Compliments of
Paul C. Boomsliter

INTERIM REPORT

ON THE PROJECT ON

ORGANIZATION IN AUDITORY PERCEPTION

AT THE

STATE UNIVERSITY COLLEGE

ALBANY, NEW YORK

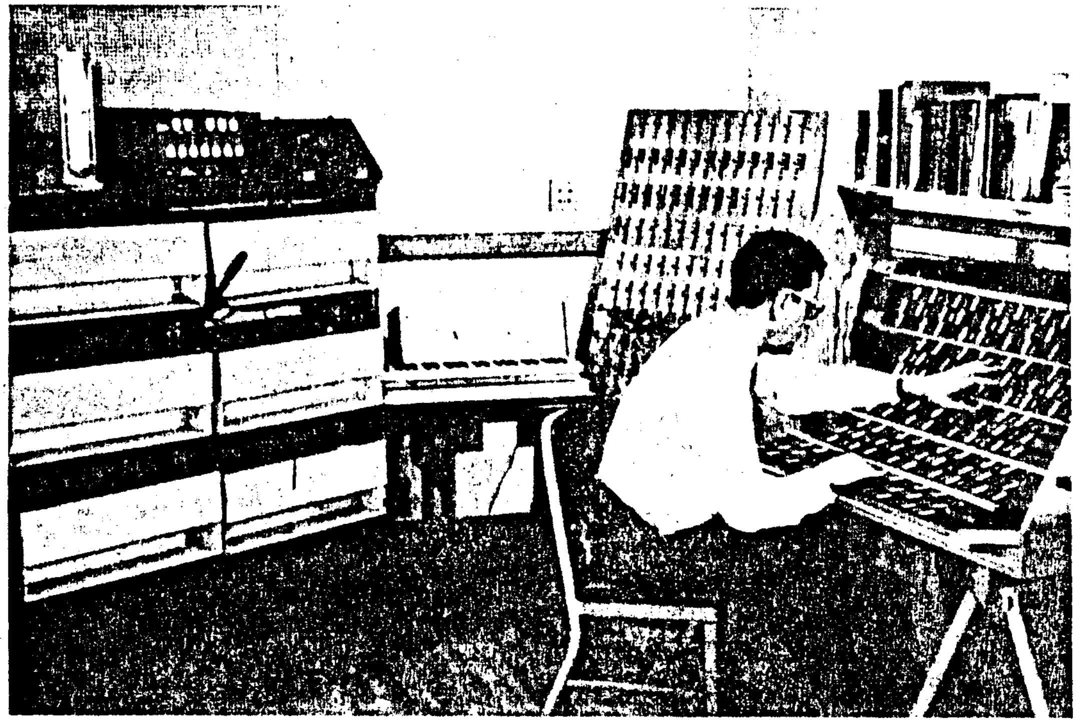
WITH RECORDED ILLUSTRATIONS

PAUL C. BOOMSLITER

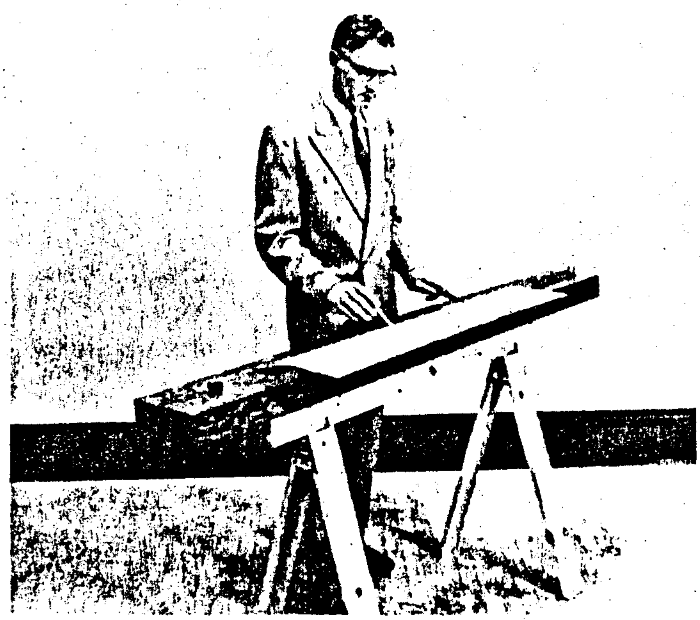
and

WARREN CREEL

FEBRUARY, 1962



Paul Bloomsliter at the Referential Organ, 1961



Warren Creel with the Modified Monochord, 1958

The Referential Organ

The six resonator boxes on shelves at the left contain six three-octave Estey reed organ actions; each reed is opened for sounding by a solenoid, keyed electrically. The blower motor is behind the actions. The U-shaped instrument atop the actions is a manometer, for measuring air suction. The two cases beside it contain a Stroboconn, a stroboscopic instrument for measuring musical frequencies, accurate to one cent, which is used in tuning the reeds. In the center is a conventional Estey reed organ action, with reeds tuned to equal temperament. A connection board stands in the corner.

Dr. Boomsliter is seated at the keyboard, each tab of which is an electrical switch connected with a solenoid. This view shows the five levels of the present keyboard, each level containing three rows of key tabs. The bottom level can slide into three different positions. The plugs to change the connections for each position are not shown. Fig. 1 following page 19 provides detail of one octave of a single level of the referential keyboard.

The Modified Monochord

This instrument has completely variable pitch, governed by the position of a sliding steel in the hand of the player. The diagram under the string shows the pitch for each position of the steel. The diagram can be pushed sidewise to bring various strips, representing different scales, under the string. The tuning system of the referential organ was taken from notes chosen by ear, by musicians playing melodies on this instrument. The modified monochord and its use are discussed on page 13 and following pages.

Acknowledgments

The referential organ was built in the summer of 1961, financed by a grant from the Danforth Foundation. Experimental work with it has been financed in part by a grant from the Research Foundation of the State University of New York.

"It is so easy...to plant more seed in an already well-plowed field; so hard to drive a new furrow into stony ground."

Dr. Dickinson W. Richards Jr.

Interim Report

on the

Project on Organization in Auditory Perception

at the State University College
Albany, New York

This project is a study in auditory perception. It has its roots in a group of anomalies in hearing unexplained by the conventional Ohm's Law--Fourier analysis approach. It draws its form from a group of anomalies in musical tuning unexplained by conventional scale theories.

In the basic theory of hearing, this inquiry has led to a view of tonal perception as organized by nerve processes operating in time, an aspect of acoustic stimuli which has heretofore received little attention in acoustics. A discussion of this hypothesis and its bases was published by the authors in an article, "The Long Pattern Hypothesis in Harmony and Hearing," in 1961.[1]

In music, this inquiry has led to a systematic structure encompassing logically the many deviations from standard scales that we have found in the tuning preferences of musicians playing melodies with complete control of tuning.

The two prongs of the inquiry are mutually supporting and intimately related. It appears that both operate on the same principles. The study of musical patterns is expected to develop measurable dimensions of neural organization in hearing, the original purpose of the study. But it also should contribute to the theory of music directly.

In the present stage of the inquiry, musician subjects are asked to pick out their preferred tunings of melodies on a specially constructed reed organ. To the present time, no musician subject has selected or preferred classical just tuning, equally tempered tuning, Pythagorean tuning, or tunings in any other known scale. The preferred tunings all fall within the system devised in this project, a system which we call "referential just intonation."

"Referential just intonation" was found by study of tunings selected by musicians on an instrument providing for totally variable frequency--a long string, struck with a steel, with the pitch controlled by the place of strike. The musicians were kept informed of the frequencies struck, and asked to repeatedly check them until they were absolutely satisfied. The notes chosen have thus far fallen within three sub-systems -- major, minor, and blue -- forming a larger system of tuning by linked roots and just ratios to them. These roots are organized according to the "power-of-two" relation which has been repeatedly observed by psychologists studying the phenomenon of the tonic. The "power-of-two" relation, in turn, is explicable from the idea of a coding pattern operating through time, in accordance with the "long pattern hypothesis" of the original study.

It should be emphasized that these tunings were not developed theoretically, but taken directly from musicians' choices. The system was developed from the specific notes struck, and incorporates these.

In substance, the data indicate that a melodic sequence has an organized unity, with all notes of the sequence related by simple whole-number ratios to a common reference note. Typically, the reference notes change from phrase to phrase of a melody. The reference notes are linked in systems of simple whole-number ratios, with different systems of linkage for major, minor, and blue. Other linkages seem likely, but have not yet been explored.

The accompanying recording provides samples of referential just tuning in each of the linkages so far studied. The examples are:

"If I Didn't Care," blue linkage, theme song of the Ink Spots quartet, tuning selected by Joseph Boatner, bass and arranger for the quartet.
"Empty Bed Blues," blue linkage, tuning selected by the investigators, from a Bessie Smith recording, used on Leonard Bernstein's LP What is Jazz?
"In the Evening by the Moonlight," major linkage, tuning selected by Henry Sullivan, barbershop choral director and arranger.
Mozart's "Serenata Notturna," major linkage, tuning selected by Edgar Curtis, symphony conductor and head of the Music Department at Union College.
No. 6 of Bela Bartok's Fifteen Hungarian Peasant Songs, minor linkage, tuning selected by Edgar Curtis.

The recording also includes a demonstration of an experiment on the problem of key, tonal center, or root.

The implications of this inquiry require considerable discussion. This, and the details of the tunings used, etc., follow in the body of the report. At this point, however, you should listen to the accompanying recording, not for its details, but for the over-all flow of the patterns.

Each reed tone of the experimental organ is actually smooth in that it is free from either intensity or frequency vibrato. The recording was made on tape, and even the best tape recorders use a friction drive which introduces flutter. The effects are dulled thereby.

* * * * * * * *

Scales Derived from Melodies

The science of musical scales is the oldest laboratory science because the monochord, used by Pythagoras and before his time, is the first measuring laboratory instrument devised by man. It is fair to say that, throughout the history of the science, nearly all investigators in the Western tradition have begun with scales, setting them up by one formula or another, after which they tested the scales by trying melodies in them. (Max Meyer[2] must be mentioned as an exception, and Harry Partch;[3] there may have been others, but they were not numerous.) Keyboard instruments tend to enforce this approach, since the instrument must be tuned in the scale first. It is not possible, on a conventional keyboard instrument, to play a melody in an unforseen scale.

An assumption of this investigation is that a scale is the framework of a melody, or, to put it another way, that the auditory organization of a melody is the source of the scale <u>for that melody</u>. This implies that different melodies may have different scales, and that one should inspect the melodies to find the scales. (The theory of Indian music takes it for granted that different melodies have different <u>ragas</u>, or scales, hence the limitation of our remarks to "the Western tradition" in the preceding paragraph).

Musicians playing melodies on our referential organ have been outspoken in their surprise at discovering the false sound of the so-called "true scale" for some melodic sequences. The reader has already listened to the first melodic example in the recording, comparing the opening phrase of "In the Evening by the Moonlight," in classical just intonation and in referential intonation.

The referential pattern found for "In the Evening by the Moonlight" is not an exception, rather it is a typical and systematic pattern. A number of different musicians, guided only by the preferences of their ears, have chosen this system for a number of different standard melodies in the major mode. These musicians tried, and rejected, equally tempered and classical intonation for these melodies. Most musical theory assumes as a basis the classical just scale, sometimes called the true scale or the natural scale (formula given, and discussed, in an appendix). To date, no example of this scale called "natural" has been found in nature in our experiments, nor have we found even one single example of choice of the equally tempered scale, which might be supposed to be produced by cultural conditioning. (We can say nothing about the intensive professional conditioning of a professional pianist, since we have not yet had one as subject).

Reading versus Hearing

While musicians, we have learned, find it hard to believe the patterns found in this project when they read about them, they have no trouble at all when they sit at the keyboard, produce the patterns themselves and hear them. The theory of music would be very different if musicians commonly had access to apparatus with a choice of accurate tunings, instead of being restricted to tempered keyboards.

As an example of the issues raised by accurate tuning, let us take this opening phrase of "In the Evening by the Moonlight." The referential tuning was chosen by Henry Sullivan, a high school music teacher, barber-shop choral director, and barber-shop quartet singer. The opening notes, as written, are Sol, Do, Mi, Mi Mi, Mi Mi Mi. Mr. Sullivan's chosen tuning uses the same Sol, then the special notes we call Day and May rather than Do and Mi. The names coined in this project for special notes, to escape tedious repetition of complex ratios, are covered in a table in one of the appendices. The notes Day and May are higher than Do and Mi, in each case by 21.5 cents, a bit more than one-fifth of the difference between adjacent notes on the tempered piano. Readers acquainted with scale theory will recognize 21.5 cents as a Dydimic comma; in referential tuning this comma is a frequent distinction, but not by any means the only distinction between notes.

In conventional theory this phrase, Sol Do Mi, is running up the major chord. This is a melodic sequence. We can hear as a matter of plain experience that the notes belong together; they are linked. It seems plausible that they are linked by membership in the good standard major chord.

Instead Mr. Sullivan's ear rejected the opportunity to run up the plausible chord Sol Do Mi, and chose Sol Day May instead. All listeners have had the same experience, so we assume that the reader did. Such examples, and there have been an abundance of them, compel the conclusion that melodic organization is not directly analogous to the organization of a chord. If the two types of organization rest on a common base (and we think they do) it is applied in a different manner.

As an overview first, before detailed analysis, we can note a characteristic difference between classical just intonation and referential intonation. Listeners are likely to report that the classical tuning sounds duller in general effect than the referential tuning. The best evidence on this comes from an abundance of comparisons back and forth at the keyboard. The recording offers only a limited opportunity for this, although better than nothing.

The classical note Mi has the simple ratio 5/4 to Do; the note May stands in the complex ratio 81/64 to Do. The note Day has the ratio 81/80 to Do.

Now the note May need not be viewed as a relative of Do, acting through this ratio 81/64, but as a relative of relatives of Do, related by a simple ratio to other notes linked in turn by a chain of simple ratios to Do. The same chain accounts for Day, which has the impossible direct ratio, 81/80 to Do, but stands in a simple relation to the same chain of references, extended in time, that accounts for May. Thus simple ratios, but in extended reference, can account for Day and May. The classical notes Sol Do Mi are linked directly by simple reference. Details of the system of extended reference are supplied in a later section.

The <u>direct</u> reference sounds dull, the <u>extended</u> reference sounds right, and not dull. Is the direct reference perhaps too easy to interest an auditory system capable of much greater complexity?

Mr. Sullivan used Day, a high first, as the second note of his melody, but ended on Do, the orthodox tonic. He had two different tunings of the first scale step. The use of more than one tuning for what is ostensibly the same scale step is covered in an appendix.

In conversations with musicians we have found a rather wide-spread hypothesis that tuning varies with the up-or-down direction of a melody--that is, that notes tend to be sharpened when the melody is going up, and flattened when it is falling. That hypothesis does not suggest additional notes in the scale, but takes the traditional notes for the norm, as modified by melodic motion. Because the hypothesis seems to be widespread, we want to make it clear that this is not what we are talking about. Our data consist of the notes chosen by musicians on apparatus that makes choice available. The chosen tunings depart from traditional tunings, but not according to melodic motion up and down, as may be seen in the examples included in this report. The chosen tunings do seem to be related by just intervals to linked references; thus they represent a form of just intonation, although not the formula of the classical just scale.

Definition of Just Intonation

A just interval in music is defined as a ratio of frequencies in small whole numbers. (The term is related to right or justice.) Thus if Do is 264 cycles per second, a just or pure tuned Sol will be 396 cycles per second. Sol has three waves for every two of Do; the ratio is 3/2, in small whole numbers, the ratio of the perfect fifth in music. The Pythagoreans had no way of counting cycles per second, but they observed the same ratio inversely, 2/3, for the fifth in string length on the monochord. Their observations on the connection between mathematical relations and concord of sweet sounds contributed strongly to the subsequent development of scientifi mathematics and unscientific numerology.

The reason for the relation between concord and small whole number ratios is not known to this day. It involves the theory of hearing, and how we hear is not known. Our long pattern hypothesis, published in the article previously cited, holds that perception of pitch is not instantaneous, but involves processes of accumulation and neural coding taking place through time. In this hypothesis the long pattern of a succession of waves is regarded as one of the components of the sensation of pitch. If notes stand in a ratio of 3/2, they share a common long pattern, since three waves of one take the same time as two waves of the other. (The long pattern article reports a staggered frequency experiment, supporting the concep that long patterns are involved in organization of sensations of pitch, and discusses a number of aspects and related phenomena.)

Any scale tuned by a formula of just intervals is a just scale (as opposed to a temperament, and there are other temperaments than equal temperament--for example, mean-tone temperament). Many just scales have been devised. J. Murray Barbour says, in his detailed history, Tuning and Temperament,[/] "The Pythagorean scale may be thought of as the limiting form of just intonation, since it has a great many pure fifths, but no pure major thirds." And again, "In the foregoing pages there have been presented more than twenty different monochord in authentic just intonation, i.e., with pure fifths and major thirds. . .and yet each has a right to be called just intonation!"

Detailed formulas for the Big Three (Classical just intonation, Pythagorean, and equally tempered), plus details of referential tuning, are supplied in an appendix.

The Reputation of the Classical Just Scale

The classical just scale is taken as the standard in most writings on music theory. The tempered scale is a compromise, regarded as something to approximate the classical scale, with the advantage of modulation into many keys, without mechanical complications and without intolerable mistuning.

Many excellent musicians think that classical just tuning is used by performers who are free to control pitch, such as string quartets and capable vocal groups. We hold that this opinion has a foundation in good observation, even though it is not correct in all details.

Just tuning, or pure tuning, has a characteristic sound. A piano tuned in the classical just scale provides harmonies of great beauty and distinctive character, far better than from a piano in tempered tuning. The pure tuning has a recognizable effect.

Good musicians hear this recognizable effect in the melodic intervals of good vocalists and string players. They hear it as just tuning, and may well assume that it is classical just tuning, since few books on scales mention the existence of any other kind of just tuning.

Many different research projects making measurements from performance have shown that vocalists and string players do not in fact use the classical just scale. The impression of purity in the hearing of good musicians remains. Our explanation is that the note heard as in pure tuning is indeed tuned in a pure simple ratio to a reference, although not necessarily to the preceding note of the melody.

Measurements from Performance

Measurements of tuning in performance have produced surprising results, which have been well described by Zuckerkandl.[5] Regarding certain experiments: "Their object was to determine if people who sing tend more to equal temperament or to just intonation. To this end, it was only necessary to have the same melody sung by a number of people and to register their tones by a measuring instrument such as an oscilloscope.

"The result of these measurements was highly unexpected. It went so far beyong the limits of the original question as to render it meaningless. What appeared was that the singers sang neither in just intonation nor in equal temperament--they simply sang unimaginably off pitch. And this was equally true of all the singers, trained and untrained, unmusical and highly musical. ...Such facts can no longer be discussed in terms of poor intonation; the singers simply sang different notes than those which the text prescribed." (Italics supplied).

A paragraph later, he said, "But the most significant thing about the results of this experiment is that it required the intervention of the measuring instrument to reveal these grotesque distortions of pitch, these false tones. The audience, which included experienced musicians, had not noticed them at all. ...It is not until physics intervenes, with its measuring instruments, that the false tones are brought to light--to the surprise of the investigators, to the astonishment of the musicians. But what do 'wrong' and 'false' mean here--where obviously, so long as the approach is purely musical, so long as no measurements are undertaken, everything is right and nothing is wrong or false? For what is musically right, musically wrong, the court of last appeal is the ear of the musician, not the physicist's apparatus."

Unlike some investigators, Zuckerkandl recognized that the discrepancies could not be explained away as human error, because they were too large--" the human ear, under other circumstances, immediately and unfailingly perceives discrepancies that are mere fractions of those here established."

Other studies have produced similar results. Zuckerkandl is saying that music in fact uses notes that are not dreamed of in our philosophies. He did not carry his work to definition of these notes; in fact, Zuckerkandl accepted the hypothesis that variation in tuning is modification of the classical scale as the norm, and that modification comes from upward or downward motion of the melodic line.

In connection with all measurements of tuning, it must be kept in mind that we really have no reason to expect to find the classical scale in melodic performance by singers, because that scale was not taken in the first place from melodic performance by singers. The classical just scale comes from a definition, recorded in monochord string lengths by Claudius Ptolemy, in Alexandria, about 150 A.D. Certainly it is possible to define and talk about the classical just scale, or about helium carbide. It is necessary to find a specimen of the classical just scale or of helium carbide in order to show that they exist.

Any physical scale can be set up by a tuner, but it does not <u>exist as a formula for auditory organization</u> unless human listeners use it as an organizing matrix for musical patterns. A false physical scale may serve to convey a melodic pattern, by grace of the ear's power in approximation, but it will sound somewhat mistuned or dull, because of the clash between the notes of the formula and the tuning called for by the listener's auditory organization. The sensation of mistuning or dullness is a clue that the formula note is not the genuine auditory pattern, and it cannot be used as a guide to what is happening in auditory organization.

We should define some terms here, not to create distinctions but to report distinctions that can be found in fact. A physical scale is any formula at all that can be set up by a tuner; it need not be a valid reflection of musical patterns. A psychological melody scale is an assortment of notes used by the human sense of hearing to produce the organized patterns that we call melody. A melody involves a sense of key or a tonal center. Confusion follows if we assume that every physical scale can be a psychological melody scale. Human listeners, guided by context, can and do reinterpret physical scales to convey melodies, even though the raw material of the hearer's melodic organization is only approximated in the physical scale.

Analysis of melodic structure in music theory requires, as the first step, finding the psychological melody scale, by experimental methods. An assumption that a usable physical scale must be the psychological melody scale will lead to error. In musical performance, the psychological melody scale has advantages--it sounds better, although an approximate scale, supported by the context, may do.

It need not follow, as a dictum, that every good psychological scale must be a psychological melody scale. Some composers are careful to avoid setting up a sense of key or tonal center--in twelve-tone music, for example.

Even in conventional music it is obvious that a maximum of organization is not the aim. Conventional patterns frequently operate by raising a doubt about the organization, then clearing it up. Our investigation, in contrast, is a study of auditory organization and tonal centers, and should not be interpreted as setting up rules for musical aims.

An American study of the type mentioned by Zuckerkandl is that by Paul C. Greene.[6] He used recordings by unaccompanied violinists. He measured the melodic intervals from note to note and compared his results with those to be expected from the classical just scale, the tempered scale, and the Pythagorean scale. Greene referred to the classical just scale as "the natural scale," as may be seen from his title. He found that the measured intervals used by the violinists did not match any of the three scales, but they averaged closer to Pythagorean intervals than to either classical or tempered. He did not attempt to derive scales from the tunings that his subjects had used.

Measurement from performance is certainly valuable, but it raises a number of questions about what should be measured. Are the most significant measurements from note to note, or from note to tonic, or from note to reference, or all three? These different choices in measurement may supply different data. Our own experiments do not supply tunings from performance, but rather a record of choices, the tunings chosen by musicians in selecting the tuning of a melody. These choices may not necessarily be identical with measurements from performance, because it is possible that a variety of additional factors operate in performance. This can be tested only by experiment, which we hope to do as means and equipment can be obtained.

The Tonic Phenomenon, Roots, and Linked References

Max Meyer[7] published in 1901 a description of the power-of-two phenomenon. Take any two notes related by a simple whole-number ratio, and play them as a small two-note melody. If one of the numbers is a power of two, (that is, of the series 1--2--4--8--16--etc.) the ear will select this as the satisfactory ending note of the melody, and will feel unsatisfied by an ending on the other note. If neither number of the ratio is a power of two, the ear will establish no marked preference. Perhaps one or the other note will be preferred as ending, but on a much less definite basis.

The recording demonstrates the power-of-two phenomenon with the combinations 9/8, 5/4, and 6/5. The notes represented by 8 and 4 are selected by the ear as resolving the melodic pattern. Listeners have reported that no such definite resolution takes place in the case of 6/5.

A number of psychologists since Max Meyer have reported experiments on the power-of-two, and their work will be found reported in Max Schoen's The Psychology of Music.[8] As Schoen summed up the results: "Obviously, then, a pure power of two is preferred by far over other tones as a final tone."

The reader observed in listening to the recording that each power-of-two note asserted itself strongly as master of the combination. This is the tonic phenomenon. The tonic of a scale is the note Do, the note that normally resolves a melody by serving as the satisfactory ending note. Most musical theorists will agree with Donald N. Ferguson:[8] "the principal clue to all understanding of tone relations is the sense of tonic or key." Yet it is a fact that music theorists in general, including Ferguson, have made little or no use of the power-of-two phenomenon as a clue to the sense of tonic. More discussion of this will be found in an appendix, and the recording illustrates an example used by Ferguson, which seems paradoxical for one theory of resolution, but in fact serves as an excellent demonstration of the power-of-two in resolution

Perhaps the power-of-two phenomenon has been neglected because it seems to be an isolated oddity. We are quite right to doubt that the human ear calculates squares or cubes or any other powers of any number. The phenomenon is demonstrable as a fact, but one feels uncomfortable about it in the absence of an explanation.

Our own hypothetical explanation for the power-of-two may be found in our article on the long pattern hypothesis. Summarizing it here, we must begin with the octave phenomenon. Human listeners find a family resemblance in notes related by 2/1, the interval of the octave. If 100 cycles be taken as the note C, then a musician will classify a 200 cycle note as C again, an octave higher, and 400 cycles will be another C, and 800 cycles, and so on. The musician's nomenclature is based on a resemblance which we all experience in hearing these notes. We know what he means when he says that they are the same note, in different pitches. In this terminology, a note and a pitch are not the same thing, since the same note has many different pitches.

In terms of long patterns, it can be said that all of these different C's are regular subdivisions of the same long pattern. They are halvings and re-halvings, the simplest possible subdivisions. Thus, the time for one cycle of a 100 cycle tone is one hundredth of a second. The 200 cycle tone has two cycles in this same time; the 400 cycle tone has four cycles, and so on for the 800 cycle tone, etc.

The observable family resemblance of the octave phenomenon exists, regardless of any theory.

Next we take a combination of two notes, 300 cycles and 200 cycles. They stand in the ratio 3/2, the musical fifth; 200 is Do and 300 is Sol. They share a common long pattern since three waves of the higher pitch take the same time as two waves of the lower pitch. This shared pattern is one hundredth of a second.

In the cited article we suggest the possibility that simple ratios give us a sensation of harmony, of belonging together, because they share a common long pattern. If an accumulated long pattern is a component in the sensation of pitch, which it may be, then two notes with a common long pattern have a certain ingredient of pitch in common, and could well be sensed as related.

The hypothesis assumes that this shared long pattern is the organizing characteristic of the combination. Now which tone will dominate the combination? The 200 cycle tone <u>is</u> the tone of the long pattern, in the sense that, reduced by octaves, it becomes the long pattern; an octave down from 200 cycles is 100 cycles. Since 200 cycles is the tone of the long pattern, it is <u>the</u> tone of the combination.

The 300 cycle tone <u>fits</u> <u>into</u> the common long pattern, but it cannot <u>be</u> the long pattern, since reduced by octaves it becomes 150 cycles, and then 75 cycles.

Thus, in the combination 3/2, the note represented by 2 is the trade-mark of the combination, since it is a regular, or octave subdivision of the shared long pattern that makes it a combination. The only numbers in ratios that can thus be octave subdivisions of long patterns are 1, 2, 4, 8, 16, etc. The reader has heard the resolving effect of powers of two on the record. It is not necessary to think of the ear as calculating squares and cubes to account for the phenomenon; it may be produced by long patterns in auditory organization.

Suppose we take a ratio of 6/5, which the reader has heard on the record. Neither is a power of two, neither gives the sensation of a root comparable to that produced by a power of two. The ear may arrive at a choice, but it is not of a clear sort.

As a caution, regarding terminology, the word "root" is not always used in the same sense in music theory. In a combination such as 6/5, a musician may label one or the other as root of the combination, and for good reasons, although he does not hear in either note the same phenomenon of resolution that is experienced from a power of two. One cannot assume uniform terminology.

The phenomena of roots and tonics are clearly crucial in musical structure, as theorists recognize. Hindemith[9] says, "What are root tones? In all harmonic units, intervals, or chords, the constituent tones seem to have unequal harmonic values. Some seem to us to be more important and to dominate the rest of the sounding entity. These roots, as they are called, are the carriers of the unit's potentialities in respect to its membership in a row of successive harmonies. Overtones, although providing excellent measurements for the size of the intervals, do not explain this fact. The way in which the overtones determine the qualities of intervals resembles the entries in those columns of a doctor's file that describe the patient's physical appearance, his age, his size, his complexion, revealing nothing of his inner constitution, his mental capacities, and his potentialities as a citizen."

Hindemith asks what are root tones, but leaves the question unanswered, except for his testimony that he observes them in action as the carriers of organization, and this is good testimony, with which other musicians would agree.

Musicians know, and perhaps some other readers do not know, the function of the tonic, or Do of the scale. A typical melody could well be described as a path of variations around a tonic. The start and finish are clear. A melody quickly establishes a note as the tonic, giving us the sense of key, without which the pattern would be unintelligible noise. After development of the pattern, which builds up an auditory tension, a normal melody ends on Do, relieving the tension. This relief of tension is called resolution. Melodies ending on anything but Do are rare, and the other endings produce a very special effect.

A melody begins by establishing a tonic, and ends on the tonic. It would seem to follow that some form of connection to the tonic is maintained in between, and it should be possible to find it if it is sought by the right methods. Further, it would seem that this connection winds up some spring of tension, which is unwound by the ending on the tonic. The situation supplies good specifications on what to seek, before the start of the search. In spite of this promising beginning, the search has been unsuccessful. Hindemith says,[10] "It is an astounding fact that instruction in composition has never developed a theory of melody."

The present inquiry has produced the framework of a theory of melody. Additional work is needed to carry it beyond the present stage, but it is serving surprisingly well for a large variety of melodies. The theory is a logical extension of the power-of-two phenomenon.

The theory of melody, and the scales or referential tunings that accompany it, are so logical that they might be taken as a structure contrived from the power-of-two phenomenon. In fact the concept was reached empirically, by working backward from tunings to structure, and it arrived step by puzzling step, like a film cartoon of a house being drawn from the roof down. The theory can be understood best from an inspection of a sample of the data from which it sprang.

In our experiments beginning in 1956 we used (and to some extent we still use) a long string of thin piano wire on a sounding board, with pitch governed by a guitar steel held on the string. A felt damper on the wrong end of the string kills vibration there, so the sound comes from the segment between the steel and the bridge. A practiced player can "think the melody" and his hand will automatically go to the right place on the string. If the note is not exactly right his hand will slide the steel to the place called for by his ear. A diagram below the string shows the notes for each position of the steel with satisfactory accuracy.

Good musicians were asked to play standard melodies on this string, and their choices of notes were recorded. At this point we did not tell the musician, "Well, you've made your pattern, now go away, we will analyze it." We asked him to repeat it, to try alternative patterns, to make sure that this was what his ear wanted.

Specifically, we will suppose the musician was playing something in a minor mode. The fourth of the classical just scale, in C, the note F or Fa, with the ratio 4/3 to Do, is shown as 352 cycles per second in the table in the appendix. The player would slide the steel to a higher pitch, a position about 3/8 of an inch closer to the bridge on our string, easily detectable and very different in sound. Within the limits of accuracy of our instrument, this seemed to be a ratio 27/20 to Do, with 364.4 cycles per second. We use the name Fay for this high fourth, 27/20.

In sum, the musician subject used Fay rather than Fa for this minor melody. We asked him to repeat it, and he chose the same tuning consistently, which suggested that this was not an accident. We asked him to try Fa instead, and compare the melody several times, with the classical tuning and then with his special tuning, to make sure of his preference. He did so, and declared positively that Fay was right, and Fa sounded wrong.

Experiments with other musicians and other minor melodies produced the same result. It was apparent that something special was happening to the tuning of this note, consistently, in minor contexts. (Meanwhile special tunings of other notes were accumulating, of course.)

A ratio like 27/20 is an embarrassment to theory, because it is not made of small whole numbers. But the ratio 27/20 is a pure fifth, 3/2, above 9/5, which is a good minor note (9/5 is a pure-tuned B flat). Thus Fay is a good just ratio above 9/5. The classical note Fa, 4/3 above Do, is a bad ratio above 9/5 (40/27, in fact).

In terms of just intervals, Fay should sound good, and Fa should sound bad, if the listener is hearing in reference to 9/5 rather than to Do. In addition, it was observable that this note Fay occurred in melodic sequences in which all of the notes stood in good just ratios to 9/5. Examination of the recorded tunings of other melodies showed a similar tendency: the notes of a melodic sequence would all stand in just ratios to some one note, as if this note were the reference for the tuning of that sequence.

The reader may well remark that 9/5 is not a strong ratio, and seems like an unusual choice for an anchor point in melodic structure. On the other hand there is something unusual about the over-all sound of the minor mode, and it may well be characterized, and indeed, created, by something special in the anchor points.

The note Fay 27/20 is not limited to minor patterns, but only in minor patterns will it be found as a relative of 9/5, as indicated by the fact that all of the other notes in the sequence stand in simple ratios to 9/5.

The first stage of the data, then, drawn like a roof with no walls or foundation, was the occurrence of off-pitches that sounded completely proper to musicians, and were stubbornly maintained by them, and these pitches were in wildly complex ratios that certainly could not be relatives of the tonic.

The second stage was that the notes used by musicians did not necessarily stand in pure harmonic ratios to each other; that is, the step between successive notes often were pure thirds, or pure fourths, etc., but sometimes they were not pure harmonic intervals, and the lack of a pure ratio in the melodic step did not hurt the sound in the least. Everything sounded good, as it should, since every note was chosen by a musician to sound good.

The third stage was that the notes of a melodic sequence would often stand in pure ratios to some one note. This is the reference note phenomenon, which has already been described, and it was the first development that made sense. It accounted for unity in a melodic phrase, and gave a basis for the sound of pure tuning from these ratios that looked so puzzlingly complex.

The fourth stage was that certain tunings, and certain reference notes, seemed to be connected with certain musical flavors, or types. The B flat 9/5 as a reference note in minor has already been mentioned. Other reference notes came to be expectable from other types of music, and some of them are puzzles yet, in that the types do not yet have names, and we are not yet ready to attempt a classification of them.

The fifth stage came with a search for a system in the intervals between reference notes, rather than between notes of the actual melody. This involved a great deal of inspecting and juggling, cutting and fitting. In truth, mere detailed inspection for system didn't work; the good leads were found by following the sequences of reference notes used by specific melodies, and this didn't work either until the melodies were divided into three classes: major, minor, and blue. At long last this approach yielded three systems, in which the previously puzzling tunings were seen to be ranked in orderly arrays. By hindsight it seems to be a system that should have been found easily.

This system, which will be described in the following pages, is an interim scheme, which we expect to revise and improve, and we have experiments planned for testing a number of revisions. On the other hand, it has done far better than we had hoped for, by providing satisfactory patterns for a much larger number of melodies, in more types of music, than we had expected. All the work so far has been on standard melodies; we have done nothing on experimental or exotic patterns, which might very well call for additional systems of linkage.

The simplest system of linked roots, and probably the strongest because it is the simplest, is the major chain of references. It is linked by a succession of fifths, the ratio 3/2. This is the strongest of all musical ratios except the octave (and of course linkage by octaves would be no change at all).

The tonic note, Do, is the first major reference.

The second is Sol, a 3/2 above Do.

The third is Re, 3/2 from Sol, 9/8 from Do.

The fourth is Lay, 3/2 from Re, 27/16 from Do.

The fifth is May, 3/2 from Lay, 81/64 from Do.

The sixth is Tay, 3/2 from May, 243/128 from Do.

As the reader perhaps has guessed from our coined sol-fa system, the notes Lay, May, and Tay, are tunings somewhat sharper than La, Mi, and Ti, respectively.

The names are strange, the ratios complex, yet the reader will recall that May, and Day, sounded familiar, not strange, in the context of "In the Evening by the Moonlight."

The foregoing description is too brief to cover the concept of the linked chain, but the reader will find that it is amplified by an abundance of illustration. The keyboard diagrams are laid out by the linked references. The notation of each melody is at the same time a graph of the references with the ratio of each note to its reference.

The minor chain is similar in organization, a system of references linked by the ratio 3/2, except for the first step, which is 6/5. This different fork in the road makes all the references different.

The tonic note, Do, is the first minor reference.

The second is ri, 6/5 above Sol.

The third is li, 3/2 above ri, 9/5 above Do.

The fourth is Fay, 3/2 above li, 27/20 above Do.

The fifth is Day, 3/2 above Fay, 81/80 above Do

The sixth is Say, 3/2 above Day, 243/160 above Do.

The minor first linkage, 6/5, produces a different set of references although everything thereafter is the strong link, 3/2. This is consistent with everyday experience in listening to melodies: a minor melody sounds minor throughout, not merely on certain minor notes. A melody does maintain a link to the tonic, so a system of structure should be a scheme of organization of linkage to the tonic. It is consistent with our experience that the organization should be minor throughout.

The blue chain again uses the strongest ratio of linkage, 3/2, except for the first step, which is 5/4.

The tonic note, Do, is the first blue reference.

The second is Mi, 5/4 above Do.

The third is Ti, 3/2 above Mi, 15/8 above Do.

The fourth is fi, 3/2 above Ti, 45/32 above Do.

The fifth is dah, 3/2 above fi, 135/128 above Do.

The sixth is sum, 3/2 above dah, 405/256 above Do.

Various indications of these structures could be cited if space allowed. For example, a high proportion of blue melodies begin on Mi, which is 5/4, the first characteristic note of the blue reference chain.

Of course we do not think of linked references as ratios in the brain, any more than we think of the power-of-two phenomenon as mental arithmetic with exponents. We conceive a reference system as some type of matrix, used by the auditory system for organization, which can be crudely represented in mathematics until better ways of describing it are learned. We think of organization as using long patterns. We think of extended reference as perhaps organization by a very long pattern, a pattern far too long to be manageable directly, but one that can be encompassed because it is composed of regular subdivisions. An illustration, not directly analogous, is provided by a musical rhythm span of 64 beats, too much to handle, but managed easily as a regular march time when it is four lines, each of four measures, with four beats to each measure.

We think of matrices as linkages of ratio subdivisions, and we assume that various systems of linkage are learned and culturally conditioned, since musical modes accompany cultures and frequently are named for them (Scotch pentatonic, Hungarian Minor, the Lydian mode, New Orleans blues). As for the simple ratios that can be linked in various matrices, we think of them not as cultural variables, but as results of the inherent operation of the auditory system by long patterns. We take it that we learn to like the patterns that we are exposed to, within the limits of what we are inherently equipped to organize

The Referential Keyboard

The specific content of this general discussion can be seen best in the referential keyboard, which was set up by these principles, and in musical patterns played on it. The keyboard was designed to provide linked references, and a complete assortment of relatives for each reference. The assortment could be complete only within limits, since the supply of ratios is theoretically infinite. As a tentative cut-off point we decided to use no ratios to the base 7 and beyond (that is, not 8/7 or 9/7, etc.) and no ratio more complex than 9/8. These limits were chosen as a practical limitation for the first step. There is good reason to suppose that more complex patterns are musically useful. For example, Harry Partchll uses ratios like 11/8, and even beyond, not in mere paper calculations but in physical fact, since he works by ear and plays his patterns on accurately tuned instruments. We excluded them from the first keyboard because we wanted to start with a simple beginning for standard music, within patterns that had been found for such standard music on the string equipment, and not at all with the idea of excluding them from later tests.

The following ratios come within these limits, and were used for the keyboard, with note names for the first row of the keyboard as shown on the table.

1/1	the reference, Do.
9/8	the second, Re.
7/6	the septimal or subminor third, rie.
6/5	the minor third, ri.
5/4	the major third, Mi
4/3	the fourth, Fa.
7/5	the septimal or subminor fifth, fen.
3/2	the fifth, Sol.
8/5	the minor sixth, sen.

5/3 the major sixth, La.

7/4 the subminor 7th or natural 7th, lie.

9/5 the acute minor 7th, li.

The foregoing list includes <u>all</u> the mathematically possible ratios within the limits. It is not a scale, but a note and all of its direct-ratio relatives, within the tentative bounds set. It happens to include the classical just scale except for Ti 15/8, but was not drawn from any scale.

Fig. 1 is a photograph of one octave of the first level of the keyboard. Each key is a metal tab, which makes contact as a switch when pressed. The current operates a solenoid, opening the appropriate reed on the reed organ actions. The keys are spaced according to musical cents, and the cent values for each note are shown in Fig. 1.

The keys are in three rows, according to relationship to the reference. The first row contains all ratios to powers-of-two of the reference: 9/8, 5/4, 3/2, 7/4. In long pattern terms, these make common long patterns with two waves of the reference, or octaves of two waves. Tabs in the second row are ratios that make common long patterns with three waves of the reference, or octaves of three waves: 7/6, 4/3, 5/3. Tabs in the third row make common long patterns with five waves of the reference: 6/5, 7/5, 8/5, 9/5.

(For ease in fingering, the keyboard has duplicates of the 1/1 tab in the second and third rows, labeled 3/3 and 5/5. These add nothing in theory, and would not even be an advantage in fingering if these metal tabs were flexible through their whole length, so they are left out in the later diagrams. Similarly, 3/2 is duplicated in the second row as 9/6.)

We can consider 1/1, the reference, as Do in the key of C. It is always taken as the tonic for this keyboard.

Each level of the keyboard has these three rows, and all levels are exactly alike. The keyboard at present contains five levels. Fig. 2 is a diagram of the five levels, set for major reference.

The major pattern is a sequence of linkages by 3/2. The bottom level is the starting point; 1/1 here is the tonic. The second level is placed with its 1/1 over 3/2 on the first level. Thus 1/1 in the second level is Sol, as marked. Electrically and musically it is a complete duplication. The contacts connect to the same wire, and the solenoid for this reed does not know which of the tabs has been pressed.

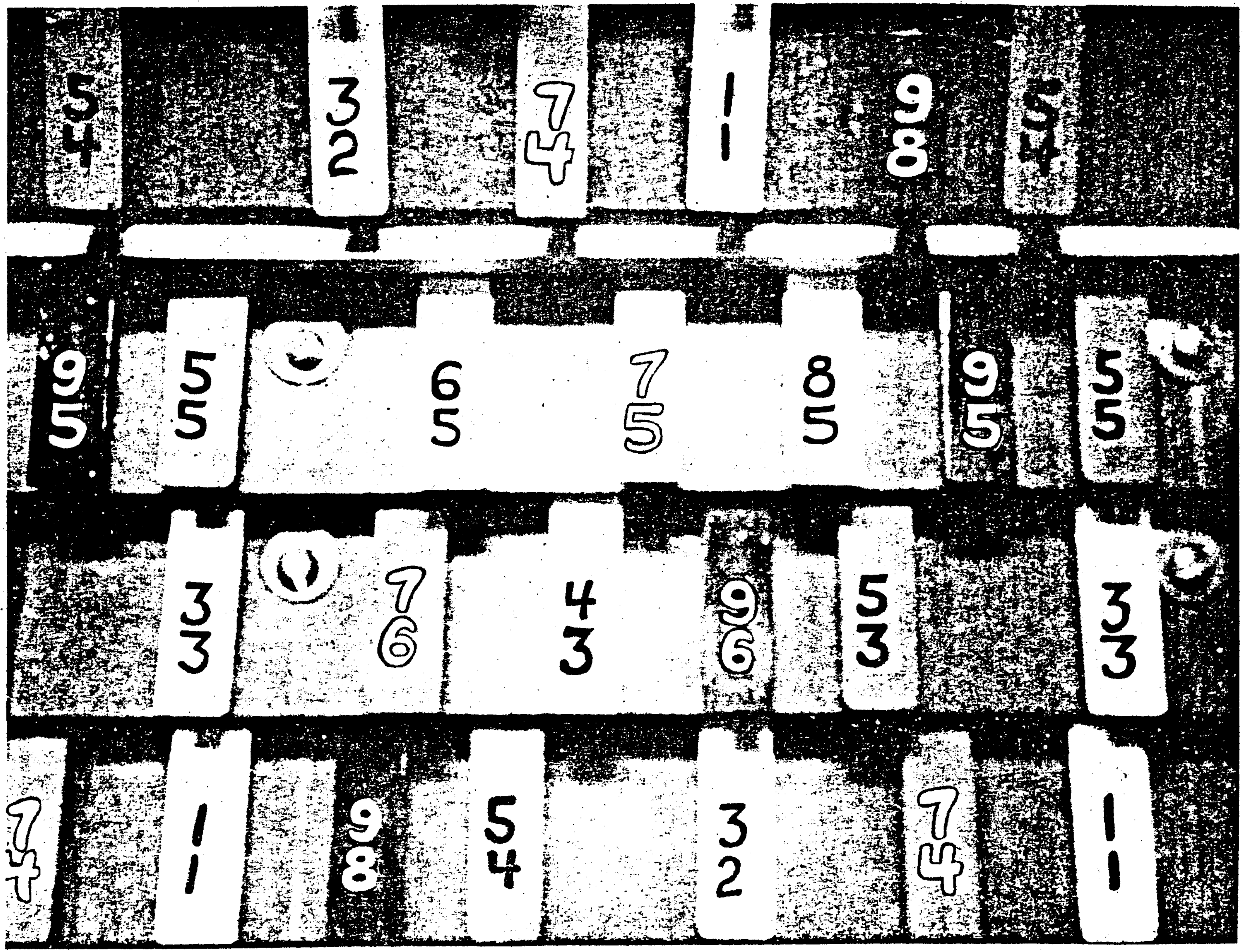

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Fig. 1

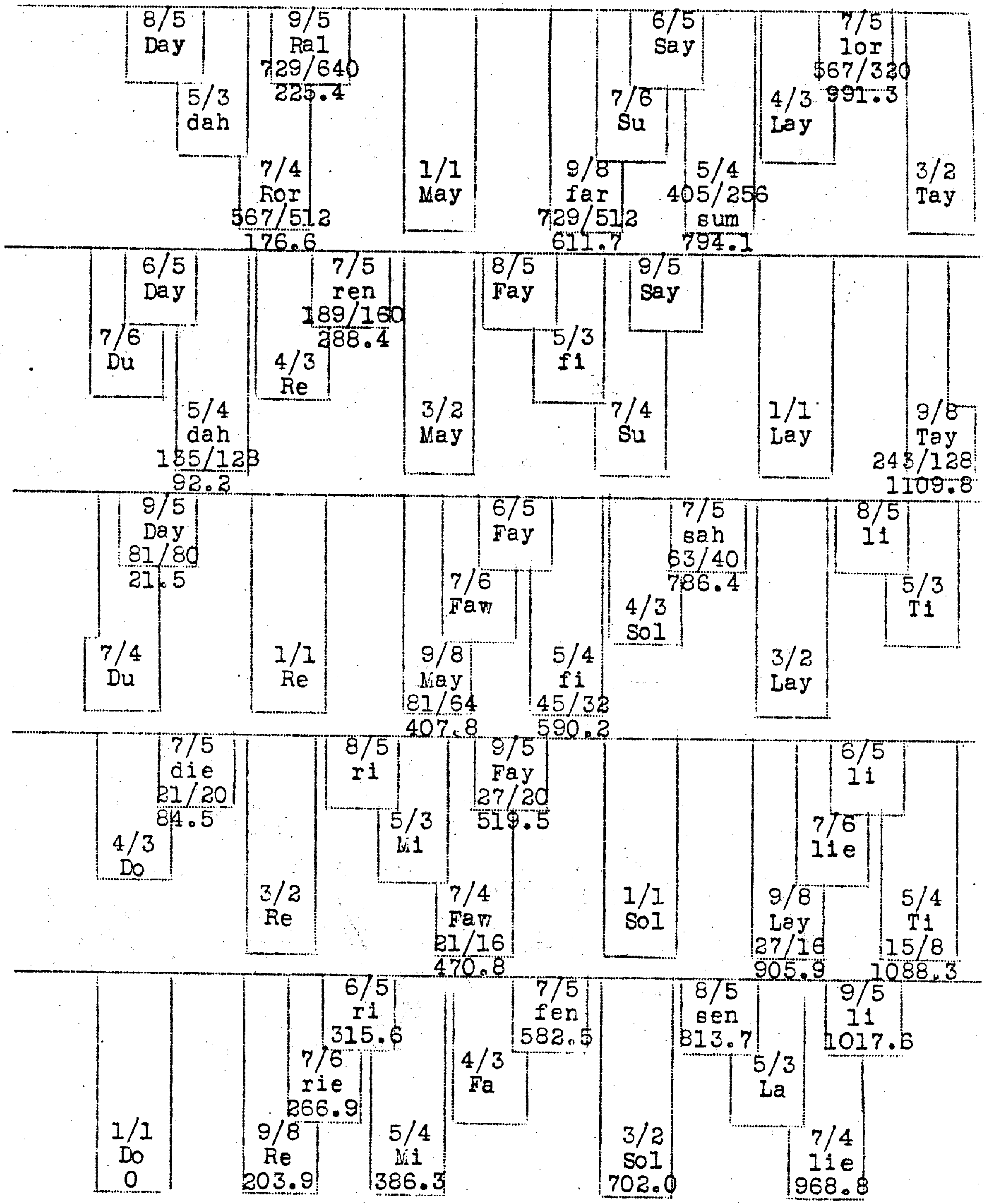


Fig. 2

Major Keyboard, by Reference Ratio, Absolute Ratio, and Cents

The tabs in level 2 have the same ratio markings as in level 1, but now they represent ratios to Sol, the reference of this level. Thus 5/4 in the second level is 5/4 above Sol, making it the classical note Ti, 15/8 above Do.

The reader will find several duplications. In the second row, 5/3 over Sol is the note Mi, identical with 5/4 over Do in the first row, also 3/2 in the second row is the same as 9/8 in the first, and so on.

There are certain differences. In the second row, 9/8 over Sol is Lay, sharper than La in the first row. Again, 9/5 over Sol is Fay, sharper than Fa in the first row.

The theory of classical just intonation calls for such changes in tuning with a modulation from the key of the tonic to the key of the dominant (Sol). We find that musicians choose the changed tunings without modulation, seemingly from change of reference in the melody.

The same process takes place from level to level as the keyboard goes up. In major patterns, 3/2 of each level becomes 1/1 in the next, making certain notes hold over, and bringing in some new tunings. Details of the tunings, the ratios to the reference and to Do, and the cent values, are shown in the figure. In brief, the keyboard supplies a chain of references, linked by 3/2, with a supply of ratio relatives for each reference. We began with five levels because the string patterns seemed to indicate that they would be enough, but we left room to add a sixth. We now know that we guessed wrong; we need a sixth and a seventh.

Before taking up minor and blue linkages, we should illustrate major patterns with a melodic pattern, the opening phrase of "In the Evening by the Moonlight." Fig. 3 supplies the notation.

The melody begins on Sol, which is 3/2 on the first level, and 1/1 on the second, and 4/3 on the third. The ratios in the squares above the notes show this; every note is written as a ratio in its Referential Keyboard Level.

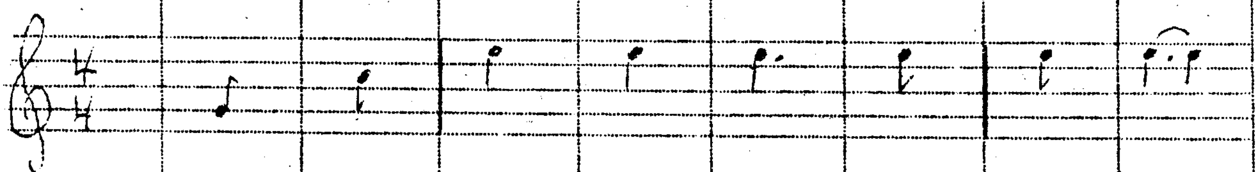
Below the notes are written the name, the cent values, and the absolute ratio to Do of each referential note, the corresponding classical notes, and the corresponding tempered notes.

This pattern, chosen by Henry Sullivan, goes into extended reference promptly. It goes from Sol, the first note, to Day rather than Do, to the surprise of Mr. Sullivan, and to us. He did not expect Do to sound wrong as the second note, but it did, and he had to find the right note as musician subjects do, by poking around until he hit a tab that sounded

In the Evening by the Moonlight -- 1

Words and music by James A. Bland, 1880

Referential Keyboard Level								
6								
5			1/1	1/1	1/1	1/1	1/1	1/1
4		6/5						
3	4/3							
2	1/1							
~~3/2~~ Major 1	3/2							



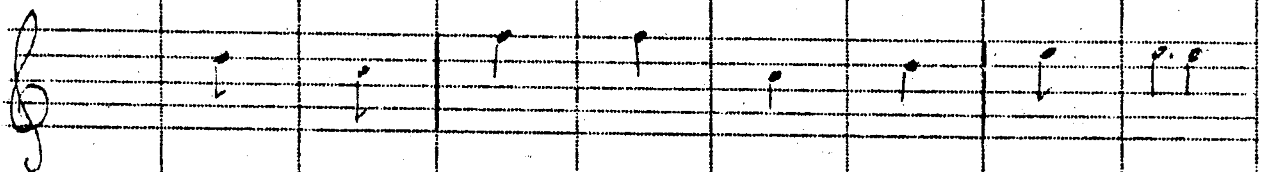
	In	the	eve--	ning	by	the	moon	light
Ref.	Sol	Day	May	"	"	"	"	"
Cents	702.0	81/80	81/64					
Abs. Ratio	3/2	21.5	407.8					

Class.	Sol	Do	Mi					
Cents	702.0	0	386.3					
Abs. Ratio	3/2	1/1	5/4					

Temp.	G	C	E					
Cents	700	0	400					

Fig. 3 - Part 1

Referential Keyboard Level								
6								
5	9/5	8/5			8/5	9/5	1/1	1/1
4			8/5	8/5				
3								
2								
3/2 Major 1								



	you	could	hear	those	voic--	es	sing--	ing
Ref.	Ral	Day	Fay	"	Day	Ral	May	"
Cents	225.4	21.5	519.5				407.8	
Abs. Ratio	729/640	81/80	27/20				81/64	
Class.	Re	Do	Fa		Do	Re	Mi	
Cents	203.9	0	498.0				386.3	
Abs. Ratio	9/8	1/1	4/3				5/4	
Temp.	D	C	F		C	D	E	
Cents	200	0	500				400	

Fig. 3 - Part 2

right to him. At that time he did not know the system of the keyboard, although after a few melodies he caught on to it.

In our terminology, this melody "goes up on an ambiguous note." The note Sol has ambiguous reference, or multiple reference, as the diagram shows; it is in the first three levels. The shift from Sol in row 1 to Day in row 4 seems to be a leap of four levels in reference. On the other hand, Sol is also in row 3, so a shift to row 4 is only a shift of one level by that method of accounting. What is the right method? It would be folly to conjecture at this stage, for one cannot assume that this keyboard provides a picture of auditory organization. It merely is a fact, to be filed for future reference, that melodies often go up and down the keyboard on notes like Sol or Re that extend over several keyboard levels.

What is the procedure of a musician subject, like Mr. Sullivan? They think the melody, then as they work out each note they hunt on the keyboard for the tab that supplies the tuning they have in their heads. They are not searching for the closest approximation, and in all the patterns given here they say that these are good tunings, not just something near.

In our illustrations, we have smoothed the fingerings, showing the notes of a sequence in a single level when they actually are, although the musician made the choices by ear, and not by the easy fingering that the illustration may show.

We claim no implications for the reference levels shown in the illustrations. For example, the note May is written here as 1/1 in row 5. A glance at Fig. 2 will show that this same note is 3/2 in level 4, and 9/8 in level 3, and it would be 4/3 in level 6 if we had a level 6. Does a valid picture of auditory organization make one level better than another for writing this, or should it be considered, in an auditory sense, as simultaneously in all of these, potentially? We do not yet know, we need more data, and we see no point in pretending to be definite about something still uncertain. We write it in the fifth row because it makes good fingering with the following note, which cannot come below the fifth row.

Some readers may not be familiar with musical cents. A cent is one hundredth of a tempered half step. It is a proportional hundredth, calculated logarithmically, by a formula which may be found in Helmholtz. Twelve hundred cents make a full octave. Thus C is zero cents, C-sharp is 100 cents, D is 200 cents, and so on. If the reader thinks of 20 cents as 20% of the difference between piano notes he can form a concept of the tuning differences indicated.

Turning now to other matrices, a minor organization is characterized by 6/5 as the first link, after which the

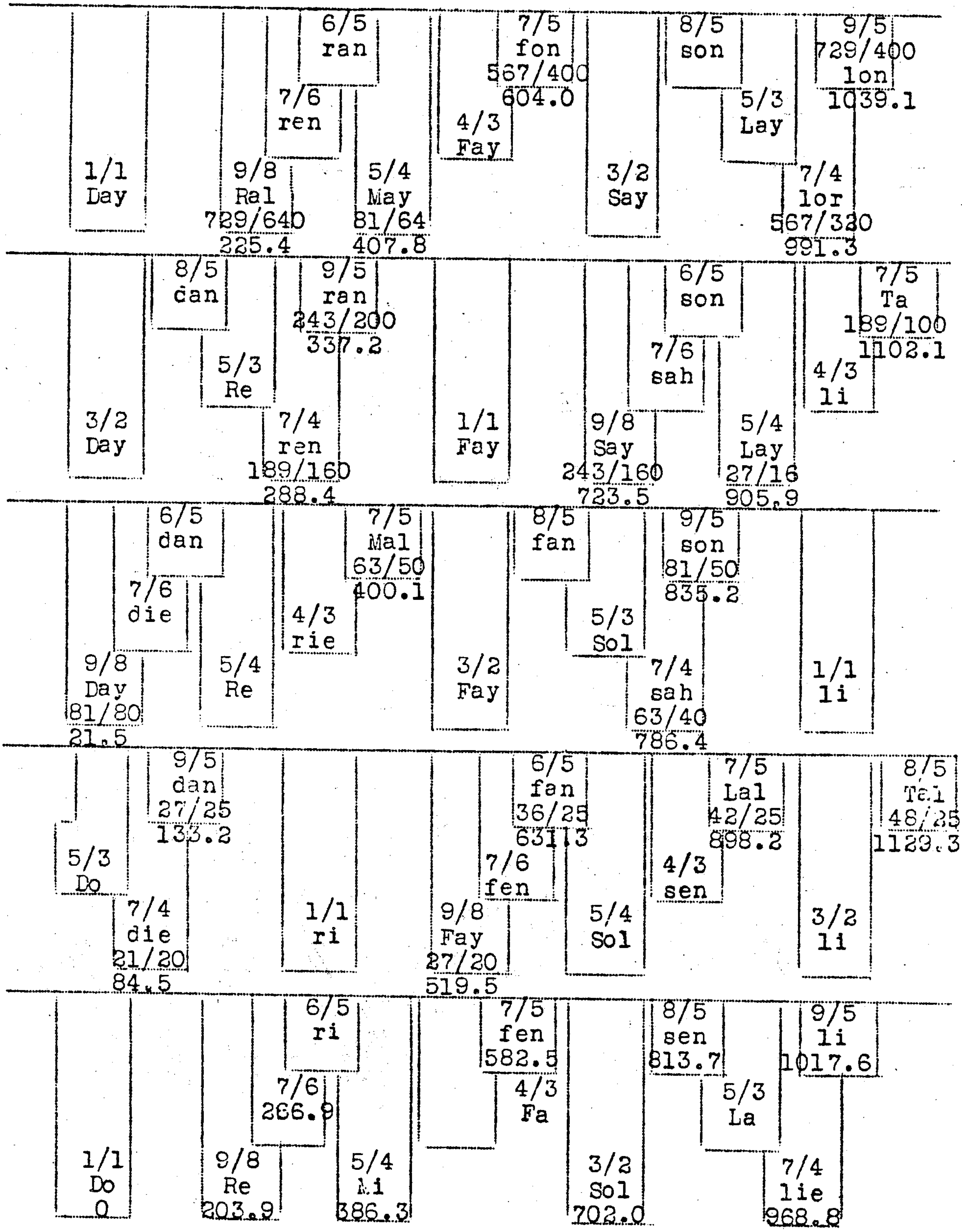


Fig. 4

Minor Keyboard, by Reference Ratio, Absolute Ratio, and Cents

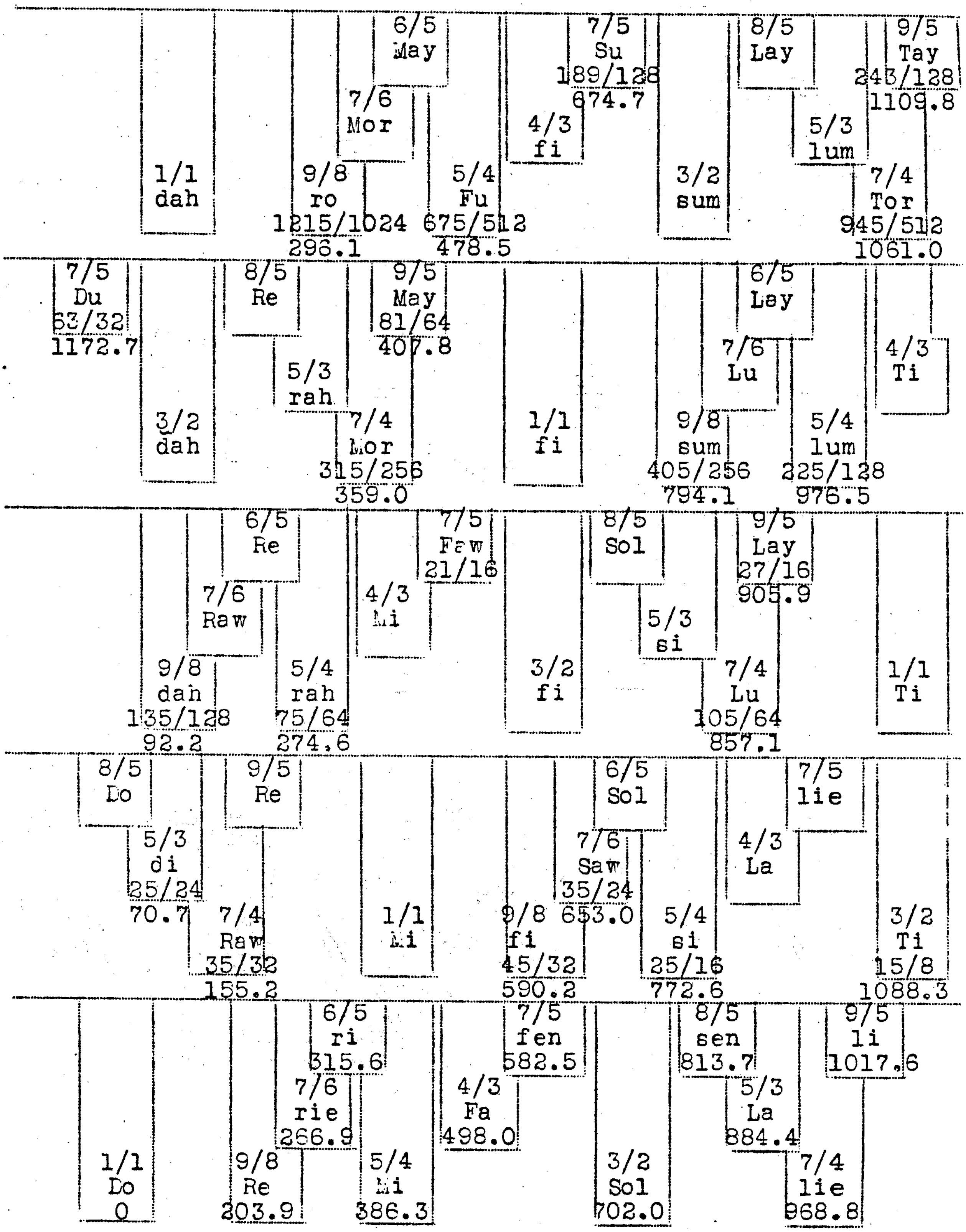


Fig. 5

Blue Keyboard, by Reference Ratio, Absolute Ratio, and Cents linkages are by 3/2. Fig. 4 diagrams the keyboard set for minor patterns. The ratio 6/5 in the first keyboard becomes 1/1 in the second. Our keyboard allows this section to slide, so we move it into this position.

After the first keyboard, all levels are linked by 3/2, as in major, but the relationships to Do are changed throughout as the figure shows. Ideally, we should leave the first keyboard alone and change all the others. To save money, it was more practical to leave the top four alone and change the bottom one. We slide the bottom keyboard and change two gang Cinch-Jones plugs to make new connections for the first row. This changes the absolute pitch of the tonic to E rather than C, but gives the right relationships for minor at minimum cost.

Fig. 5 diagrams the keyboard for the blue matrix, in which 5/4 becomes 1/1, after which all linkages are by 3/2. Again the change is made by the expedient of sliding the first keyboard, putting 5/4 under 1/1 on the second level, and changing the connections with the plugs. The pitch for blue melodies then becomes D sharp for the tonic, but this makes no difference for examination of patterns.

Appendix A gives the notation and referential tuning for the melodies in the recording:

Empty Bed Blues (blue)

If I Didn't Care (blue)

Mozart's Serenata Notturna (major)

One of Bartok's Hungarian Peasant Songs (minor).

They may be tested by mathematics or logic, but we urge the reader to test them by ear. They were chosen by good musicians working carefully by ear; they embody, not merely individual notes, but auditory organization in sequences, and it is possible to hear the patterning, to hear the unity in a group of notes. We suggest that the reader play each melody several times, following the score by eye, and that he listen with the intention of hearing connections, unity, pattern, relationships among notes of a sequence. The patterning, which can be heard, is the primary material of this report, more significant than any comments on paper.