Prof. dr A. D. FOKKER

BEEKBERGEN[?]

De Hertsweide[?]

Engelanderweg 32 – Telef. 05766-2366[?]

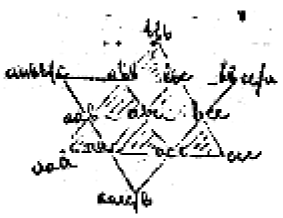
October 1st 1970

Dear doctor Wilson

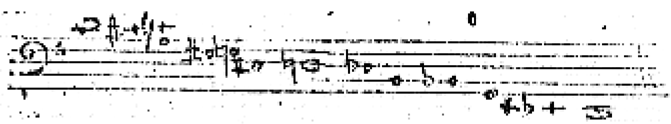
These last weeks I have been playing with combining musical systems. Perhaps you might be interested in it, and therefore I have pleasure to communicate with you. The element of a system is chose to be a chord of three notes, having intervals a/b, b/c and c/a, and you can write it as a triangle

We can multiply these numbers at pleasure, and write the chord

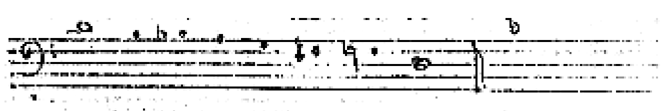
I choose the number abc as the centre of my system and I surround it by triangles,



representing either the same chords, with a : b : c, or the inverse ones with 1/a : 1/b : 1/c. You see what I mean. There are thirteen notes in all. You may leave out the central one, and have a scale of twelve notes, or you may leave out the six outmost notes, and have a scale of seven notes. Even with the classical numbers 4, 5, 6 you get a scale not known in classical music. For instance, choosing a as central note



This includes a coherent scale of seven notes only



I leave to you the pleasure of finding the scales resulting from the numbers [5, 6, 7], and other ones.

In fact the results are the same which I gave in my book *Neue Musik mit 31 Tönen* [New music with 31 notes], under the name Kreisspiegelungen eines Dreiklangs[?], mirroring cycles. Here, the system has been represented in a diagram, like a "mandala". Last year you sent me your mandalas. I am so very sorry I have lost them, unfortunately. If you would kindly send me another set of them, I would be very grateful.

C.J. Jung, when writing on mandala, stresses the fact of fourfold symmetry. Your mandalas, and mine, use other numbers.

Dear doctor Wilson, I shall very much like to hear from you again.

With best wishes

Sincerely yours,

A.D. Fokker

Ervin M. Wilson

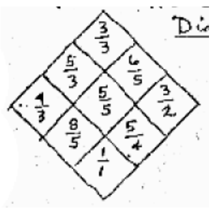
1219 Poinsettia Drive

Los Angeles, California

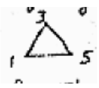
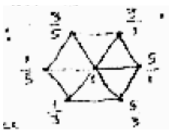
90046

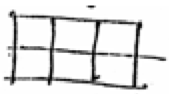
Dear Prof. Fokker,

Thank you for your correspondence of Oct 1970. The mirroring cycles you describe are, indeed, a pleasure to listen to. The lesser cycle I've given particular attention to. In his book, Genesis of a Music, Harry Partch diagrams the members of the smaller, 7-toned cycle, with 4, 5, 6 values thus:



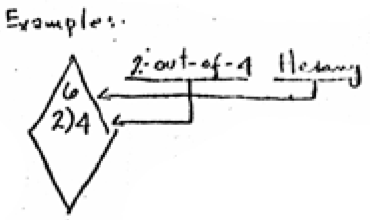
He labels this the Incipient Tonality Diamond. He appreciates the self-reciprocating character of the figure, but gives no indication that he understands its cyclic nature. The advantage of the modular geometries is that they very frequently remind the eye of symmetrical sequences that might too easily be overlooked. Assigning functions to the triadic module thus:

 we get the figure:  which gives the functions equivalent to the ratios in his "diamond". It is in accordance with Harry's precedent that I label this figure and its analogs in tetradic, pentadic, hexadic, heptadic, and ogdoadic etc tone-space "Diamond". These figures are self-mirroring and cyclical, "spherical", or "hyper-spherical" in the higher numbered orders of tone-space. They are all node- or tone-centered (even if we drop the central tone as you suggest) and their structure suggests a pervading tonic in the musical style. These may all be elaborated upon in concentric layers as you have done in your 13-toned figure. The figure I label Hexany-Diamond

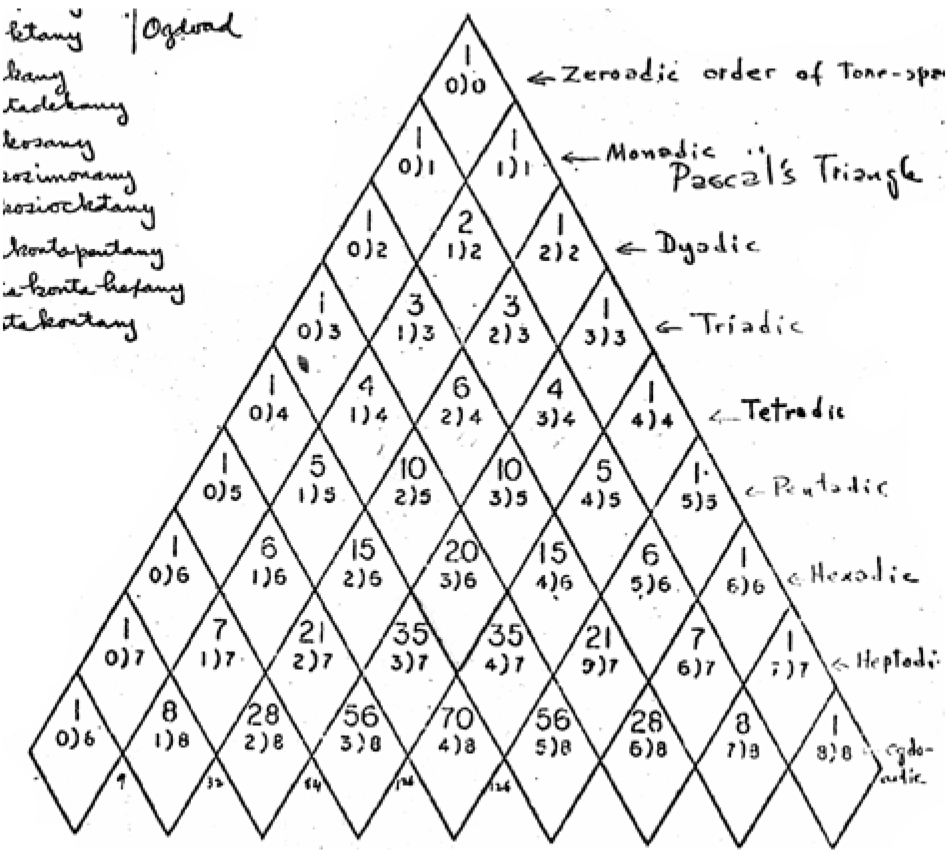
is a minimal symmetrical elaboration about the tetradic diamond.* I make no attempt to treat the diamonds as combinational sets. However, they are complementary to the combinational sets in the same way a  is complementary to a  in a cross-hatched grid  etc.

The combinational sets (which I have suffixed with “-any”) are interstice-centered, and do not imply a musical style oriented to the tonic. The combinational sets, like the diamonds, give circular, “spherical” or “hyper-spherical” patterns. However, these are not universally self-mirroring. The self-mirroring sets are, in fact, restricted to the series, (1-out-of-2), 2-out-of-4, 3-out-of-6, 4-out-of-8, etc. I abbreviate these figures, and label them thus; 2)4 hexany, 3)6 eikosany, 4)8 hebdomekontany. It is these figures which are of exceptional interest to me.

*(The figure I label “Mandala” is an elaboration upon the 2)4 Hexany. 4 tetrads plus their 4 reciprocal tetrads flank the 8 facets of the Hexany. The hexany, itself, implies the number 4, as it is formed by combinations of 2-out-of-4.)



No. of Tones	Name of combinational set	(Primary module)
1	Monany	Monad
2	Dyany	Dyad
3	Triany	Triad
4	Tetrany	Tetrad
5	Pentany	Pentad
6	Hexany	Hexad
7	Heptany	Heptad
8	Oktany	Ogdoad
10	Dekany	
15	Pentadekany	
20	Eikosany	
21	Eikosimonany	
28	Eikosiocktany	
35	Triakontapentany	
56	Pentakontakexany	
70	Heptakontany	



I use Pascal's Triangle to codify the combinational sets. I extend the meaning of the word "combination" to include 0-out-of-N, 1-out-of-N etc to N-out-of-N. (OVER)

A. Each combinational-set contains the combinational-sets above it. Example: the 2)4 hexany contains the 1)3 triany and the 2)3 triany, and consequently, the 0)2 monany, the 1)2 dyany, &the 2)2 monany; and as the next consequence, the 0)1 monanay & the 1)1 monany; and as a final consequence the 0)0 monany.

B. Each of the combinational sets derived from a given primary module may be said to belong to the (horizontal) order of tone-space identified by that module (tetradic tone-space derives from tetrad etc). Each of the combination sets in a given order of tone-space, when taken as a subset of a given (applicable) "matrix" set (in a higher order of tone-space) will variegate into a spectrum of varieties. Each of the combinations in a given order of tone-space will variegate into the same number of varieties in regards to the given matrix combination-set within which it occurs.

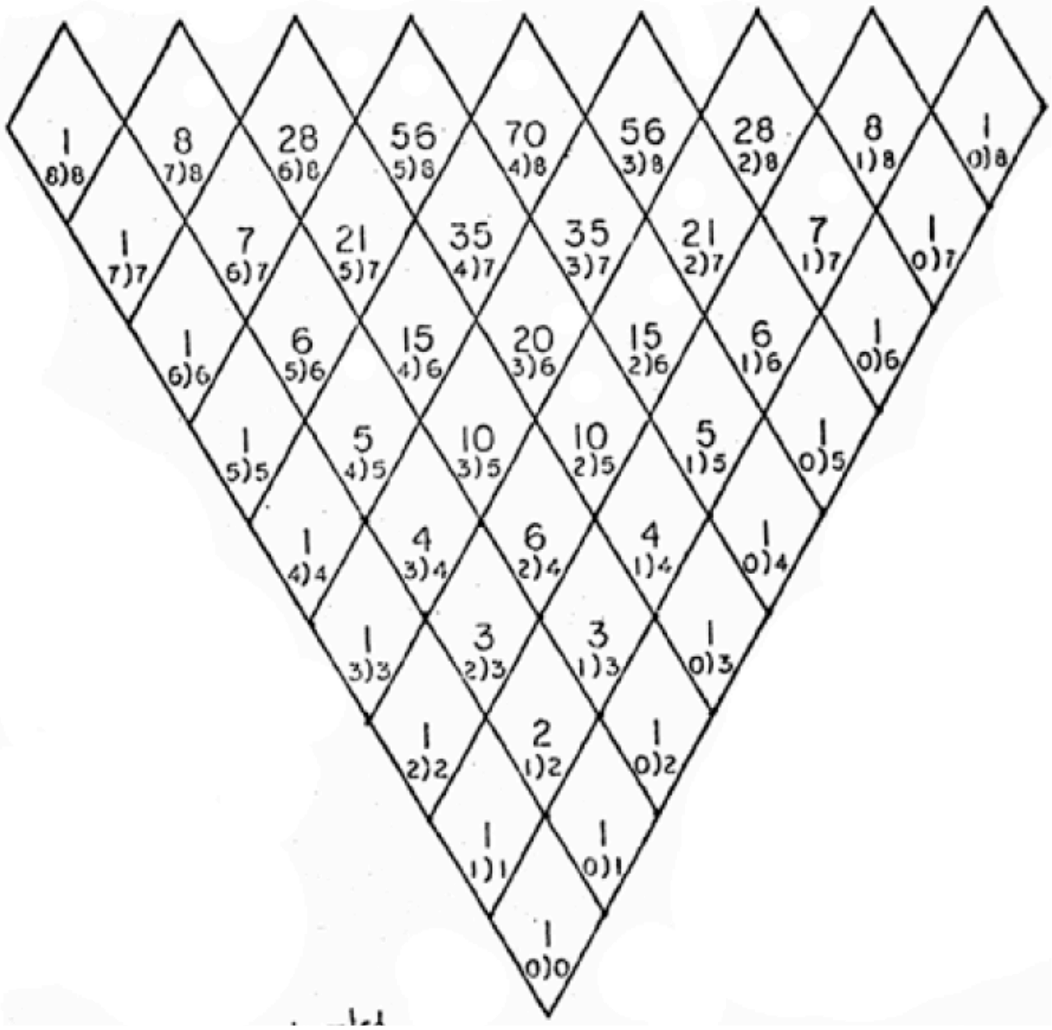
C. The number of such varieties (according to B) can be found by relating the order of tone-space (in which the combination occurs), in a descending vertical sequence to the numbers, in sequence, along the base of the triangle formed by that order of tone-space containing the matrix combination-set.

Example in the 2)4 hexany matrix:

The zeroadic combination,	0)0 monanay	has 1 "variety"
The monadic combinations,	0)1 monanay & 1)1 monany	have 4 varieties
The dyadic combinations,	0)2 monany 1)2 dyany 2)2 monany	have 6 varieties
The triadic combinations,	1)3 triany & 2)3 triany	have 4 varieties
The tetradic combination,	2)4 hexany	has 1 variety

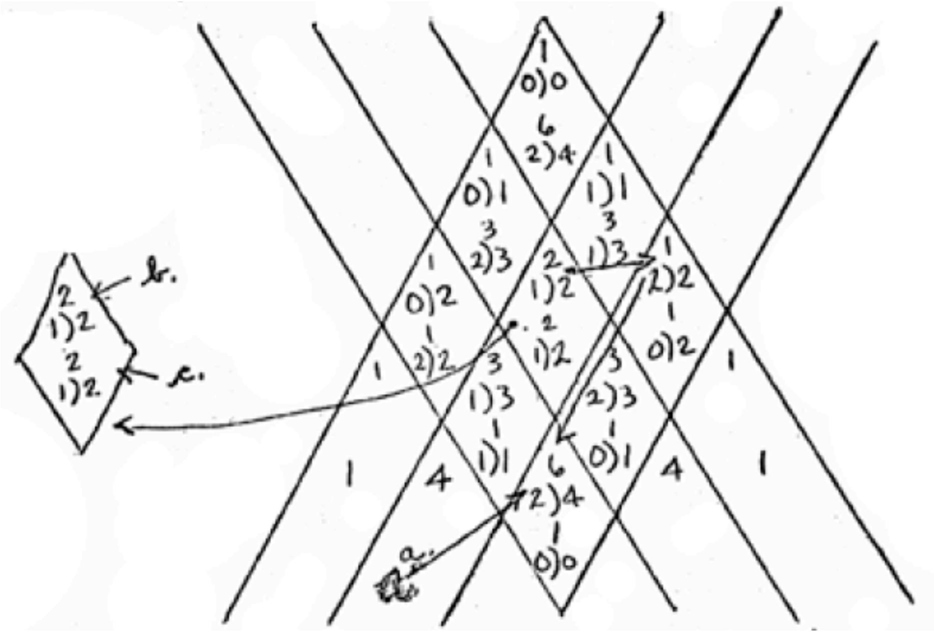
[↑ looking straight across tetradic line (KG)]

Pascal's Triangle, Inverted



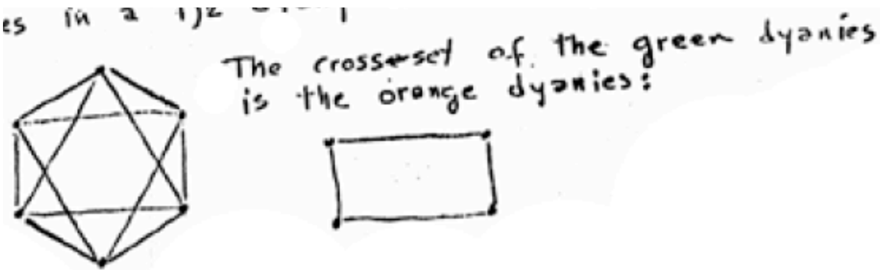
If I superimpose this inverted triangle over the upright triangle in such a way that the nadir of the inverted triangle corresponds with a given combinational-set of the upright triangle, the overlapping area of the two triangles will give the sub-combinational-sets of said, given combinational-set. (See example on another sheet)

D. Each of said varieties of the sub-set (according to B & C) will have a certain number of occurrences within the context of the matrix set. Each set of occurrences will form a combinational-cross-set to the sub-set being considered. The number of occurrences, and combinational-cross-set which these form, may be located by generating on inverted triangle (rotated 180 rather than mirrored) from the matrix set. Example for 2)4 hexany matrix:

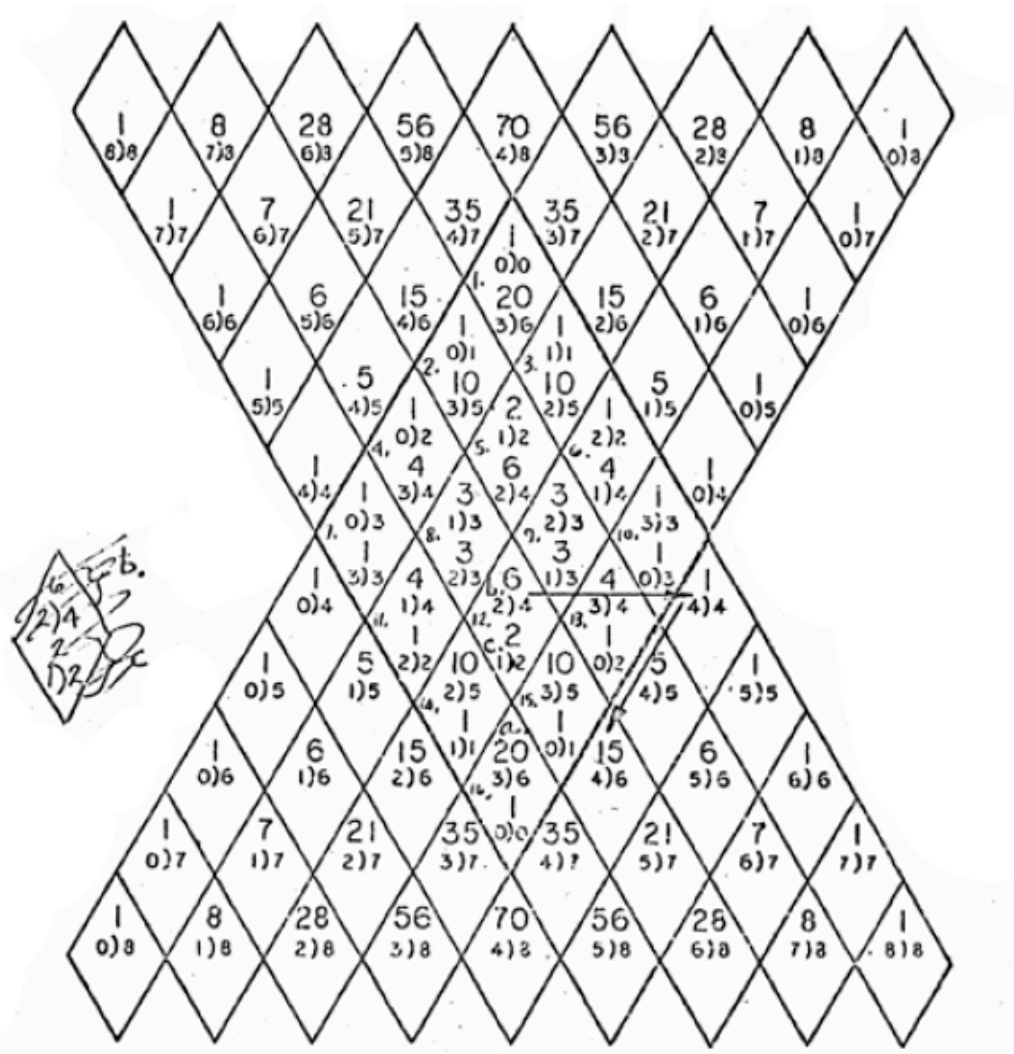


Thus, in the 2)4 hexany matrix (a.) the 1)2 dyany sub-set (b.) has 6 varieties [(green arrows) according to statement C] each of which occurs 2 times in a 1)2 dyany cross set (c.)

The cross-set of the green dyanies is the orange dyanies:



Superimposition of the inverted triangle & upright triangle with the nadir of the inverted triangle corresponding to the 3)6 eikosany of the upright triangle.



Example: In the 3)6 eikosany matrix (a.) the 2)4 hexany subset (b) has 15 varieties (arrow) each of which occurs 2 times in a 1)2 dyany cross-set (c.). See other side of sheet for complete list of subsets.

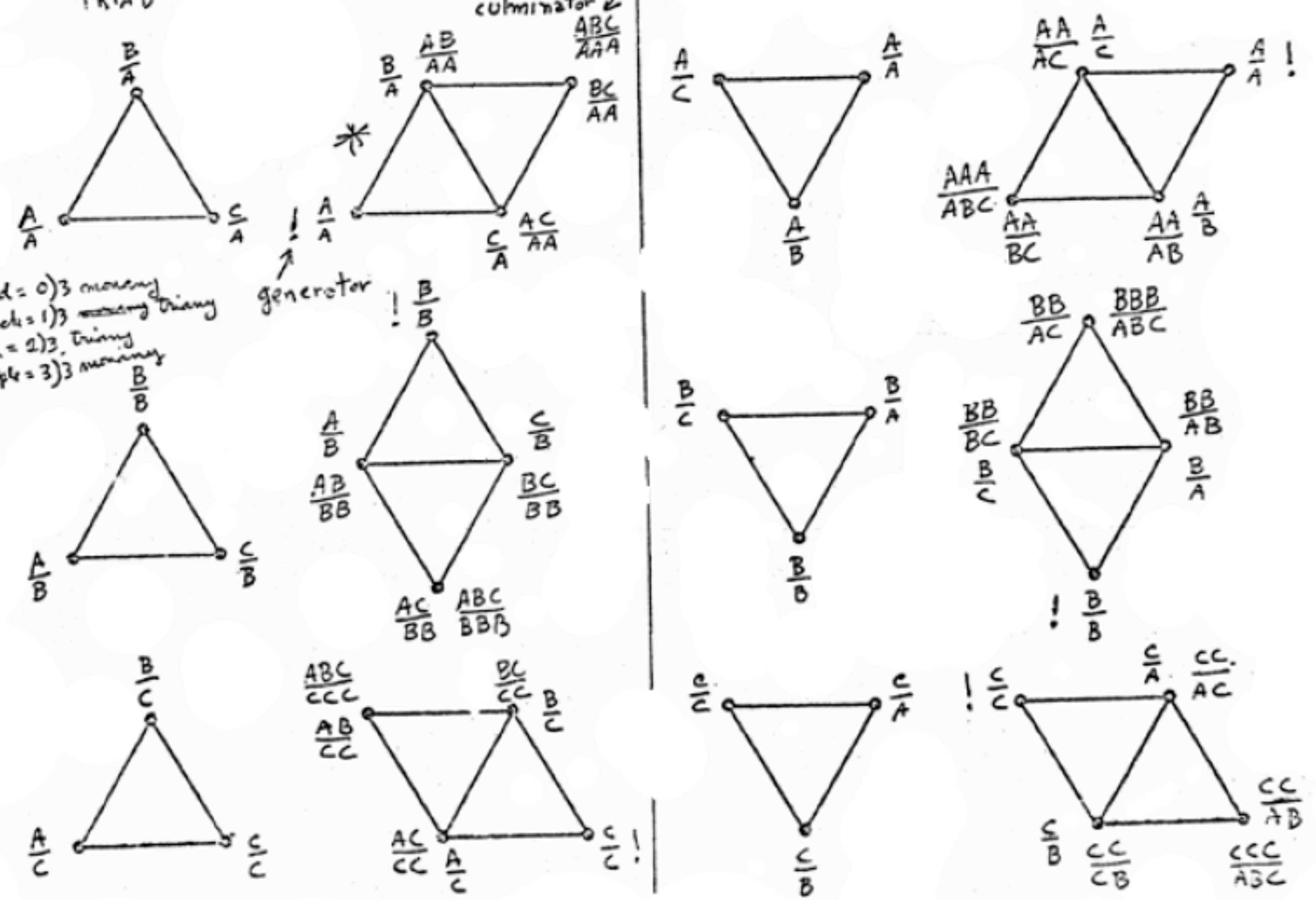
3)6 Eikosany Matrix:

Rhombus No. Red	Subset	Varieties of Subset	No. of occurrences each variety	Cross-set
1.	0)0 Monany	1	20	3)6 Eikosany
2.	0)1 Monany	6	10	3)5 Dekany
3.	1)1 Monany	6	10	2)5 Dekany
4.	0)2 Monany	15	4	3)4 Tetrany
5.	1)2 Dyany	15	6	2)4 Hexany
6.	2)2 Monany	15	4	1)4 Tetrany
7.	0)3 Monany	20	1	3)3 Monany
8.	1)3 Triany	20	3	2)3 Triany
9.	2)3 Triany	20	3	1)3 Triany
10.	3)3 Monany	20	1	0)3 Monany
11.	1)4 Tetrany	15	1	2)2 Monany
12.	2)4 Hexany	15	2	1)2 Dyany
13.	3)4 Tetrany	15	1	0)2 Monany
14.	2)5 Dekany	6	1	1)1 Monany
15.	3)5 Dekany	6	1	0)1 Monany
16.	3)6 Eikosany	1	1	0)0 Monany

TRIAD

TRIADIC COMBINATIONAL SERIES'

& RECIPROCALS

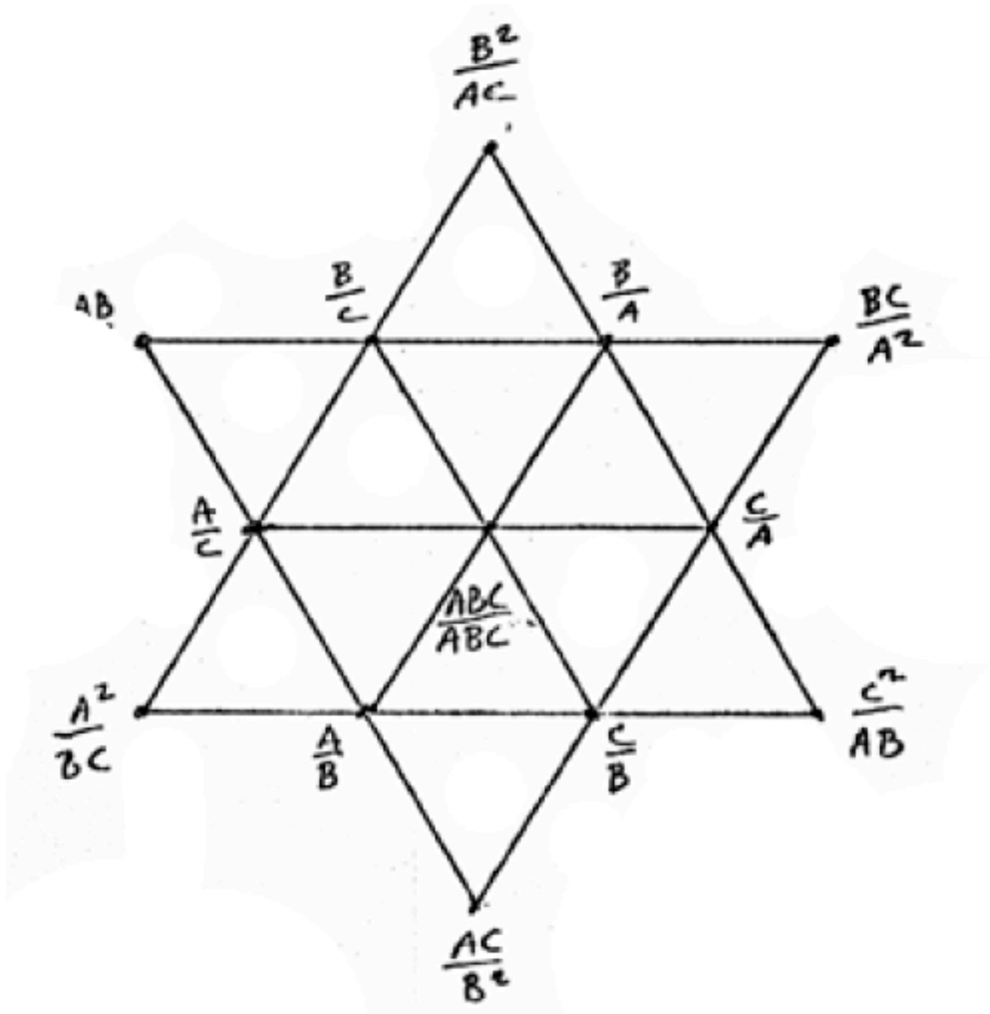


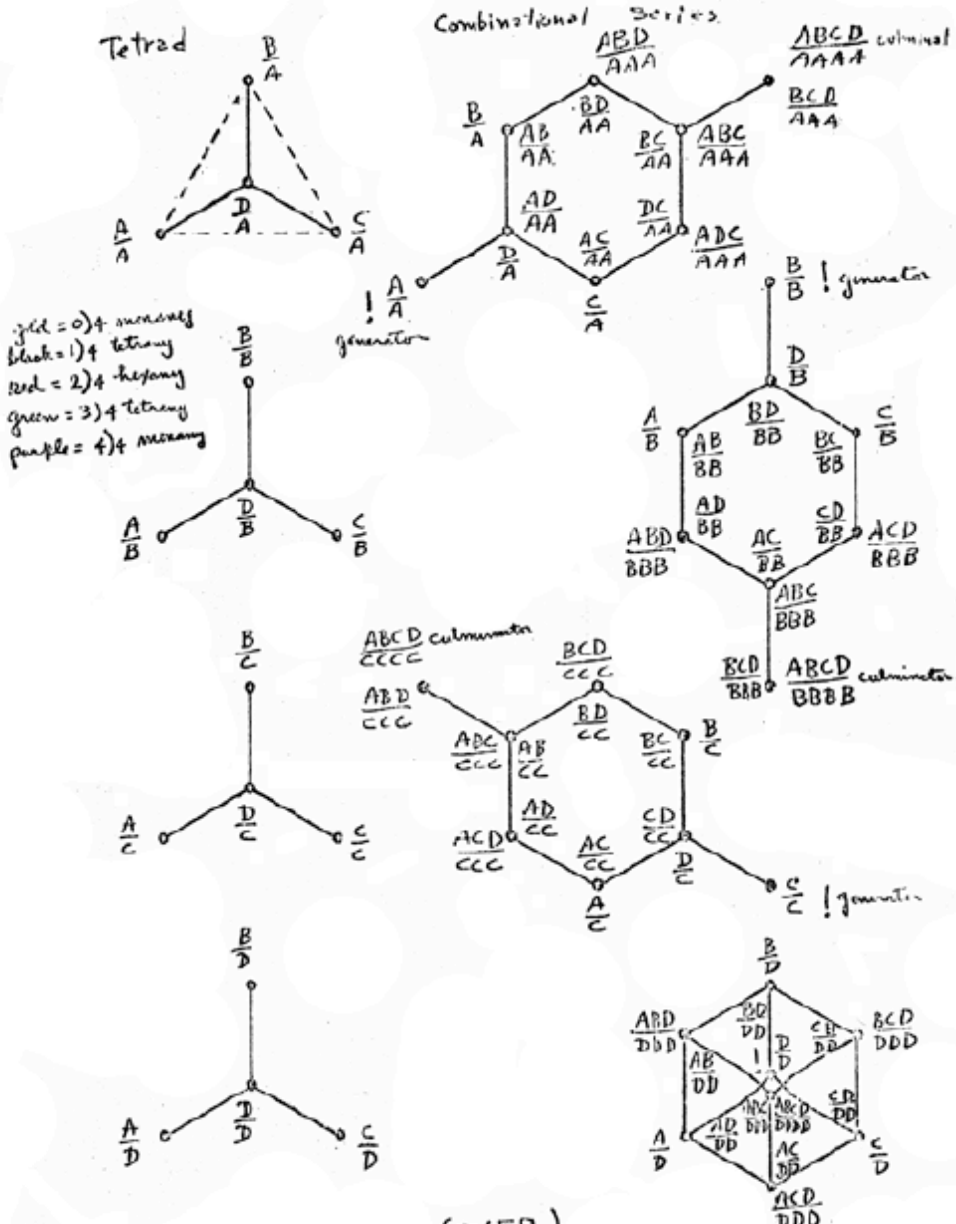
gold = 0)3 ~~money~~
black = 1)3 ~~money~~ thing
red = 2)3 thing
purple = 3)3 money

(OVER)

If, in the asterisked figure, I assign to A, B, C values of 1, 3, 5, the resultant figure is identical to Euler's 1·3·5 genus, the generator being identical to his fundamental, and the culminator to his guiding-tone. However the remaining variations of the combinational series are significantly different from the Euler genus.

If I form a 6-fold aggregate of the triadic combinational series I arrive at your mirroring cycle.

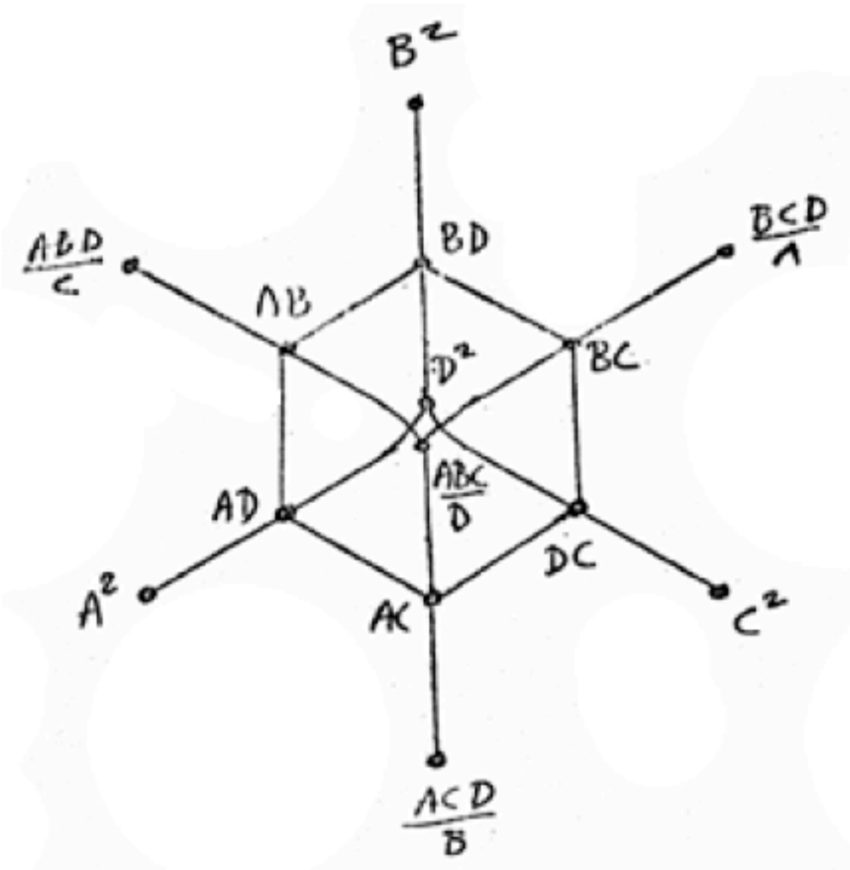



Tetrad
Combinational Series
ABCD/AAAA culminat
BCD/AAA
ABD/AAA
BD/AA
B/A
AB/AA
BC/AA
ABC/AAA
AD/AA
DC/AA
ADC/AAA
D/A
AC/AA
C/A
! A/A generator
B/B ! generator
gold = 0)4 monany
black = 1)4 tetrany
red = 2)4 hexany
green = 3)4 tetrany
purple = 4)4 monany
D/B
BD/BB
A/B
C/B
AB/BB
BC/BB
AD/BB
CD/BB
ABD/BBB
AC/BB
ACD/BBB
ABC/BBB
BCD/BBB
ABCD/BBBB culminator
ABCD/CCCC culminator
BCD/CCC
ABD/CCC
BD/CC
ABC/CCC
AB/CC
BC/CC
B/C
AD/CC
CD/CC
ACD/CCC
AC/CC
D/C
A/C
C/C ! generator
B/D
ABD/DDD
BD/DD
CD/DD
BCD/DDD
AB/DD
D/D
ABC/DDD
ABCD/DDDD
A/D
AD/DD
AC/DD
C/D
ACD/DDD
D/D

(OVER)

(I'm using, here, an abbreviated lattice for drafting simplicity)

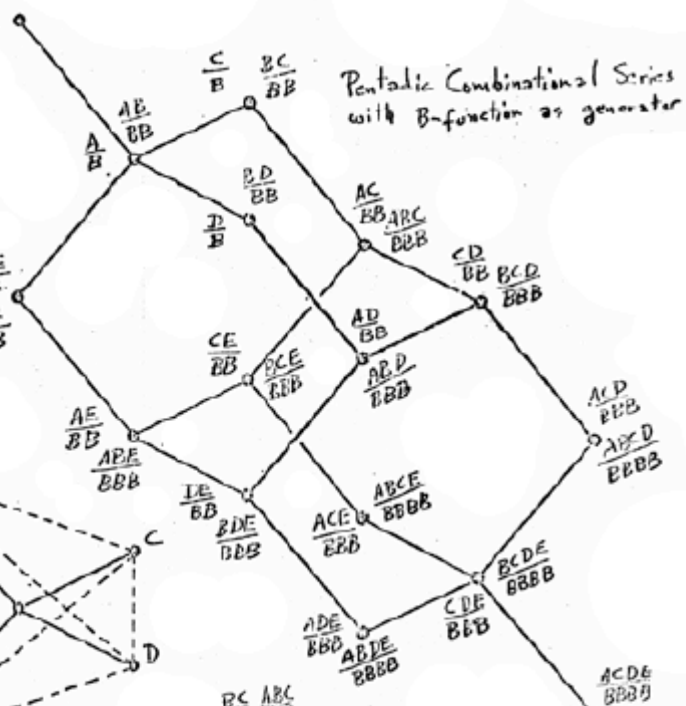
The 4 tetradic combinational series (and their 4 reciprocals, not shown) may be superimposed on the 6 tones of the 2)4 hexany (in red) resulting in the figure I call mandala. Analogs to this figure exist in hexadic and ogdoadic, in all even-numbered orders of tone-space.

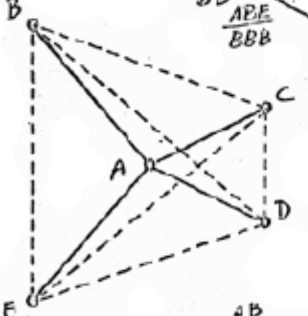


This figure is identical to its reciprocal, that is self-mirroring. The tetrads progress in a "spherical: pattern, or, more specifically, in an octahedral pattern. (See separate sheet for complete lattice) generator | $\frac{B}{B}$

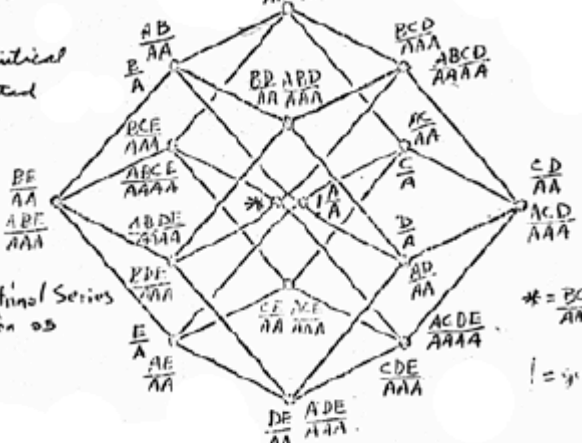
Pentadic Combinational Series with B-function as generator

gold = 0)5 Monany
black = 1)5 pentany *
red = 2)5 dekany
green = 3)5 dekany
purple = 4)5 pentany
orange = 5)5 monany.





* 1)5 pentany is identical to generating pentad



Pentadic Combinational Series with A-function as generator

* = $\frac{BCDE}{AAAA}$ &

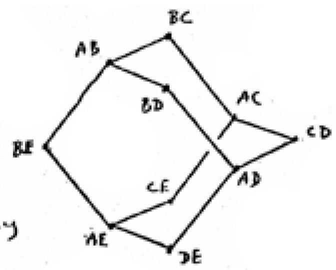
| = generator

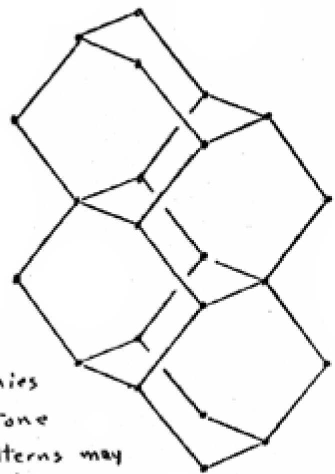
(OVER)

This lattice is intended to be constructed and viewed in full-space. The generating module is an abbreviation of the centered-tetrahedron, using only the solid lines from corners to center; as shown. When C, D, & E functions are used as generator (not shown) the results are structurally similar to the B-function generator (shown). Of extraordinary interest, to me, is the structure developed when the center of the centered-tetrahedron is taken as the generator (in this case the A-function, shown). This structure when seen in full space is the rhombic dodecahedron, plus dual centers at the generator & culminator. I had already used the 14-points of the dodecahedron as a nucleus for scale development long before I observed its crystallographic implications.

The 2)5 dekany, as can be observed, is this structure, which viewed in full space is visually most pleasing.

2)5 dekany

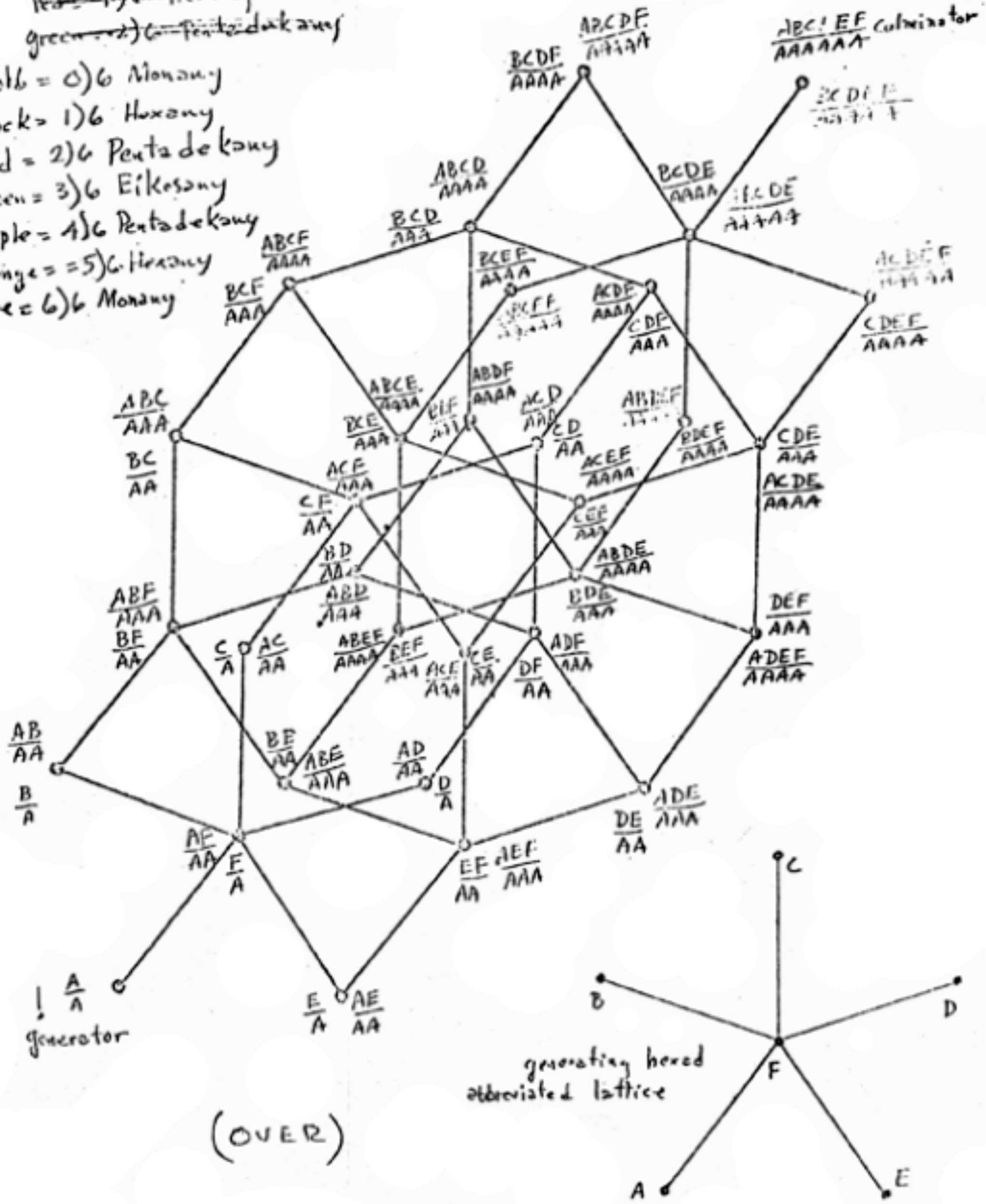




I have used this combination of dekanies to construct a 22-tone structure. Such patterns may be extended indefinitely, to fill space.

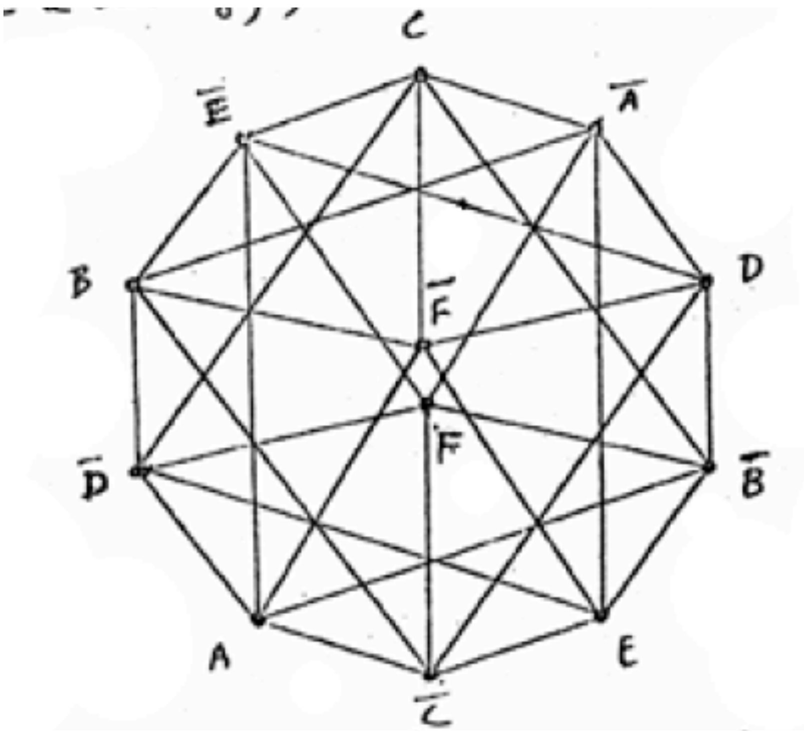
~~gelb = 0)6 Monany~~
~~red = 1)6 Hexany~~
~~green = 2)6 Pentadekany~~

gelb = 0)6 Monany
black = 1)6 Hexany
Red = 2)6 Pentadekany
Green = 3)6 Eikosany
Purple = 4)6 Pentadekany
Orange = =5)6 Hexany
Blue = 6)6 Monany



(OVER)

The eikosany may be generated from each of the six functions of the hexad (A-function shown), and from each of the six functions of the reciprocal hexad. On the flat the lattice yields a ten-fold symmetry. However, if one could view the eikosany in hyper-space one would find a 12-fold symmetry (analogous to the eight facets or sides of the 2)4 Hexany) corresponding to the 12 possible generators. To illustrate, the eikosany may be surrounded by 12 pentadekanies, each of which shares a dekany in common with the eikosany. In the diagram the points (.) signify a pentadekany (or a dekany), identified by its immediate generator.



The 30 connecting lines indicate where the pentadekanies (or dekanies) share a hexany in common.* This figure I call the hexadic hypersphere; and it may be the basis for mirroring progressions. Since a perfectly regular hyperspherical progression is impossible in linear time, the hypersphere is meaningful, only, as a set of alternate possibilities.

*Also, the connecting lines, numbering 30, correspond to the number of hexanies in the eikosany.