THE SEPTENDENE

as derived by relative rotations of the $[\frac{9}{8}, \frac{10}{9}, \frac{16}{15}]$ tetrachord

© 1977 by Eru Wilson
Foreshortened Bosanquet Keyboard to $\frac{1}{2}$ its original depth, showing the Partch ratios issued by Ery Wilson 1977
C. Ivor Darley

Dear John,

A couple of interesting examples of the micro-comma \(3^{5/8}(\approx 0.001628)\), first in fusion, and second in articulation (at least theoretically).

\[
\frac{3}{2} = 10 \text{ units} \\
\frac{5}{4} = 5 \text{ units}
\]

17-Tone constant structure by fusion of micro-comma, \(3^{5/8}(\approx 0.001628)\)

This may also be viewed as a chain of alternating Major & minor 3rds, beginning and ending with Major 3rd, fusion of 1st and last times (\(3^{5/8}\))

Parallel 22-tone constant structure

The comma \(3^{5/8}\) appears 5 places

In a musical circumstance the \(3^{5/8}\) occurs most readily as the difference between the Chromatic Semitone \(135/128(\approx 0.076816)\) and the Pythagorean Semitone \(256/243(\approx 0.075187)\), \((=, 001629)\) done this way

A system taking the \(3^{5/8}\) (approx) as its smallest unit might be appropriate, and a continuation of the idea of using the comma as the smallest unit.
A division of 612 has a unit of 0.001634. The 256/243 = 46 units, 135/128 = 47 units, 81/80 = 11 units yielding a whole-tone with 104 units. Which is nice. But the system gives efficient 7's & 11's as well. From your tables:

\[
\begin{align*}
3/2 &= 357.997 + 0.003 \\
5/4 &= 197.020 - 0.020 \\
7/4 &= 494.101 - 0.101 \\
9/8 &= 103.994 + 0.006 \\
11/8 &= 281.172 - 0.172 \\
\end{align*}
\]

The distribution of efficiency is interesting because 3 is the most efficient and 5, 7, 9, 11, (& 13) become less efficient in that order, (which is probably the way it should be.)

The number 612 factors into \(2^2 \times 3 \times 17\). This makes it useful as a reference system. The 12-tone system occurs in degrees:

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Divisions of 2, 3, 6, 9, 18, 36, 54, 51, 66, 153, 102, 306, 17 — what have I missed? The 17-tone degree is 36 units. We have an interesting comparison of Fifths:

- 12T: 357 units
- "Perfect": 358
- 17T: 360

It is a strange coincidence that the difference between the Perfect Fifth (584962) and the 12T Fifth (583333) is 001629, which is identical to the 385 microcomma. (The last place is questionable.)

Harry mentions the notable cycle 306, which, however, neglects to give us efficient 5's & 11's.
The 612 Tone Fifth is \(0.5849673\)
The Perfect Fifth \(0.5849625\)
difference \(0.0000048\)

The defects of the Eikosary (13.5, 7.9, 11) in 612 range from +.009 for 13.9 to
-1.293 for 5.7.11. (13.9 is the only member with a positive defect.) The center of this defect
range is -1.142, and it would be desirable for the base member to have approx that defect.
(440) 11.5 in has a defect of +1.122, and (264) 13.411 has a defect of -0.149 which is only a little
nearer the center. In the event of a table of
pitches to 612 my preference would be a base
of 264.

What would such a set do besides give us
a slithering glissando of 12-tone scales? (51
to be exact!) Well, it would also give us a
slithery gliss of of 36 17-tone scales! Its usefulness
however, does not lie in the area of equal division.
What we would get is a highly accurate set
of 11-limit pitches in a full spectrum of keys.
About the worst thing that could happen would be
superimposing the 11/8 6 times with a falsity
at the last place about equal to that of the
12T Fifth. (6 \times -1.172 = -1.032 units). A highly
workable circumstance, 51 pitches to a sheet, 12 sheets,
easily reproduced & utilized. A set of factorized, just
equivalents might be listed parallel. That's a tricky
job. Notation & Keyboards? Clue: 612 is twice
the Pythagorean cycle 306.
The comma $3^{4/5}$ divides into 11 parts. Flattening the Fifth by one part $1/11$ comma gives 12-tone equal temperament. By $2/11$ comma we have the temp. between $1/5$ & $1/4$ comma:

<table>
<thead>
<tr>
<th>F</th>
<th>C</th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
<th>B</th>
<th>F#</th>
</tr>
</thead>
</table>

Flattening by $3/11$ comma we have the temperament nearer the $1/4$ comma than the $1/3$ comma:

<table>
<thead>
<tr>
<th>F</th>
<th>C</th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3/11</td>
<td>0</td>
<td>-3/11</td>
<td>-6/11</td>
<td>-9/11</td>
<td>-13/11</td>
<td>-15/11</td>
</tr>
</tbody>
</table>

Sharpening by $2/11$ comma gives us the 17-tone Diat.

Sharpening by $1/11$ comma gives a Fifth a little over 29T.

Sharpening by $3/11$ comma is just under 22T. But gives a $5/4$ much too flat, (at the 9th Fifth) a skhism to be exact.

That surprising because he erred?

Sharpening by $4/11$ comma gives the $5/4$ 1 skhism ($4/11$ comma) sharp. Sharpening by $3^{8/9}$ skhism is indicated.

Further subdivision of 612 would give a series of skhismic temperaments (or temperaments, since it would be cyclic). The $1/8$ skhism tuning is close enough to $1/6$ that compromise could be made. Well its something to kick around.

Yours,

Erv