Appendix B

Resolution and the Power-of-Two

The closing demonstration of the recording illustrates a problem on resolution raised by Donald N. Ferguson in A History of Musical Thought,[12] a standard and up-to-date text. Resolution, the feeling of satisfying a musical tension, as when a melody ends on the tonic, the note Do, is a central problem in music, and deserves detailed inspection. As Ferguson says, "the principal clue to all understanding of tone relations is the sense of tonic or key."

Hindemith was quoted in a previous section to the effect that partials (overtones) do not explain musical roots. Ferguson presents, as a paradox, the failure of the phenomena of discord and concord to account for resolution and the sense of tonic. His specific example happens to serve strikingly as an illustration of resolution in accordance with the long pattern hypothesis, acting through the process that we call, as a shorthand term, the power-of-two phenomenon.

The reader has heard in the recording that Ferguson's example does not accomplish resolution as would be expected from a theory of discord and concord. It does resolve in another way, under the power-of-two phenomenon.

As Ferguson presents the problem:

"How far can the sense of tonic be explained on the basis of the perception of consonance and dissonance?

. . . .

"Physicist and musician agree that discord implies motion toward concord. To the physicist it would appear that for the dissonant interval F-B, the perfect concord F-c should offer a more satisfactory resolution than the imperfect concord E-c. But this supposition the musical ear denies. Even when we approach from F-B the concord F-c, but still more with the inversion of these intervals (B'-F to C-F), we feel that F is actually not concordant with C at all. .....Exactly as if it were a dissonant note, it wants to go to E--or at least we want it to; and all the physics in the world will not convince us that the perfect concord is a more satisfying tone combination than the imperfect. The musician's way of looking at discord and concord differs, apparently arbitrarily, from the physicist's way. Thus, although it appears true that the physical fact of discord relationship establishes for us the sense of tonic in our scale, it must be admitted that the implications of physical science are also denied in the structure of our scale; and that physics does not fully account for the sense of tonic."

In the example, as played in the recording, B♭-F is Ti-Fa. It is followed by C-F, which is Do-Fa, and is not a successful resolution. The ear wants C-E, which is Do-Mi, and does accomplish the resolution.

The situation is clear on the referential keyboard, with the ratios lettered on the keys. The original interval, Ti up to Fa, is the ratio 64/45, not a harmony. It needs resolution. The classical Fa is related to Do as 4/3, with Fa having four waves to three of Do. Thus Fa stands in a strong harmonic ratio to Do, <u>but not to Do in its capacity as root</u>--it forms a common pattern with <u>three</u> waves of Do, not with two, or four, or eight, etc.

The note Ti, with the ratio 15/8 to a Do below it, stands as 15/16 to the Do immediately above it. Ti is notable as a leading tone, a successful next-to-last tone in a melody, introducing Do in its capacity as tonic. It is related to an octave subdivision, 16 waves, a power-of-two. With Ti-Fa then, we have two notes that clash with each other, but share a relationship to a third note, Do.

The interval Do-Fa, as Ferguson rightly says, does not resolve the tension. It sounds the Do, which Ti called for, and sounds Fa, closely related to Do, <u>but not to Do as a root</u>. Do is the <u>3</u>, and Fa is the <u>4</u> in this combination. So far as the long patterns are concerned, this should not be Do up to Fa, in the key of C, but Sol up to Do in a different key, the key of F. C up to F is a perfectly good concord in the key of C, but not a resolving pattern that ends the tension by coming to rest on C. This is what we would expect from the long pattern relationships, and this is what we hear in fact, as demonstrated in the recording.

The pattern Ti-Fa is resolved successfully by C-E, which is Do-Mi. This is the ratio 5/4, in which Do is the <u>4</u>, the power-of-two note, and Mi is the 5.

The ratio 4/3 is a closer <u>concord</u> than 5/4. By a tradition inherited from the Pythagoreans, 4/3 is rated among the perfect concords, and 5/4 is classed among the more complex intervals, to which the classification of concord was stretched later, but with the reservation <u>imperfect</u>. The classification is correct enough in the sense that 5/4 is more complex. Starting from the proposition that "discord implies motion toward concord," as Ferguson does, 4/3, a good concord, should fill the bill. It does not do so, casting doubt on the concept that resolution amounts to discord moving to concord.

The situation becomes different if we assume that a musical context sets up a long pattern and operates around it. The tension of the pattern can be resolved by sounding the note that *is* the long pattern, in one of its octave subdivisions, and sounding it *in its capacity* as the long pattern. C-F, or Do-Fa, does sound the note Do, but not in its capacity as a tonic, for Fa makes a pattern with three waves of the Do, not with any power-of-two, or octave subdivision of the long pattern of the musical context.

In contrast, C-E, Do-Mi, 5/4, sounds Do *as* the tonic. Thus the resolution can be explained by long patterns, and not by discord moving to concord.

It must be added that although Fa is 4/3, with a ratio of 64/45 to Ti, on paper and in physical vibrations when so tuned on the organ, we cannot take for granted that this is the whole story. Possibly the ear treats the note as an approximation for something more codable. Good procedure requires examination of this possibility.

We have made some informal tests which suggest that the ear does code the note Fa otherwise in some contexts. Formal testing is yet to be done.

The informal tests relate to the dominant 7th chord, of which the Ti-Fa interval is a part. The familiar Amen chord is the dominant 7th, resolved to a major chord. The dominant 7th is Sol-Ti-Re-Fa, and it resolves to Sol-Do-Mi. These contain the Ti-Fa and Do-Mi that we are discussing. In tests at the organ, several musicians, including some choir singers, have expressed a preference for the natural 7th over Sol in the dominant 7th chord. This is the note Faw, 7/4 over Sol, and 21/16 over Do. Faw is markedly flatter, by 27.2 cents, than classical Fa. One well-trained choir singer, on hearing the Amen chord as Sol-Ti-Re-Faw, exclaimed, "That's the way we sing it!"

Perhaps, under some circumstances, the auditory system codes Ti-Fa as Ti-Faw. The notes Ti and Faw stand to each other in the ratio 7/5, highly complex, not a smooth staple harmony, but codable. Ti below Do is 15/16, while Faw above it is 21/16, both related to an octave subdivision of Do, a power of two, and hence both calling for it as resolution.

This is stated for the record as a possibility. More testing is needed on this pattern. In either case, whether Fa or Faw is coded, the long pattern accounts for the resolution.

The three notes Sol, Ti and Re can be said to lead to the tonic, in the sense that they relate to it as tonic, by relating to an octave subdivision of it, a power-of-two. Sol is

3/2, Ti is 15/8, Re is 9/8. It is quite in order for this group to introduce Do as tonic.

Fa 4/3 in its own right relates to Do otherwise than as tonic, and would set up a clash with the remainder of the chord. This may be what happens. On the other hand, if this note is coded by the auditory system as Faw, 21/16 to Do, this would be the fourth note leading to Do by relating to an octave subdivision. This may well happen in the presence of Sol, Ti and Re, and not in another context.

We have quoted elsewhere Hindemith's remark that it is astounding that instruction in harmony has never succeeded in producing a theory of melody. Ferguson's example provides a specific illustration of the problem. One might suppose that it would be possible to account for motion from one note to another by partials, or by a principle of discord and concord, but when these principles are applied to a specific case they do not work out. We experience auditory demands for notes different from those predicted by the theories.

In contrast, in this example, the long pattern hypothesis accounts for the notes that the ear demands.

End of Appendix B

Appendix C

A Basis for Musical Organization in Everyday Hearing

As this report was being written an article was published in the Scientific American for April, 1962, supplying evidence in support of one aspect of the long pattern hypothesis. The article, "Attention and the Perception of Speech," by Donald E. Broadbent,[13] mentions experiments in which components of a sound were heard as coming from the same source if they had the same pulse (long pattern, in our terms) but were heard as from different sources if the pulses differed.

This is the aspect of hearing which we discussed as basic in our article on the long pattern hypothesis. In brief, the long pattern hypothesis calls attention to the fact that we detect a sound as coming from a source. We hear the voice of a person, the growl of a dog. The growl calls for action. Our problem is not to contemplate the separate components of the growl, but to escape the dog or take counter-measures. He growls, not in a soundproof room, but in a noisy environment. Our eardrums are vibrating in literally hundreds of other ways, with which the dozens of components of the dog's growl are mixed.

Our first task is to separate the components of the growl from the other vibrations, and organize them as belonging together, coming from a source. They share a characteristic that can mark them as belonging together: all harmonic partials from a complex vibration share a common long pattern, which is the same as the frequency of the first partial or fundamental.

We said in the long pattern article:[14] "The frequency concept of pitch assumes the rule: one frequency--one sensation. Our experience in normal listening is described better by the rule: one source--one sensation. This gives us efficient listening for sensing the environment, and it can be explained by a common long pattern theory, while the frequency theory fails to explain it."

The long pattern hypothesis regards organization by common long patterns as necessary for meaningful hearing of ordinary sounds. Harmonic relationships in music are extended applications of this long pattern organization. Notes in harmonic ratios, like 3/2 or 5/4, share common long patterns, and this may be the reason for our perception of them as related.

Thus musical organization can be regarded as a by-product or extension of the essential aspect of everyday hearing that enables us to sense the environment by the rule: one source--one sensation.

As Broadbent describes his experiment: "Peter Ladefoged of the University of Edinburgh and I have been experimenting with a device that was developed by Walter Lawrence of the Signals Research Development establishment in England. Our version of the apparatus sends a series of electrical pulses (analogous to pulses from the vocal cords) through two filter circuits, each of which passes primarily one frequency. The waves from one filter circuit, which are like those from the largest human speech cavity, are mixed with waves from the other, which imitate the frequencies produced by the second largest cavity. Together the two wave trains are heard as quite acceptable vowel sounds that can be changed by tuning the filters to different frequencies. Varying the pulse rate used to excite the filters alters the apparent pitch or intonation of the "speech": it rises with faster pulse rates and falls with slower ones.

"When the same pulses excite both filters, a listener hears the output as readily identifiable vowel sounds. This is true even when the low frequency is fed into one ear and the high frequency into the other. But if the two filters are pulsed at slightly different rates, the "speech" becomes unacceptable and the listeners say that *they are hearing two sounds coming from two sources rather than a single vowel sound*. (Italics supplied.)

"Other experiments on the fusion of sounds at the two ears conducted by Colin A. Cherry and his colleagues at the Imperial College of Science and Technology in London, also support the idea that when the rate of pulsing, or modulation, is the same for two sounds, the hearer perceives them as one sound. It seems reasonable to suppose, therefore, that a man can listen to one person and ignore another primarily by selecting from the mass of sounds entering his ears all those frequencies that are being modulated at the same rate. Since it is most unlikely that the vocal cords of two speakers would vibrate at exactly the same rate at any moment, modulation would almost always provide an important (if not the sole) means of separating a pair of voices."

The human ability to organize tone through interference is discussed at some length in our long pattern article. The point, which receives welcome support from the Broadbent article is that organization by long patterns--organization through time--should not be thought of as a specialized musical aspect of hearing, but a constant, basic necessary process, used all the time as a necessity for making sense out of sounds.

The patterns of music being found and explored in this project suggest that organization by long patterns is capable of some sophisticated applications and extensions. Even without analysis we know that we experience music as strong,

Appendix D

Doubts about Pure Tones and Ohm's Law of Acoustics

The classical just scale is sometimes called "the natural scale." This term never rested on any evidence that this scale could be found in natural music, but on the argument that it had a natural basis in the partial series. It has never been a true statement that musical notes, of any system, are represented by the partial series. Diagrams in harmony books showing notes as drawn from the partial series are produced by fudging the figures. Some books include footnotes acknowledging that certain notes are fudged, others leave this out.

To make matters worse, the partial series itself has been losing status during recent years, because it does not supply answers to the problems of hearing. The present direction of thinking is indicated in the following quotation from a book, Waves and the Ear,[15] by three Bell Telephone Laboratory men, van Bergeijk, Pierce, and David. "It is perhaps unfortunate for the student of hearing that Fourier ever discovered his famous theorem that all sounds, however complicated, can be considered as built up of many simple sine waves. It was only natural that hearing should be investigated with the simplest possible sounds; so almost all our knowledge is based on measurements with sinusoids. What is unfortunate about all this is that sine waves are also the most meaningless sounds that one can think of. The sounds that are meaningful to us--speech, music, traffic noise--are anything but sinusoids, and the belief is rising (from such contradictions to Ohm's Law as we have seen) that the ear is more than an organ to break down complex sounds into sine waves. The ear and brain seem to operate on patterns of sound, and ordinarily we are not aware of the constituent sine waves in the pattern. I think that the most exciting and profitable work is going to be done in this field of pattern recognition. But how do you go about it? That, you will recognize, is the vital, strategic question, and people are presently working with it, trying to find a stimulating question that may start them on the way."

The findings in our project suggest that nerve coding operating through time is a promising candidate for investigation in connection with pattern recognition, and that the phenomena of music, examined by the proper methods, raise some stimulating questions in this area of the theory of hearing.

As recently as 1937, Stevens and Davis, in Hearing,[16] considered it proper to leave out discussion of theory, on the grounds that Ohm's Law of Acoustics and the Fourier theorem described the basic conditions of all hearing, including perception of speech sounds.

Thirteen years later, in 1950, Ira Hirsch, in his book The Measurement of Hearing,[17] was saying, in contrast, "So far as the relation between the audiogram and DISCRIMINATION LOSS is concerned we can only point out that here lies one of the greatest points of ignorance in contemporary clinical audiology." The audiogram, of course, is a test of hearing by sine waves, the score made in hearing various pure tones across the ear's range. It was no longer possible to assert that an individual's ability in useful hearing, in discriminating, even a simple speech sound, such as an S, could be measured by testing him on pure tones which might be supposed to make up that speech sound.

When Hallowell Davis, one of the authors of Hearing, was making up his index of social adequacy in hearing, in the year 1948, he used in essence only one audiogram tone, a thousand cycle pure tone, and that only to measure how loud a sound had to be to reach his subject's attention. The remainder of the test is a list of words, measuring the ability to hear speech by the score on speech, without any attempt to use sine wave scores as indicators of performance in hearing.

Some recent work suggests that we recognize speech sounds by organizing our perception of their formants, or patterns, which may come in various frequencies, as in music a major chord is recognized as the same pattern whether in high or low register. From Waves and the Ear,[18] again: "In spite of this difference in size of vocal tracts and the corresponding difference in the sounds produced, the phonetic value or color of the same vowel pronounced by different speakers can be recognized or perceived without difficulty by the listener. The key to this process lies in the ratio of the resonant frequencies which are about the same when different speakers utter the same vowel. Thus vowels are similar to musical chords, which maintain their identity when played in various keys."

The quotation illustrates the new approach to the theory of hearing. As for an underlying similarity in hearing vowels and chords, it cannot be taken as established until more is known about how hearing operates, but it represents a promising path for investigation. It is possible that the present project will come full circle and lead back to speech processes, which were the starting point. Our project began as an inquiry into some problems in speech, and in hearing testing. It seemed that certain phenomena in music should offer clues. The original string apparatus was built, in 1956, not for experiments on scales, but for use in reading about scales. It was devised as a simple system for producing with adequate accuracy the notes printed as formulas in books and articles on scales. It was a do-it-yourself system of auditory illustration, serving the same purpose that the recording serves for this report i.e., letting the reader hear the sounds.

To our surprise the patterns of music turned out to be not as printed, and sine wave theory proved irrelevant to the organization found.

End of Appendix D

Appendix E

A Physicist-Musician on Pure versus Tempered Tuning

The following quotation represents the point of view of a physicist who is also a musician, writing in 1960, about musical tuning. Arthur H. Benade is a professor of physics at the Case Institute of Technology, and an accomplished flutist. He says in his book, Horns, Strings and Harmony:[19] "To the Greeks, and to many people who came later, the magic of pure number was so potent that they were quite willing to settle for the numerical relations they found, without ever asking <u>why</u> they came out that way. Some musicians with more logical correctness than physical knowledge have seized upon the Pythagoreans' need to know the musical answer beforehand as evidence that the numerical method of organizing music is a sort of peculiar accident. They are tempted to assert that all musical rules are a matter of custom and agreement, and that there is no absolute right or wrong way to go about choosing notes in music. My own belief is that music is <u>very</u> strongly affected by the manner in which things vibrate, <u>and</u> by the manner in which our ears work."

At a later point Dr. Benade discusses keyed instruments, such as the trumpet and clarinet, or his own instrument, the flute, which allow the performer some leeway in sharping or flatting the pitch by lipping the instrument, although they are keyed to play in equal temperament. He then adds a comment about the need for variations from equal temperament, speaking presumably from his own experience as a flutist. The comment is not unusual--it is rather the routine report of instrumental musicians--but it gains force for our topic by coming from a musician who <u>is</u> also knowledgeable <u>in</u> acoustics.[20] "Yet when one of these [keyed wind instruments] is used along with a piano, a skillful player will sometimes unconsciously depart from even temperament for the purpose of getting a more musical compromise. This necessity for continually adapting to the piano's even temperament puts somewhat of a burden on the instrumentalist who plays with the piano."

No musician subject in our project has selected equally tempered tuning as satisfactory for any melody, and none of them has been surprised at this, since there is a general awareness among good musicians that equal temperament has drawbacks.

End of Appendix E

Appendix F

The Concept of the Scale as an Orderly Array

The concept that a musical scale is an orderly array of notes is surprisingly potent in shaping theories of music. It leads a theorist to begin with the unspoken assumption that the right answer, when he finds it, must be some single row of notes.

The Latin scala means ladder, and a musical scale traditionally is assumed to be something like a ladder, with only one rung to each step. The reader has observed that the selections made by musicians in this project often require alternative notes, two or more rungs at some steps. Tradition has shaped the concept of one right note to a step, and undoubtedly it is reinforced by the fact that music sounds like an orderly array of notes when we hear it. On the referential keyboard we see that each level of the keyboard is indeed an orderly array. A typical melody uses several orderly arrays, linked orderly arrays, and only when we consolidate the several arrays into a single line (as was done in the recording for "Empty Bed Blues") do we get a close crowding of alternative notes, providing several rungs for a step. They are not so consolidated in a melody. If we want to use the ladder metaphor, we should say that the structure of a melody uses several ladders, with steps spaced apart, not that there is a single ladder with several rungs per step.

Paul Hindemith as a writer on theory does not gloss over facts; he faces issues clearly, which is an advantage in quoting from him. He states the present issue in definite terms which show how the concept of the orderly array can be a roadblock to analysis.

In Hindemith's Craft of Musical Composition[21] he partially abandons the partial series as the source of the scale, since he finds overtones beyond the sixth unsatisfactory, so he bases his scale on multiples of the first six partials. At certain points the calculations offer a choice of more than one note, and he feels it necessary to select one, rejecting the other for all contexts. This is illustrated in his discussion of the 7th overtone. "The seventh overtone of C, --B-flat (448) Hertz cannot be used. If we attempted to apply the same procedures to it as to its predecessors, we should arrive at terrifying results." He then lists a number of these results, which are alternative notes, a surplus of rungs. For example, he speaks of "B-flat (112) of which we may say in advance that it suits our purposes less well than the B-flat (113.78) which we shall arrive at by other means. (The superiority of the latter will appear from the consideration of the distances between tones, which we shall soon undertake.)"

We do not use overtones in calculation, but the notes he is speaking of as derived from the 7th overtone are the same, in some cases, as the septimal notes on the referential keyboard (notes standing at 7/4, 7/5 or 7/6 over the reference). Joseph Boatner chose some septimal notes in "If I Didn't Care." There are some in "Empty Bed Blues." Various other musicians have used septimal notes in various contexts on the referential organ. The reader heard a septimal note, 7/4 over Sol, in the experiment on the dominant 7th chord in the recording. These notes sound correct and good to our musician subjects in contexts that call for them.

Hindemith illustrates the effect of the concept of the orderly array when he rejects one B-flat because "it suits our purposes less well" than another. Clearly he holds that it is necessary to make a choice between these notes, and that the choice must be made for the purposes of music <u>in general</u>. The traditional concept of the scale rejects "in advance" the concept that musical contexts vary, and that one tuning may suit our purposes for one context, while another tuning will be called for by a different context.

As for distance, Hindemith says in the next paragraph, "In the distances between the tones, there must be some clear order. The smallest interval thus far is the minor second between E and F, and it should not be hard to establish this as the smallest interval in our scale. But the new smallest interval E-flat 74.66 to E-flat 76.8 6 would assert its claims. And since it would not do to provide only one or two tones of the scale with auxiliary tones which were simply slight flattenings of the original tones, every tone of the scale would have to be provided with a similar auxiliary. And these auxiliaries would have to have other tones at a similar distance below them, and so on, until we should have a hundred or more separate tones to the octave. Such a structure would be impractical, and musical technique could not cope with it. To realize to the fullest extent how meaningless it must remain for practical music, one need only imagine a singer hopelessly struggling with such small intervals."

Hindemith supplies an invaluable description of the process of arriving at a musical scale by calculation. There is a certain false appearance of inevitability about scales handed down by tradition or authority. The choices that had to be made are not mentioned, much less the reasons for the choices. Hindemith, in contrast, gives us the choices, and the reasons.

Obviously we do not agree with his rejection of alternative notes, nor with the proposition that a singer would struggle hopelessly with the small intervals. In referential tuning, which does have small <u>distinctions</u> (though not small

Hertz?

intervals, unless one thinks of all the referential levels as consolidated to a single row) it does not appear that the tuning process is a struggle, nor even very complicated. At this point it appears that a performer, such as a singer, has a single reference for a melodic sequence, and holds to that one reference, singing a set of relatives to it, all simple relatives, in simple ratios.

So much for logic, a splendid tool in its place, but surely this is not the place for it. It would seem that musical scales should be sought in the tunings used by musicians as the first step. Mathematical processes are useful after one has the data, but the use of mathematics to create the data is so breathtakingly wrong that it need only be stated clearly to reveal itself as fallacy. It seems to be the standard practice in music theory, it has been going on for better than two thousand five hundred years since Pythagoras, but age has given it no validity.

We are not plumping for pure testing as against pure reason, for this easy alternative does not exist. Tests and measurement offer no escape from the responsibilities of interpretation. For example, it is no help to measure actual pitches used in performance and then interpret them as either matching or departing from a traditional scale that was set up by logic. Measurements yield raw data; this is an indispensable beginning, the only proper beginning, but no more than a beginning.

Martin Mayer, in The Schools,[22] comments to the point on this. " 'Research,' writes the psychologist Lee Cronbach with the assurance of a man stating something incontrovertible, 'can first reveal the essential nature of any problem and then test the efficacy of any proposed improvement in practice.' No physicist would be capable of such a statement. 'Research' in any science is raw material for judgment, and valueless without judgment. To say that people's judgments tend to be better when backed by information is by no means to say that 'research shows.' And when the subjects are such tricky and uncontrollable specimens as human beings, the information gained by research may well be wrong. Judgments based on wrong information may be worse than judgments without information."

We believe that music theory needs research, and that musicians should welcome it, but they are entirely right to be skeptical about research. We think that musicians are quite right to hold, as we know many of them do, that certain research findings are questionable because they run counter to the experience of musicians in their art.

End of Appendix F

Appendix G

A Note on Harmony

It is asserted by many music theorists that complex tunings are suitable for melody but unsuitable for harmony. Since Western music uses harmony it follows logically that Western music must use a simple unvarying scale. A comment is needed on this position because many readers will have read it.

This report contains no examination of harmony because we have not gone far enough with harmony to justify interim conclusions, but we have explored it to some extent. Some of our musician subjects have worked out harmonic accompaniments to melodies on the referential keyboard, and have found that referential harmonies sound right and proper to them.

Logically it would seem reasonable to say that if your bass stands still you are then restricted to those few notes that will harmonize with it. The situation is different if it is a linked set of references that stands still--then both bass and melody could move in tuning. Empirically, our musician subjects find that the latter is the case; if they want to use Say melodically, rather than Sol, they find that the satisfactory harmony is Day--May--Say. They do not find themselves restricted to Do in the bass, which would require Sol rather than Say as a satisfactory harmony.

Detailed discussion will require more experiment, and must wait for a future report. Meanwhile, readers interested in experimental evidence on this point may find it in <u>Genesis of a Music</u>, by Harry Partch.[23] A British music theorist told Partch that complex tuning was blocked by harmony in Western music, and also that it would make modulation impossible; he wrote out an example of a modulation which he said would be impossible in accurate tuning. Partch, who has built his own instruments in accurate tuning and composes music in his own scales, tried it, found that the modulation was accomplished, and in fact that it could be accomplished several different ways, satisfactorily not only for his own ear, but for various audiences. He said, "The contention that harmonic music with melodic subtleties is impractical and ineffective is answered in the two progressions diagrammed above."

End of Appendix G

Appendix H

A Practical Variable Keyboard

Statements similar to the following can be found in many books on scale theory: "So-called 'pure intonation' instruments ...are bound to have so many keys that they will always be too unwieldy to be of any use in musical practice." The particular wording is from Hindemith[24] but the same statement can be found in many books. It used to be true as a mechanical limitation; it is no longer true in the era of electrical switching.

Our own referential keyboard serves as a search keyboard, but it is unwieldy for performance. A practical instrument should be built with standard keyboard and fingering, because musicians are accustomed to this. It is entirely possible to set up variable referential tuning on a standard keyboard, providing for instant switching from one tuning formula to another by stops, or foot pedals, or pre-coded sequences, or in any number of ways. Limitations scarcely exist; anything at all can be done along this line.

To what formulas of tuning should the stops be set? This is the question that calls for exploration by research, not the mechanical problem.

Robert W. Young has described the current situation in the McGraw-Hill Encyclopedia of Science and Technology,[25] "The relatively recent availability of electrical and electronic musical instruments, and the possibility of rapid switching to different temperaments, have served to reopen the question of the tuning of keyboard instruments. Perhaps in another century the equally tempered scale will have been replaced--but if so, only in satisfaction of requirements of the musical ear and not of a mathematical postulate!"

Appendix I

Tables of Notes for Reference

In the following table comparing the tempered, classical just, and Pythagorean scales, the sol-fa note names for the classical just scale are the standard names, those used for the Pythagorean scale are from the coined names used in this project.

Tempered		Classical Just			Pythagorean		
Name	Cents	Name	Ratio	Cents	Name	Ratio	Cents
C	0	Do	1/1	0	Do	1/1	0
D	200	Re	9/8	203.9	Re	9/8	203.9
E	400	Mi	5/4	386.3	May	81/64	407.8
F	500	Fa	4/3	498.0	Fa	4/3	498.0
G	700	Sol	3/2	702.0	Sol	3/2	702.0
A	900	La	5/3	884.4	Lay	27/16	905.9
B	1100	Ti	15/8	1088.3	Tay	243/128	1109.8

As the cent values show, the tempered scale may well be described as a modified Pythagorean scale.

No Single Rule for Classical Accidentals

When a melody containing accidentals is played in classical just intonation it is necessary to decide on the tuning for the accidentals (black keys on the piano in C). Two different rules exist. One makes a sharp note flatter than the corresponding flat note (for example, G-sharp flatter than A-flat). An example of this formula may be found in McHose.[26] This is the system we have used, to the extent that the frequencies were available on the referential keyboard, and they usually were. A formula that makes the sharp notes sharper than the flat notes may be found in Redfield.[27]

Table of 53 Notes in the Octave
Provided by Five levels of the Referential Keyboard
in Major, Minor, and Blue Linkage

Name	Ratio	Cents	Name	Ratio	Cents
Du	63/32	1172.7	fen	7/5	582.5
Do	1/1	0	fi	45/32	590.2
Day	81/80	21.5	fon	567/400	604.0
			far	729/512	611.7
di	25/24	70.7	fan	36/25	631.3
die	21/20	84.5			
dah	135/128	92.2	Saw	35/24	653.0
dan	27/25	133.2	Su	189/128	674.7
			Sol	3/2	702.0
Raw	35/32	155.2	Say	243/160	723.5
Ror	567/512	176.6			
Re	9/8	203.9	si	25/16	772.6
Ral	729/640	225.4	sah	63/40	786.4
			sum	405/256	794.1
rie	7/6	266.9	sen	8/5	813.7
rah	75/64	274.6	son	81/50	835.2
ren	189/160	288.4			
ro	1215/1024	296.1	Lu	105/64	857.1
ri	6/5	315.6	La	5/3	884.4
ran	243/200	337.2	Lal	42/25	898.2
			Lay	27/16	905.9
Mor	315/256	359.0			
Mi	5/4	386.3	lie	7/4	968.8
Mal	63/50	400.1	lum	225/128	976.5
May	81/64	407.8	lor	567/320	991.3
			li	9/5	1017.6
Faw	21/16	470.8	lon	729/400	1039.1
Fu	675/512	478.5			
Fa	4/3	498.0	Tor	945/512	1061.0
Fay	27/20	519.5	Ti	15/8	1088.3
			Ta	189/100	1102.1
			Tay	243/128	1109.8
			Tal	48/25	1129.3

We have found some melodies that need more than the five levels now available. In many cases it is possible to find the pattern of a sixth level by using what we call a phantom first. It often happens that notes needed on the first level are duplicated in the second; i.e., in major linkage, 1/1 in the first level is 4/3 in the second. Treating the first level as the second, making 4/3 on the first level the tonic, changes the absolute pitch but makes available the equivalent of a sixth level.

This stratagem makes available nine additional ratios to the tonic:

Major

din	2187/2048	113.7
Lo	1701/1024	878.6
Lar	2187/1280	927.4

Minor

del	1701/1600	106.0
Fo	1701/1280	492.3
Fal	2187/1600	541.0

Blue

Don	2025/1040	1180.4
fel	2835/2048	562.9
lan	3645/2048	998.0

The 53 notes of the previous table plus these nine make 62 notes to the octave called for by the referential system as presently arranged, in major, minor and blue linkages.

End of Appendix I

Footnotes

1. (p. 1) Paul C. Boomsliter and Warren Creel, "The Long Pattern Hypothesis in Harmony and Hearing," The Journal of Music Theory, Yale School of Music, New Haven, Conn., April, 1961, pp. 2-31

2. (p. 3) Max Meyer, "Contributions to a Psychological Theory of Music," University of Missouri Studies, June, 1901; "The Musician's Arithmetic," University of Missouri Studies, January, 1929.

3. (p. 3) Harry Partch, Genesis of a Music, The University of Wisconsin Press, 1949.

4. (p. 6) J. Murray Barbour, Tuning and Temperament, a Historical Survey, Michigan State College Press, East Lansing, 1953, pp. 89 and 102.

5. (p. 7) Victor Zuckerkandl, Sound and Symbol, The Bollingen Foundation, New York, 1956, pp. 79 through 81.

6. (p. 9) Paul C. Greene, "Violin Performance with Reference to Tempered, Natural, and Pythagorean Intonation," in University of Iowa Studies, Studies in the Psychology of Music, Vol. IV, Objective Analysis of Musical Performance, pp. 232-51.

7. (p. 10) Max Meyer, see 1901 article, footnote 2..

8. (p. 10) Max Schoen, The Psychology of Music, The Ronald Press Co., N.Y., 1940, pp. 38 through 45.

9. (p. 12) Paul Hindemith, A Composer's World, Anchor Books, Doubleday and Co., 1961, p. 84.

10. (p. 13) Paul Hindemith, The Craft of Musical Composition, Book I, Associated Music Publishers, N.Y., 1937, p. 175.

11. (p. 18) Harry Partch, see footnote 3.

12. (p. 48) Donald N. Ferguson, A History of Musical Thought, Appleton-Century-Crofts, N.Y., 1959, pp. 5-6.

13. (p. 52) Donald E. Broadbent, "Attention and the Perception of Speech," Scientific American, April, 1962, pp. 146-8.

14. (p. 52) Boomsliter and Creel, footnote 1, pp. 12-13.

15. (p. 55) Willem A. van Bergeijk, John R. Pierce, and Edward E. David, Waves and the Ear, Anchor Books, Doubleday and Co., 1960, pp. 218-19.

16. (p. 55) S. S. Stevens and Hallowell Davis, Hearing, John Wiley and Sons, N.Y., 1938, see preface, page x.

17. (p. 56) Ira J. Hirsch, The Measurement of Hearing, , McGraw-Hill, N.Y., 1952, p. 149.

18. (p. 56) Waves, footnote 15, p. 184.

19. (p. 58) Arthur H. Benade, Horns, Strings and Harmony, Anchor Books, Doubleday and Co., 1960, p. 87.

20. (p. 58) Benade, see footnote 19, p. 132.

21. (p. 59) Hindemith, Craft, see footnote 10, p. 37-38.

22. (p. 61) Martin Mayer, The Schools, Harper and Brothers, N.Y., 1961, p. 66.

23. (p. 62) Harry Partch, see footnote 3, pp. 189-93.

24. (p. 63) Hindemith, Craft, see footnote 10, p. 52

25. (p. 63) Robert W. Young, author of articles on musical acoustics in McGraw-Hill Encyclopedia of Science and Technology, article on Musical Acoustics, p. 664.

26. (p. 64) Allen Irvine McHose, Basic Principles of the Technique of 18th and 19th Century Composition, Appleton-Century-Crofts, N.Y., p. 311.

27. (p. 64) John Redfield, Music, a Science and an Art, Tudor Publishing Co., N.Y., 1935, p. 58.