9}{3}$	$\boxed{\frac{10}{3}}$
$\frac{4}{4}$	$\boxed{\frac{5}{4}}$	$\frac{6}{4}$	$\boxed{\frac{7}{4}}$	$\frac{8}{4}$	$\boxed{\frac{9}{4}}$	$\frac{10}{4}$	$\boxed{\frac{11}{4}}$
$\frac{5}{5}$	$\boxed{\frac{6}{5}}$	$\boxed{\frac{7}{5}}$	$\boxed{\frac{8}{5}}$	$\boxed{\frac{9}{5}}$	$\frac{10}{5}$	$\boxed{\frac{11}{5}}$	$\boxed{\frac{12}{5}}$
$\frac{6}{6}$	$\boxed{\frac{7}{6}}$	$\frac{8}{6}$	$\frac{9}{6}$	$\frac{10}{6}$	$\boxed{\frac{11}{6}}$	$\frac{12}{6}$	$\boxed{\frac{13}{6}}$
$\frac{7}{7}$	$\boxed{\frac{8}{7}}$	$\boxed{\frac{9}{7}}$	$\boxed{\frac{10}{7}}$	$\boxed{\frac{11}{7}}$	$\boxed{\frac{12}{7}}$	$\boxed{\frac{13}{7}}$	$\frac{14}{7}$

Co-prime Grid showing Primes; ○ ©2003 by Erv Wilson



Ref. "Jumping Champions", Ian Stewart, Scientific American Dec 2000.
"Prime Pursuit", Ivars Peterson, Science News Oct 26, 2002.

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Note; Respective Peirce identifiers at Octave sites refer to Scale-Tree context.