

# The Marwa Permutations

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Consider the 32 Raga-scales (Thats) of N. India (fig. 1a). These may be expressed as chains of Fourths, distinguishing between Augmented Fourth (4'), Normal Fourth (4), and Diminished Fourth (4), (fig. 1b). 3 classes of That are thus revealed, (fig. 1c) based on permutations of 3 basic chains of Fourths. In class A there are 6 Thats based on 4', 4, 4, 4, 4, 4, (4) (fig. 1d). In class B there are 20 Thats based on 4', 4, 4', 4, 4, 4, (4), (fig. 1e). In class C there are 6 Thats based on 4', 4, 4', 4, 4', 4, (4), (fig. 1f). All 6 of the Thats in class A are associated with popular ragas. 15 of the 20 Thats in class B are associated with popular ragas. None of the 6 Thats in class C is associated with a popular raga. Class A and Class B are the original source of the variations and permutations that follow;

The principal features of the basic chain (linear genus) may be ordered in several interesting ways by interpolation and by the selection of the Fourth which is to be "fixed", shown in parenthesis, ( ).

Example; 4', 4 4<sub>2</sub>' 4 4 4 (4)

4<sub>2</sub>' 4 4<sub>1</sub>' 4 4 4 (4)

4<sub>1</sub>' 4 4<sub>2</sub>' 4 4 4 (4)

4<sub>2</sub>' 4 4<sub>1</sub>' 4 4 4 (4)

also; 4' 4 4 4 4 4 (4)

4 4 4 4 4 4 (4')

and; 4' 4<sub>2</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> (4<sub>1</sub>)

4<sub>2</sub> 4' 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> (4<sub>1</sub>)

4<sub>2</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> (4<sub>1</sub>)

4' 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> 4<sub>1</sub> (4<sub>2</sub>)

A diversity of linear species<sup>(shown in ratios)</sup> may be associated with a linear genus. These are analogous intonational determinants; each would impart its own, unique color to the tuning of a given set of Thats.

Example;  $\frac{64}{45} \frac{4}{3} \frac{45}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{81}{64}\right)$  and;  $\frac{45}{32} \frac{4}{3} \frac{64}{45} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{81}{64}\right)$

$\frac{35}{24} \frac{4}{3} \frac{81}{56} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{6}{5}\right)$   $\frac{81}{56} \frac{4}{3} \frac{35}{24} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{6}{5}\right)$

$\frac{45}{32} \frac{4}{3} \frac{18}{13} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{13}{10}\right)$   $\frac{18}{13} \frac{4}{3} \frac{45}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{13}{10}\right)$

$\frac{63}{44} \frac{4}{3} \frac{11}{8} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{9}{7}\right)$   $\frac{11}{8} \frac{4}{3} \frac{63}{44} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{9}{7}\right)$

$\frac{36}{25} \frac{4}{3} \frac{45}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{5}{4}\right)$   $\frac{45}{32} \frac{4}{3} \frac{36}{25} \frac{4}{3} \frac{4}{3} \frac{4}{3} \left(\frac{5}{4}\right)$

|   |   |
|---|---|
| <p>Example; <math>\frac{4'}{8} \frac{4}{3} \frac{4_2}{20} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4')</math> and;</p> <p><math>\frac{4_2}{20} \frac{4}{3} \frac{4_2}{20} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4')</math></p> <p><math>\frac{4_2}{16} \frac{4'}{56} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math> and;</p> <p><math>\frac{4_1}{32} \frac{4}{64} \frac{4_2}{45} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> <p><math>\frac{4_1}{20} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4')</math></p> <p><math>\frac{4'}{512} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> | <p><math>\frac{4_2}{20} \frac{4}{3} \frac{4'}{8} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4')</math></p> <p><math>\frac{4'}{32} \frac{4}{3} \frac{4'}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> <p><math>\frac{4'}{45} \frac{4}{32} \frac{4_2}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> <p><math>\frac{4'}{64} \frac{4}{45} \frac{4_2}{32} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> <p><math>\frac{4_2}{44} \frac{4}{7} \frac{4_1}{8} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> <p><math>\frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} \frac{4}{3} (4)</math></p> |
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These linear species are derived from 7-tone tetrachordal scales. The figures appearing on the following pages are permutation sets of these linear species, and excerpted from work in progress.

I am indebted to Amiya Dasgupta for giving me a copy of his book Applied Theory of Indian Music (North) 1977 (California Institute of the Arts, Valencia). Errors in interpretation are mine.

I am also indebted to John Chalmers for sending me a copy of his unpublished book The Divisions of the Tetrachord.





Fig 4

|       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|
| 27/20 | 45/32 | 4/3   | 4/3   | 4/3   | 4/3   | (4/3) |
| 27/20 | 4/3   | 45/32 | 4/3   | 4/3   | 4/3   |       |
| 27/20 | 4/3   | 4/3   | 45/32 | 4/3   | 4/3   |       |
| 27/20 | 4/3   | 4/3   | 4/3   | 45/32 | 4/3   |       |
| 27/20 | 4/3   | 4/3   | 4/3   | 4/3   | 45/32 |       |
| 4/3   | 27/20 | 45/32 | 4/3   | 4/3   | 4/3   |       |
| 4/3   | 27/20 | 4/3   | 45/32 | 4/3   | 4/3   |       |
| 4/3   | 27/20 | 4/3   | 4/3   | 45/32 | 4/3   |       |
| 4/3   | 27/20 | 4/3   | 4/3   | 4/3   | 45/32 |       |
| 4/3   | 4/3   | 27/20 | 45/32 | 4/3   | 4/3   |       |
| 4/3   | 4/3   | 27/20 | 4/3   | 45/32 | 4/3   |       |
| 4/3   | 4/3   | 27/20 | 4/3   | 4/3   | 45/32 |       |
| 4/3   | 4/3   | 4/3   | 27/20 | 45/32 | 4/3   |       |
| 4/3   | 4/3   | 4/3   | 27/20 | 4/3   | 45/32 |       |
| 4/3   | 4/3   | 4/3   | 4/3   | 27/20 | 45/32 |       |

|         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 256/243 | 256/243 | 16/15   | 9/10    | 256/243 | 256/243 | 256/243 |
| 256/243 | 256/243 | 16/15   | 9/10    | 256/243 | 256/243 | 9/10    |
| 256/243 | 16/15   | 256/243 | 9/10    | 256/243 | 16/15   | 9/10    |
| 256/243 | 16/15   | 9/10    | 256/243 | 16/15   | 256/243 | 9/10    |
| 16/15   | 256/243 | 256/243 | 9/10    | 16/15   | 256/243 | 9/10    |
| 256/243 | 256/243 | 256/243 | 256/243 | 256/243 | 16/15   | 9/10    |
| 256/243 | 256/243 | 16/15   | 9/10    | 256/243 | 256/243 | 9/10    |
| 256/243 | 256/243 | 9/10    | 256/243 | 256/243 | 16/15   | 9/10    |
| 16/15   | 256/243 | 9/10    | 256/243 | 16/15   | 256/243 | 9/10    |
| 256/243 | 16/15   | 9/10    | 256/243 | 256/243 | 9/10    | 256/243 |
| 256/243 | 256/243 | 16/15   | 9/10    | 256/243 | 256/243 | 9/10    |
| 16/15   | 256/243 | 9/10    | 256/243 | 16/15   | 9/10    | 256/243 |
| 256/243 | 256/243 | 256/243 | 256/243 | 256/243 | 16/15   | 9/10    |
| 16/15   | 9/10    | 256/243 | 256/243 | 16/15   | 9/10    | 256/243 |
| 16/15   | 9/10    | 256/243 | 256/243 | 256/243 | 256/243 | 256/243 |

Shrinivas  
Kafi

(Didymus  $\frac{16}{15}$   $\frac{10}{9}$   $\frac{9}{8}$ )

Fig 5 (Pythagoras  $\frac{256}{243}$   $\frac{9}{8}$   $\frac{9}{8}$ )

|     |     |     |     |     |     |           |
|-----|-----|-----|-----|-----|-----|-----------|
| 4/3 | 4/3 | 4/3 | 4/3 | 4/3 | 4/3 | (729/512) |
|-----|-----|-----|-----|-----|-----|-----------|

|         |     |     |         |     |     |
|---------|-----|-----|---------|-----|-----|
| 256/243 | 9/8 | 9/8 | 256/243 | 9/8 | 9/8 |
|---------|-----|-----|---------|-----|-----|

△ Kafi      □ Bhairavi      □ Asawari

Fig 6 (Didymus/Ptolemy  $\frac{16}{15}$   $\frac{9}{8}$   $\frac{10}{9}$ )

|       |       |       |       |       |       |         |
|-------|-------|-------|-------|-------|-------|---------|
| 27/20 | 4/3   | 4/3   | 4/3   | 4/3   | 4/3   | (45/32) |
| 4/3   | 27/20 | 4/3   | 4/3   | 4/3   | 4/3   |         |
| 4/3   | 4/3   | 27/20 | 4/3   | 4/3   | 4/3   |         |
| 4/3   | 4/3   | 4/3   | 27/20 | 4/3   | 4/3   |         |
| 4/3   | 4/3   | 4/3   | 4/3   | 27/20 | 4/3   |         |
| 4/3   | 4/3   | 4/3   | 4/3   | 4/3   | 27/20 |         |

|         |      |      |         |      |      |      |
|---------|------|------|---------|------|------|------|
| 16/15   | 9/8  | 9/8  | 256/243 | 9/8  | 9/8  | 10/9 |
| 16/15   | 9/8  | 9/10 | 16/15   | 9/8  | 9/8  | 9/10 |
| 16/15   | 9/8  | 9/10 | 16/15   | 9/8  | 9/10 | 9/10 |
| 16/15   | 9/10 | 9/8  | 16/15   | 9/8  | 9/10 | 9/10 |
| 16/15   | 9/10 | 9/8  | 16/15   | 9/10 | 9/8  | 9/10 |
| 256/243 | 9/8  | 9/8  | 16/15   | 9/10 | 9/8  | 9/10 |

○ Khamaj      ● Bilawal      X Kalyan



# Figure 9

|     |                 |                 |                 |                 |                 |                 |               |               |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|---------------|---------------|
|     | 0               | -1              | -2              | -3              | -4              | -5              | -6            | 0             |
|     | S               | M               | N               | G               | D               | R               | P             | S             |
| 1.  | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$ | $\frac{4}{3}$ |
| 2.  | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 3.  | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 4.  | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 5.  | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 6.  | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 7.  | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 8.  | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 9.  | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 10. | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 11. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 12. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 13. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 14. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 15. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 16. | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 17. | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$   | $\frac{4}{3}$ |               |
| 18. | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{4}{3}$   | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 19. | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{4}{3}$   | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$ |               |
| 20. | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$ |               |

(Hawkins  $\frac{16}{15}$   $\frac{135}{128}$   $\frac{32}{27}$ )

|     |                   |                     |                     |                 |                   |                     |                     |   |   |   |                    |
|-----|-------------------|---------------------|---------------------|-----------------|-------------------|---------------------|---------------------|---|---|---|--------------------|
|     | 0                 | -5                  | -3                  | -1              | -6                | -4                  | -2                  | 0 |   |   |                    |
|     | S                 | T                   | R                   | T               | R                 | T                   | P                   | T | R | T | S                  |
| 1.  | $\frac{9}{8}$     | $\frac{9}{8}$       | $\frac{10}{9}$      | $\frac{16}{15}$ | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{4096}{3645}$ |   |   |   | Champakali         |
| 2.  | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{4096}{3645}$ |   |   |   |                    |
| 3.  | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{9}{8}$       | $\frac{4096}{3645}$ |   |   |   |                    |
| 4.  | $\frac{135}{128}$ | $\frac{9}{8}$       | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{9}{8}$       | $\frac{4096}{3645}$ |   |   |   |                    |
| 5.  | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{9}{8}$     | $\frac{10}{9}$      | $\frac{16}{15}$     |   |   |   | Madhubanti         |
| 6.  | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   |                    |
| 7.  | $\frac{135}{128}$ | $\frac{9}{8}$       | $\frac{32}{27}$     | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Todi<br>• Lalit    |
| 8.  | $\frac{9}{8}$     | $\frac{10}{9}$      | $\frac{9}{8}$       | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   |                    |
| 9.  | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{9}{8}$       | $\frac{16}{15}$ | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Purvi<br>• Lalit 2 |
| 10. | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{9}{8}$       | $\frac{16}{15}$ | $\frac{10}{9}$    | $\frac{9}{8}$       | $\frac{16}{15}$     |   |   |   | Marwa              |
| 11. | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{4096}{3645}$ | $\frac{9}{8}$   | $\frac{9}{8}$     | $\frac{10}{9}$      | $\frac{16}{15}$     |   |   |   | Patdeep            |
| 12. | $\frac{9}{8}$     | $\frac{135}{128}$   | $\frac{4096}{3645}$ | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Chandra Kenada     |
| 13. | $\frac{135}{128}$ | $\frac{9}{8}$       | $\frac{4096}{3645}$ | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Jogiya Todi        |
| 14. | $\frac{9}{8}$     | $\frac{10}{9}$      | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Nat Bhairav        |
| 15. | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     |   |   |   | Bhairav            |
| 16. | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{10}{9}$    | $\frac{9}{8}$       | $\frac{16}{15}$     |   |   |   | Anand Bhairav      |
| 17. | $\frac{9}{8}$     | $\frac{10}{9}$      | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{4096}{3645}$ | $\frac{9}{8}$       |   |   |   | Mohan Kauns        |
| 18. | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{135}{128}$ | $\frac{4096}{3645}$ | $\frac{9}{8}$       |   |   |   | Basant mukhari     |
| 19. | $\frac{135}{128}$ | $\frac{32}{27}$     | $\frac{16}{15}$     | $\frac{9}{8}$   | $\frac{10}{9}$    | $\frac{16}{15}$     | $\frac{9}{8}$       |   |   |   | Ahir Bhairav       |
| 20. | $\frac{135}{128}$ | $\frac{4096}{3645}$ | $\frac{9}{8}$       | $\frac{9}{8}$   | $\frac{10}{9}$    | $\frac{16}{15}$     | $\frac{9}{8}$       |   |   |   | Parameswari        |

$\frac{135}{128}$  is close to  $\frac{256}{243}$

$\frac{4096}{3645}$  is close to  $\frac{9}{8}$

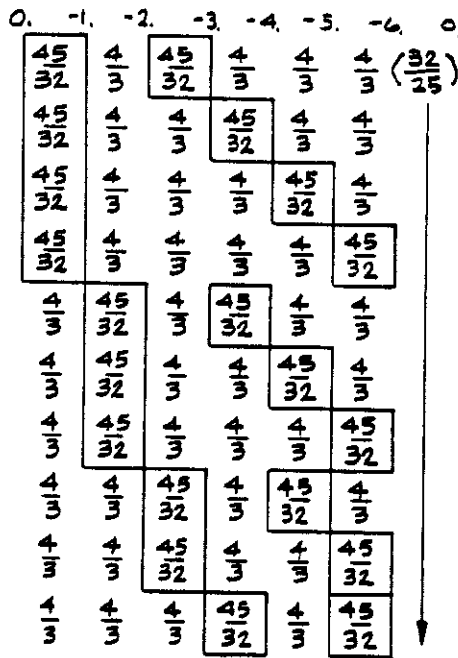
Figure 10

|     |                |                |                 |                 |                 |                 |                 |                 |                 |                  |                  |                 |                 |                  |                  |   |
|-----|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|------------------|-----------------|-----------------|------------------|------------------|---|
|     | 0              | -1             | -2              | -3              | -4              | -5              | -6              | 0               | 0               | -5               | -3               | -1              | -6              | -4               | -2               | 0 |
| 1.  | $\frac{11}{8}$ | $\frac{9}{7}$  | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $(\frac{4}{3})$ | $\frac{9}{8}$   | $\frac{9}{8}$    | $\frac{88}{81}$  | $\frac{12}{11}$ | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{112}{99}$ |   |
| 2.  | $\frac{11}{8}$ | $\frac{9}{7}$  | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{112}{99}$ |   |
| 3.  | $\frac{11}{8}$ | $\frac{9}{7}$  | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{9}{8}$    | $\frac{112}{99}$ |   |
| 4.  | $\frac{11}{8}$ | $\frac{9}{7}$  | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{63}{44}$ |                 | $\frac{22}{21}$ | $\frac{9}{8}$    | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{9}{8}$    | $\frac{112}{99}$ |   |
| 5.  | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{9}{8}$   | $\frac{88}{81}$  | $\frac{12}{11}$  |   |
| 6.  | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 7.  | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{9}{8}$    | $\frac{7}{6}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 8.  | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{88}{81}$  | $\frac{9}{8}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 9.  | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{9}{8}$    | $\frac{12}{11}$ | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 10. | $\frac{11}{8}$ | $\frac{4}{3}$  | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{9}{8}$    | $\frac{12}{11}$ | $\frac{88}{81}$ | $\frac{9}{8}$    | $\frac{12}{11}$  |   |
| 11. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{112}{99}$ | $\frac{9}{8}$   | $\frac{9}{8}$   | $\frac{88}{81}$  | $\frac{12}{11}$  |   |
| 12. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{22}{21}$  | $\frac{112}{99}$ | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 13. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{9}{8}$    | $\frac{112}{99}$ | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 14. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{88}{81}$  | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 15. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  |   |
| 16. | $\frac{4}{3}$  | $\frac{11}{8}$ | $\frac{4}{3}$   | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{88}{81}$ | $\frac{9}{8}$    | $\frac{12}{11}$  |   |
| 17. | $\frac{4}{3}$  | $\frac{4}{3}$  | $\frac{11}{8}$  | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   | $\frac{4}{3}$   |                 | $\frac{9}{8}$   | $\frac{88}{81}$  | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{112}{99}$ | $\frac{9}{8}$    |   |
| 18. | $\frac{4}{3}$  | $\frac{4}{3}$  | $\frac{11}{8}$  | $\frac{9}{7}$   | $\frac{4}{3}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{22}{21}$ | $\frac{112}{99}$ | $\frac{9}{8}$    |   |
| 19. | $\frac{4}{3}$  | $\frac{4}{3}$  | $\frac{11}{8}$  | $\frac{4}{3}$   | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{7}{6}$    | $\frac{12}{11}$  | $\frac{9}{8}$   | $\frac{88}{81}$ | $\frac{12}{11}$  | $\frac{9}{8}$    |   |
| 20. | $\frac{4}{3}$  | $\frac{4}{3}$  | $\frac{4}{3}$   | $\frac{11}{8}$  | $\frac{9}{7}$   | $\frac{63}{44}$ | $\frac{4}{3}$   |                 | $\frac{22}{21}$ | $\frac{112}{99}$ | $\frac{9}{8}$    | $\frac{9}{8}$   | $\frac{88}{81}$ | $\frac{12}{11}$  | $\frac{9}{8}$    |   |
|     |                |                |                 |                 |                 |                 |                 |                 | 0               | -5               | -3               | -1              | -6              | -4               | -2               | 0 |

(Ptolemy  $\frac{7}{6}$   $\frac{12}{11}$   $\frac{22}{21}$ )



Figure 12



(Helmholtz  $\frac{16}{15}$   $\frac{16}{15}$   $\frac{75}{64}$ )

17-Tone, Comma-Linna scale of Persia

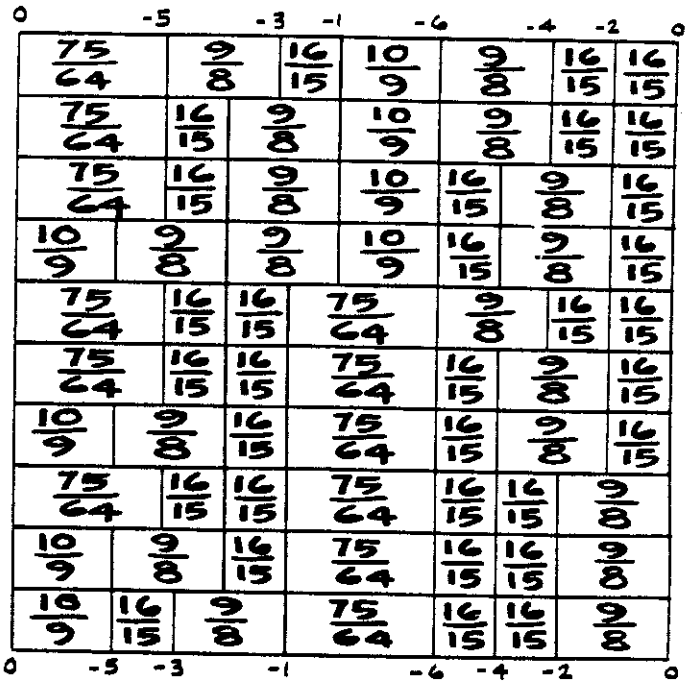
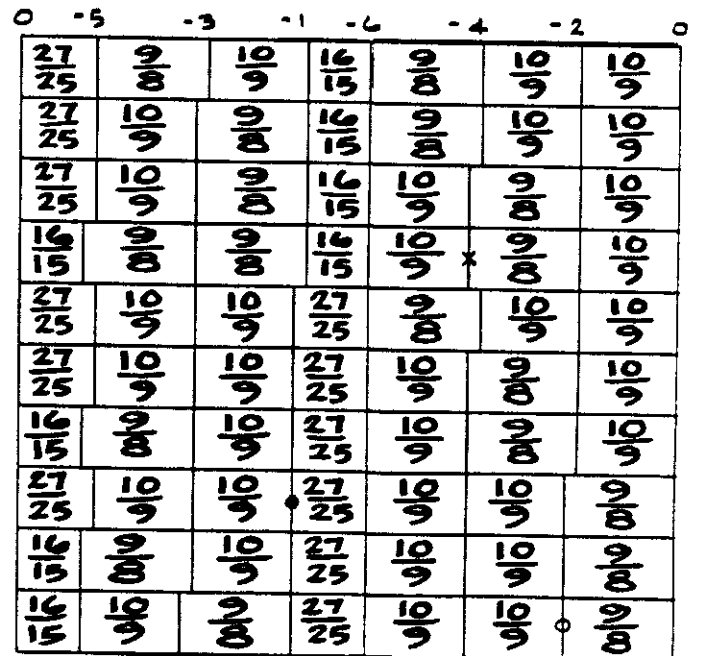
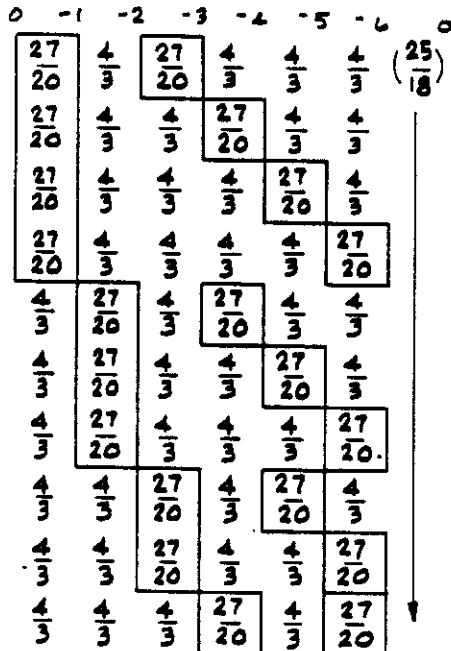


Figure 13

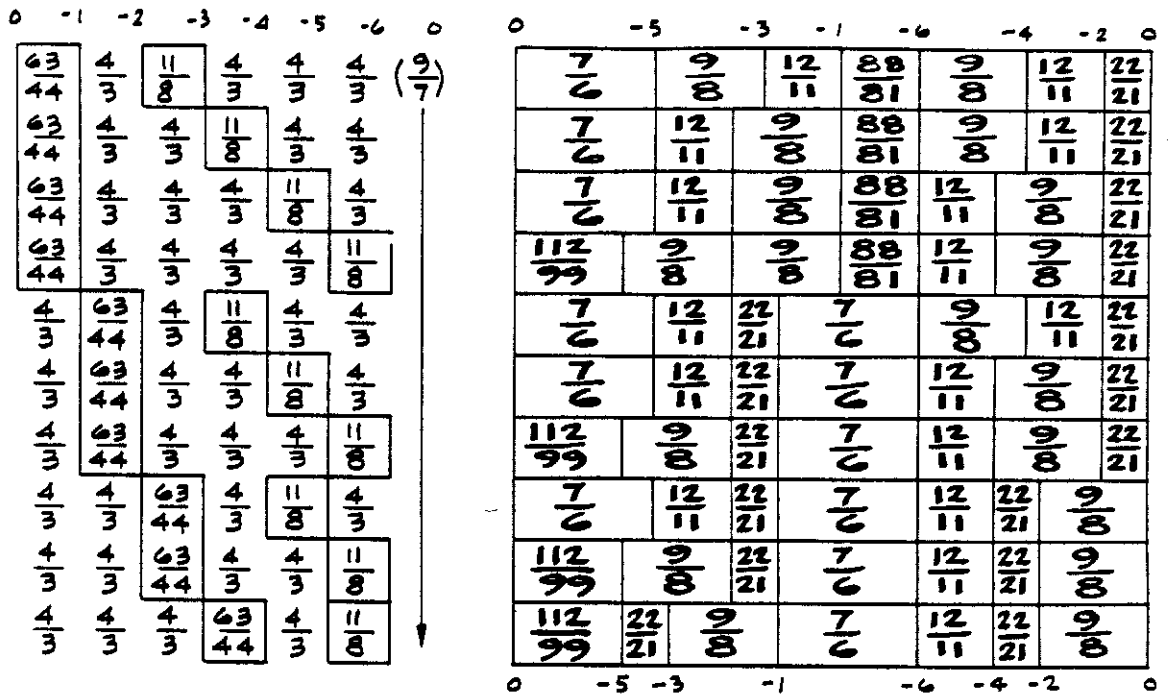
(Al-Farabi  $\frac{10}{9}$   $\frac{10}{9}$   $\frac{27}{25}$ )



x Khamaj, Shrinivas  
 ● Bhairavi, Shrinivas  
 ○ Asawari, Shrinivas

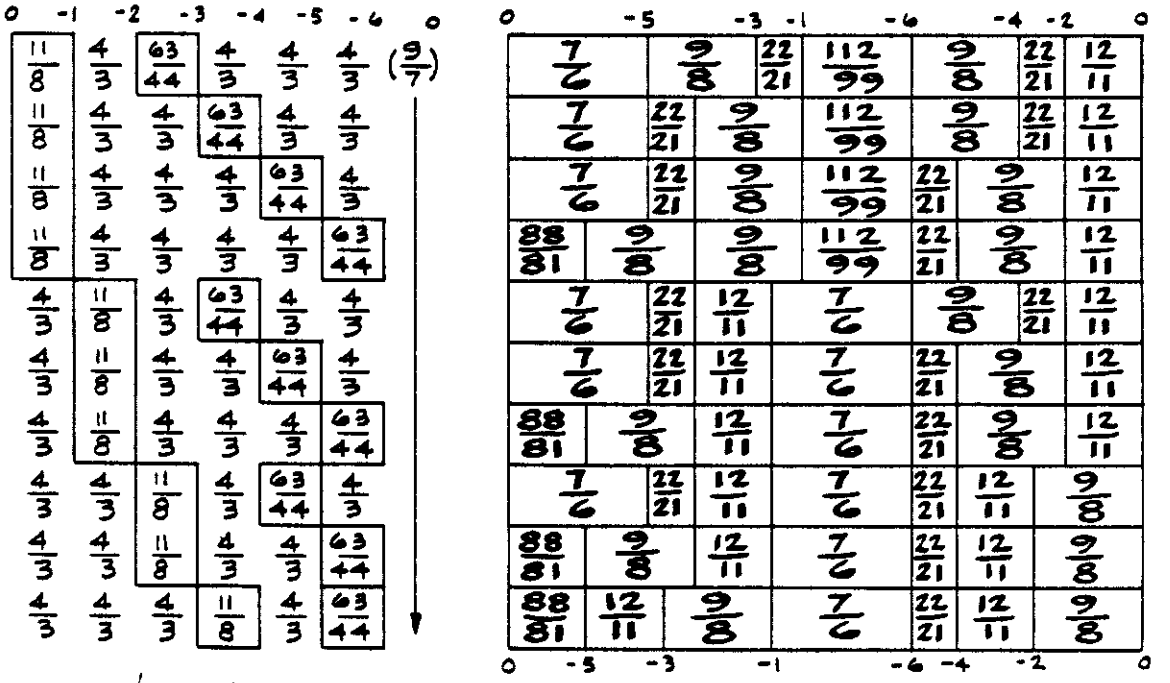


Figure 15a



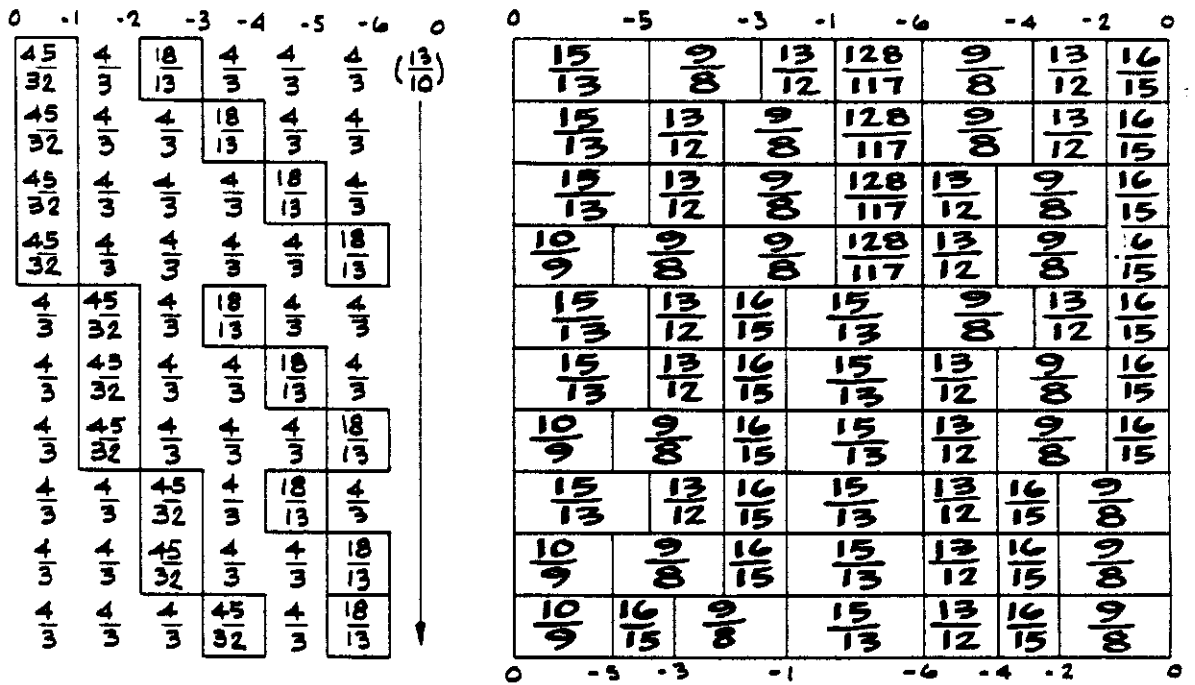
(Ptolemy  $\frac{12}{11} \frac{22}{21} \frac{7}{6}$ )

Figure 15b



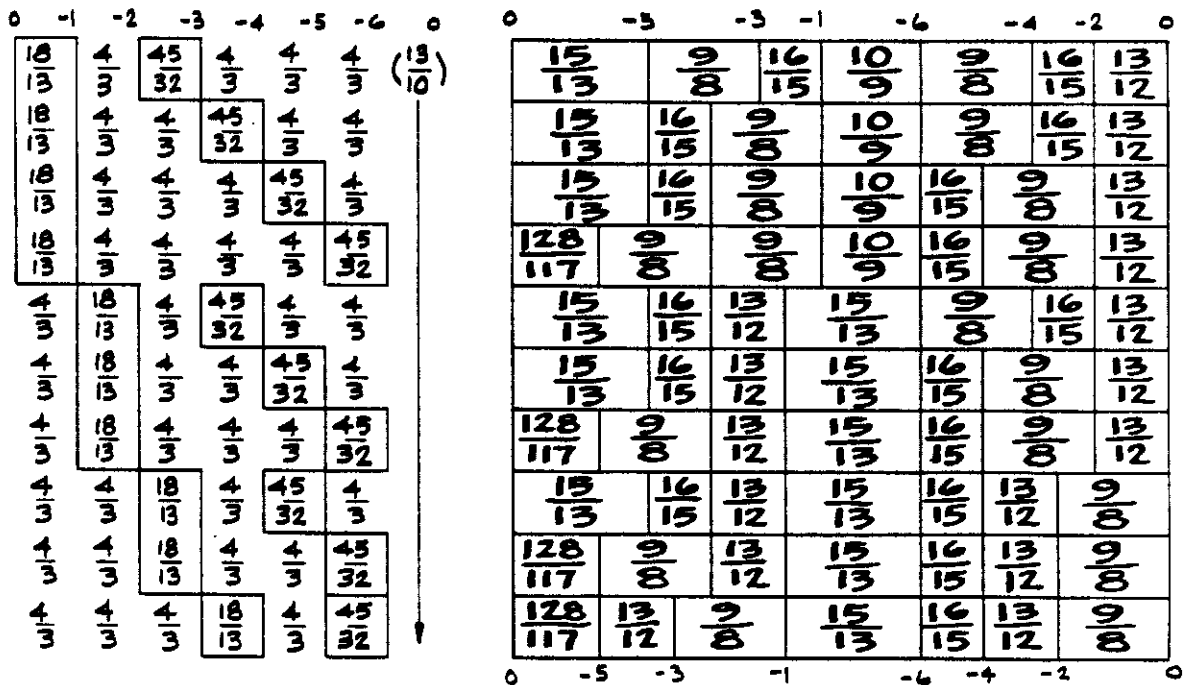
(Ptolemy  $\frac{22}{21} \frac{12}{11} \frac{7}{6}$ )

Figure 16a



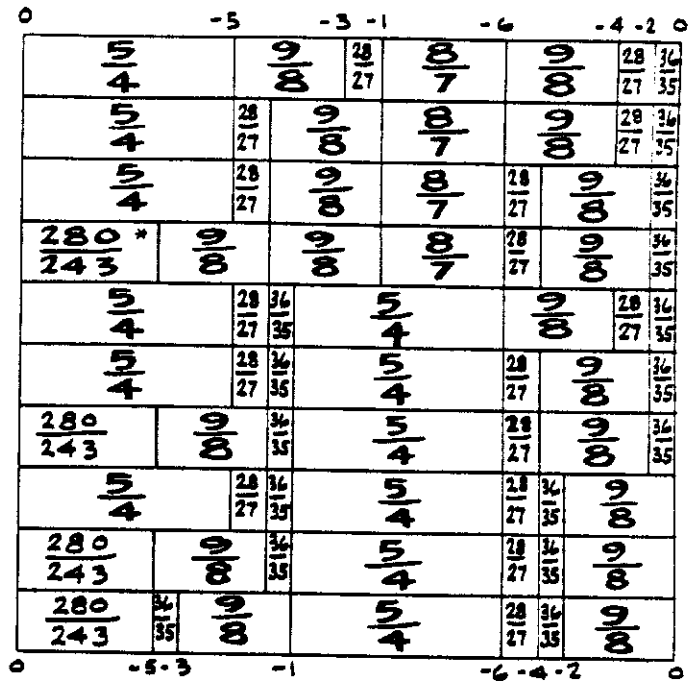
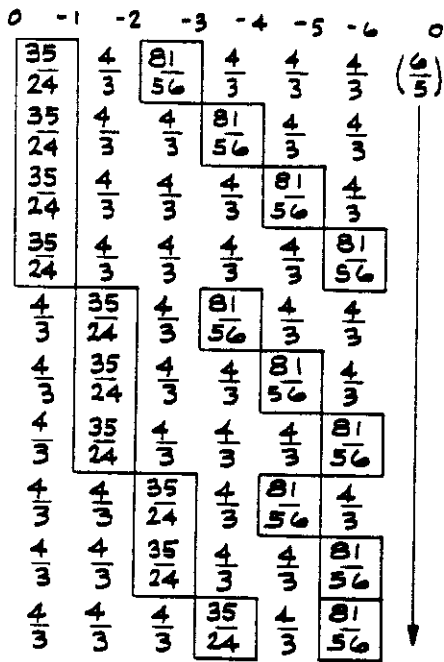
(Schlesinger  $\frac{16}{15}$   $\frac{15}{13}$   $\frac{13}{12}$ )

Figure 16b



(Schlesinger  $\frac{13}{12}$   $\frac{15}{13}$   $\frac{16}{15}$ )

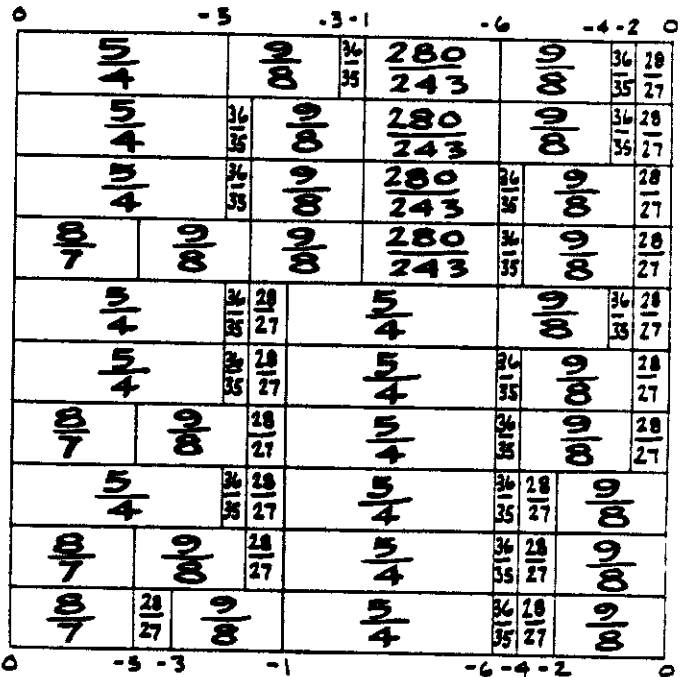
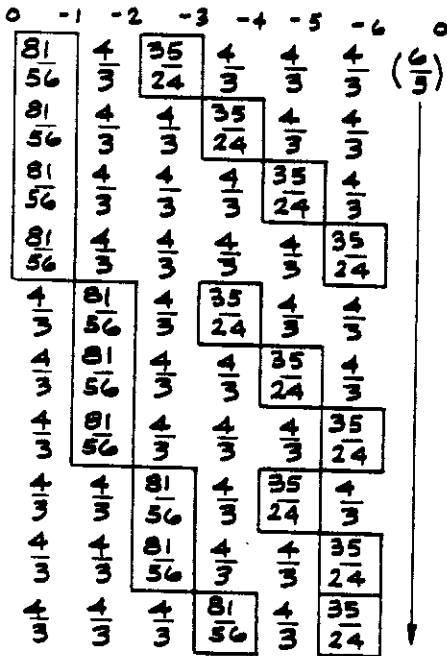
Fig. 17a



\* about 15/13

(Archytas  $\frac{28}{27} \frac{36}{35} \frac{5}{4}$ )

Figure 17 b



(Archytas  $\frac{36}{35} \frac{28}{27} \frac{5}{4}$ )

