WILSON'S 41-TONE MATRIX,
A STABLE 41-TONE SYSTEM (IDENTITIES FACTORIZED)

				13				
			$\frac{11}{3}$	11	$3\cdot 11$			
		$\frac{7}{3^2}$	$\frac{7}{3}$	7	$3\cdot 7$	$3^2\cdot 7$		
	$\frac{5}{3^3}$	$\frac{5}{3^2}$	$\frac{5}{3}$	5	$3\cdot 5$	$3^2\cdot 5$	$3^3\cdot 5$	
$\overline{3}^4$	$\overline{3}^3$	$\overline{3}^2$	$\overline{3}$	1	3	3^2	3^3	3^4
	$\overline{3^3\cdot 5}$	$\overline{3^2\cdot 5}$	$\overline{3\cdot 5}$	$\overline{5}$	$\frac{3}{5}$	$\frac{3^2}{5}$	$\frac{3^3}{5}$	
		$\overline{3^2\cdot 7}$	$\overline{3\cdot 7}$	$\overline{7}$	$\frac{3}{7}$	$\frac{3^2}{7}$		
			$\overline{3\cdot 11}$	$\overline{11}$	$\frac{3}{11}$			
				$\overline{13}$				

Here is an intriguing little matrix, the thesis of which is this: Intervals tend to be introduced into our scale in sequence of their complexity, and in 4 primary directions; (1) Harmonic, (2) Major 3/2 Pythagorean, (3) Sub-harmonic, (4) Minor 4/3 Pythagorean. Of course such a thesis is open to endless qualification and criticism. That notwithstanding, the resultant tones are conservative and realistic to an extraordinary degree, and do represent a fair cross-section of what we do or are likely to do. (the three central, horizontal rows encompass the traditional duo-vigintimal scheme of India, as reported by Fox-Strangways). Inspection reveals quite a variety of known scales, some with considerable modulation.

Now, here is the unexpected and incredible bit of intrigue; the resultant intervals dispose themselves thruout the Octave with the utmost regularity. Further, any given interval is invariably interposed by the same no. of degrees. I. E. the 9/8 occurs 25 times and consists in each case of 7 units, the 4/3 occurs 32 times and consists invariably of 17 units, and so forth. This consistant interposition is the precise (and difficult) requirement of what I lable a Stable System. Here it occured without premeditation, a coincidence of the purest ray serene. On the next sheet I'll show this same system, as ratios applied to a 41 digital keyboard (after Bosanquet).

What I'm getting at with all these illustrations is that a variety of independent approaches converge on 41 as, potentially, a vast, versatile and unexplored art-form. .....Like I was saying....just a couple of thots....

WILSON'S 41-TONE MATRIX
(A STABLE, JUST SYSTEM)

				13				
			$\frac{11}{3}$	11	$3\cdot11$			
		$\frac{7}{3^2}$	$\frac{7}{3}$	7	$3\cdot7$	$3^2\cdot7$		
	$\frac{5}{3^3}$	$\frac{5}{3^2}$	$\frac{5}{3}$	5	$3\cdot5$	$3^2\cdot5$	$3^3\cdot5$	
$\overline{3}^4$	$\overline{3}^3$	$\overline{3}^2$	$\overline{3}$	1	3	3^2	3^3	3^4
	$\overline{3^3\cdot5}$	$\overline{3^2\cdot5}$	$\overline{3\cdot5}$	$\overline{5}$	$\frac{3}{5}$	$\frac{3^2}{5}$	$\frac{3^3}{5}$	
		$\overline{3^2\cdot7}$	$\overline{3\cdot7}$	$\overline{7}$	$\frac{3}{7}$	$\frac{3^2}{7}$		
			$\overline{3\cdot11}$	$\overline{11}$	$\frac{3}{11}$			
				$\overline{13}$				

FACTOR-IDENTITIES

				$\frac{13}{8}$				
			$\frac{11}{6}$	$\frac{11}{8}$	$\frac{33}{32}$			
		$\frac{14}{9}$	$\frac{7}{6}$	$\frac{7}{4}$	$\frac{21}{16}$	$\frac{63}{32}$		
	$\frac{40}{27}$	$\frac{10}{9}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{15}{8}$	$\frac{45}{32}$	$\frac{135}{128}$	
$\frac{128}{81}$	$\frac{32}{27}$	$\frac{16}{9}$	$\frac{4}{3}$	$\frac{1}{1}$	$\frac{3}{2}$	$\frac{9}{8}$	$\frac{27}{16}$	$\frac{81}{64}$
	$\frac{256}{135}$	$\frac{64}{45}$	$\frac{16}{15}$	$\frac{8}{5}$	$\frac{6}{5}$	$\frac{9}{5}$	$\frac{27}{20}$	
		$\frac{64}{63}$	$\frac{32}{21}$	$\frac{8}{7}$	$\frac{12}{7}$	$\frac{9}{7}$		
			$\frac{64}{33}$	$\frac{16}{11}$	$\frac{12}{11}$			
				$\frac{16}{13}$				

RATIOS

ISSUED BY ERV WILSON OCT 21, 1964

WILSON'S 41-TONE MATRIX
(A STABLE, JUST SYSTEM)

				13				
			$\frac{11}{3}$	11	$3\cdot 11$			
		$\frac{7}{3^2}$	$\frac{7}{3}$	7	$3\cdot 7$	$3^2\cdot 7$		
	$\frac{5}{3^3}$	$\frac{5}{3^2}$	$\frac{5}{3}$	5	$3\cdot 5$	$3^2\cdot 5$	$3^3\cdot 5$	
$\overline{3}^4$	$\overline{3}^3$	$\overline{3}^2$	$\overline{3}$	1	3	3^2	3^3	3^4
	$\overline{3^3\cdot 5}$	$\overline{3^2\cdot 5}$	$\overline{3\cdot 5}$	$\overline{5}$	$\frac{3}{5}$	$\frac{3^2}{5}$	$\frac{3^3}{5}$	
		$\overline{3^2\cdot 7}$	$\overline{3\cdot 7}$	$\overline{7}$	$\frac{3}{7}$	$\frac{3^2}{7}$		
			$\overline{3\cdot 11}$	$\overline{11}$	$\frac{3}{11}$			
				$\overline{13}$				

FACTOR-IDENTITIES

					$\frac{13}{8}$					
				$\frac{11}{6}$	$\frac{11}{8}$	$\frac{33}{32}$				
			$\frac{14}{9}$	$\frac{7}{6}$	$\frac{7}{4}$	$\frac{21}{16}$	$\frac{63}{32}$			
		$\frac{40}{27}$	$\frac{10}{9}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{15}{8}$	$\left(\frac{45}{32}\right)$	$\frac{135}{128}$		
$\frac{256}{243}$	$\frac{128}{81}$	$\frac{32}{27}$	$\frac{16}{9}$	$\frac{4}{3}$	$\frac{1}{1}$	$\frac{3}{2}$	$\frac{9}{8}$	$\frac{27}{16}$	$\frac{81}{64}$	$\frac{243}{128}$
		$\frac{256}{135}$	$\frac{64}{45}$	$\frac{16}{15}$	$\frac{8}{5}$	$\frac{6}{5}$	$\frac{9}{5}$	$\frac{27}{20}$		
			$\frac{64}{63}$	$\frac{32}{21}$	$\frac{8}{7}$	$\frac{12}{7}$	$\frac{9}{7}$			
				$\frac{64}{33}$	$\frac{16}{11}$	$\frac{12}{11}$				
					$\frac{16}{13}$					

22-tone Scale of Indi

RATIOS

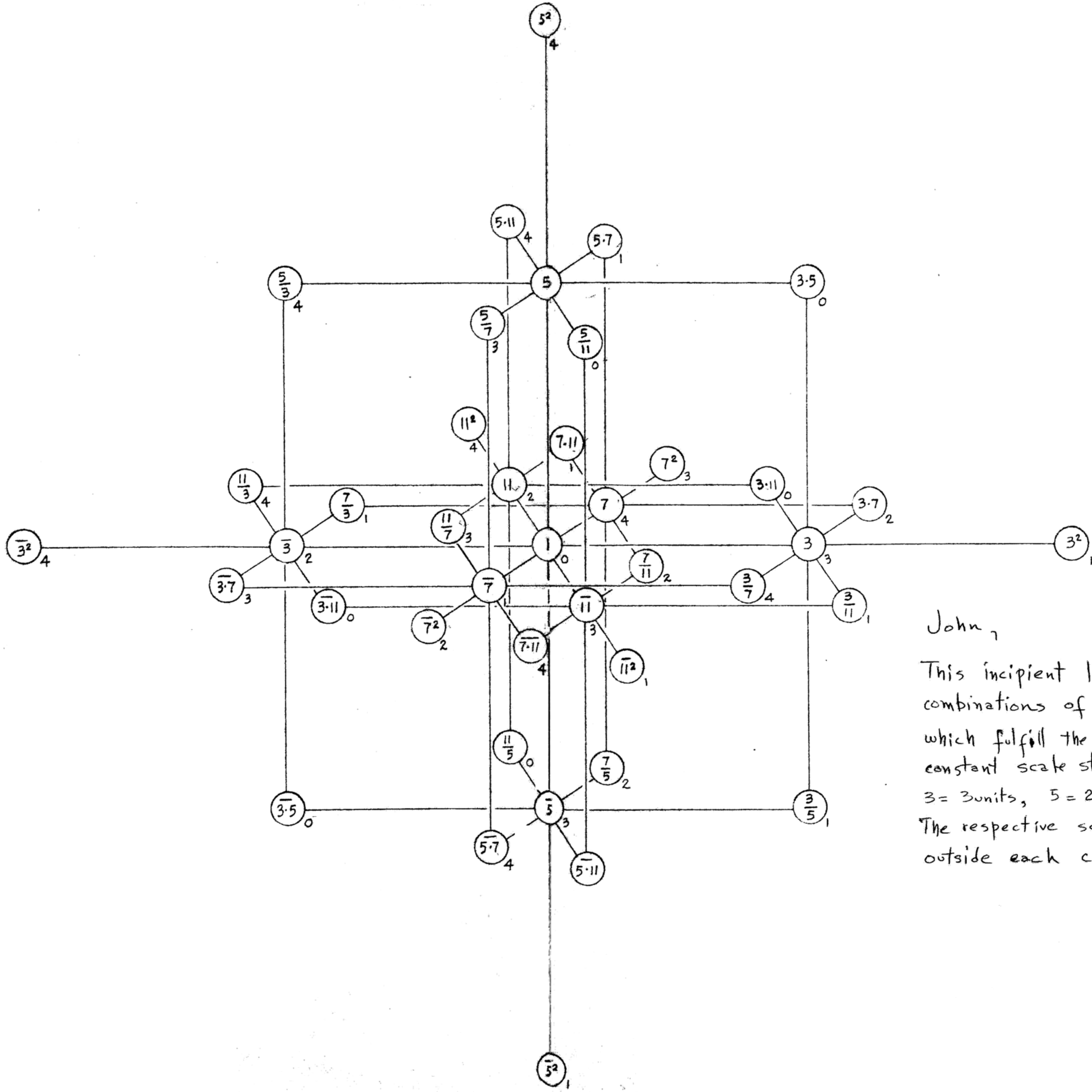
If we equate $\frac{135}{128}$ to $\frac{256}{243}$

and equate $\frac{256}{135}$ to $\frac{243}{128}$

Thus fusing the $\frac{32,805}{32,768}$ (.00163, $\log_e$)

we do have virtually a Pythagorean System in the 22-tone Indian Scale as Reported by Fox-Strangways

ISSUED BY ERV WILSON OCT 21, 1964



22 Apr 67

John,

This incipient lattice illustrates many combinations of linking factored identities which fulfill the requirements for a 5-tone constant scale structure where: 1 = 0/5 units, 3 = 3 units, 5 = 2 units, 7 = 4 units, 11 = 2 units. The respective scale degree is shown outside each circle.

Erv

A COMPARISON CHART,
41-TONE EQUAL SYSTEM
WITH
WILSON'S 41-TONE MATRIX
(A STABLE, JUST SYSTEM)
ISSUED BY ERV WILSON
OCT 21, 1964

41.	1.0000	2/1	1.0000
40.	.9756	63/32	.9773
39.	.9512	64/33	.9556
38.	.9268	256/135	.9232
37.	.9024	15/8	.9069
36.	.8780	11/6	.8745
35.	.8536	9/5	.8480
34.	.8293	16/9	.8301
33.	.8049	7/4	.8074
32.	.7805	12/7	.7776
31.	.7561	27/16	.7549
30.	.7317	5/3	.7370
29.	.7073	13/8	.7004
28.	.6829	8/5	.6781
27.	.6585	128/81	.6602
26.	.6341	14/9	.6374
25.	.6098	32/21	.6077
24.	.5854	3/2	.5850
23.	.5610	40/27	.5670
22.	.5366	16/11	.5406
21.	.5122	64/45	.5082
20.	.4878	45/32	.4918
19.	.4634	11/8	.4594
18.	.4390	27/20	.4330
17.	.4146	4/3	.4150
16.	.3902	21/16	.3923
15.	.3658	9/7	.3626
14.	.3415	81/64	.3398
13.	.3171	5/4	.3219
12.	.2927	16/13	.2996
11.	.2683	6/5	.2630
10.	.2439	32/27	.2451
9.	.2195	7/6	.2224
8.	.1951	8/7	.1926
7.	.1707	9/8	.1699
6.	.1463	10/9	.1520
5.	.1220	12/11	.1255
4.	.0976	16/15	.0931
3.	.0732	135/128	.0768
2.	.0488	33/32	.0444
1.	.0244	64/63	.0227
0.	.0000	1/1	.0000

Dec 24, 1966
Erv Wilson

To: John Chalmers

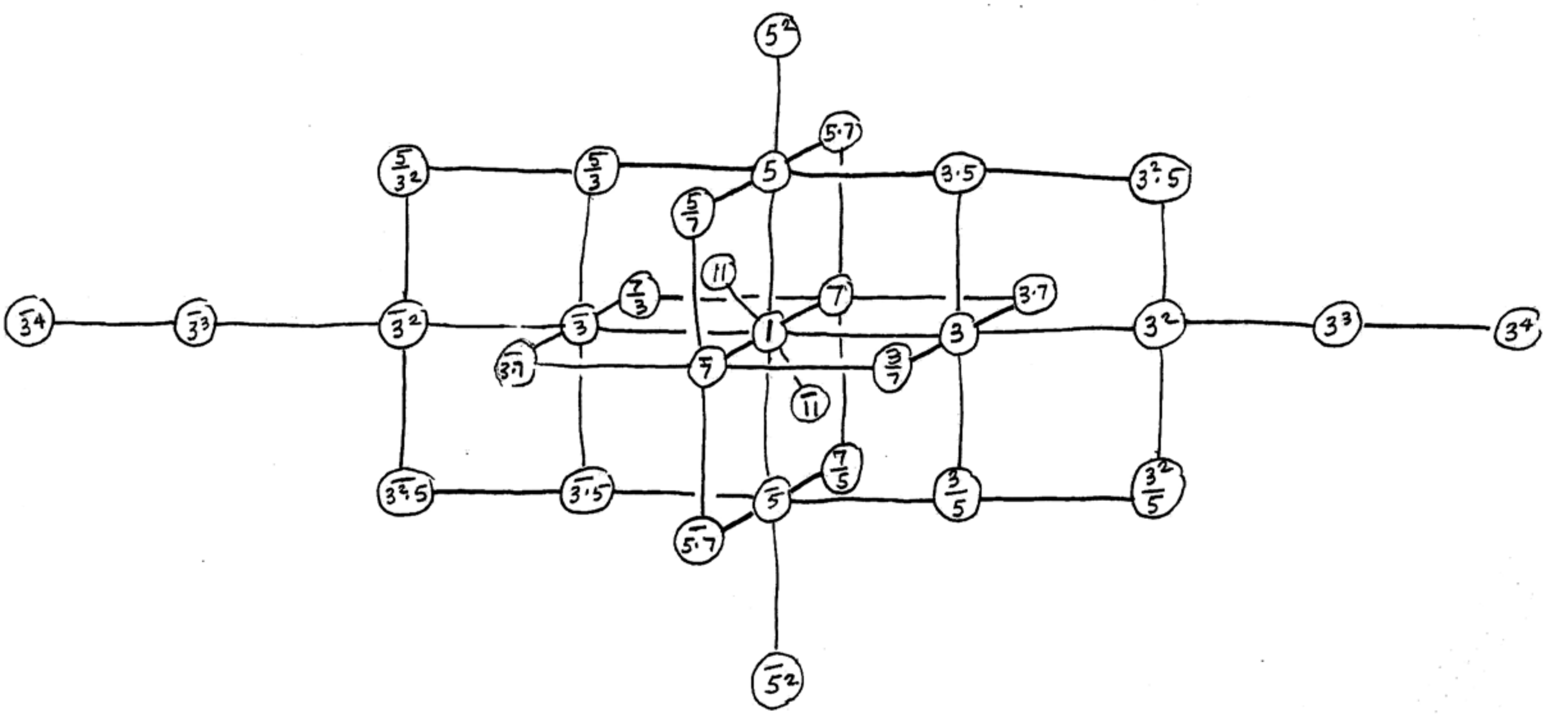
Please bear with me while I ramble.

The possibility of a polished and "complete" just table is predictable from the material you have sent (and for which I express my utmost gratitude!). The table should embody certain aesthetic properties of contour, coherence, & definition as well as the pertinent acoustical data.

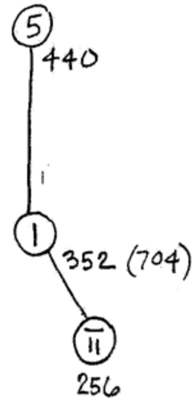
In equal temperament the question of pitch base is not critical, (tho' it is important), because any member of the equal system my be taken as tone-center without altering the acoustical relation of the tonal gestalt to each of the successive tone-centers.

In just tuning exactly the opposite situation occurs. If I have a 48-complexity tonal gestalt and I modulate from its Prime position to an 8-complexity member certain members over a 40-complexity are lost. (As compensation I get members of increased, up to 56, complexity in the opposite direction of the modulation, which may or may not be to my advantage) This difficulty is compounded if I construct my tonal galaxy from a Prime related to the Tables of Pitches by an 8-complexity interval, in which case if I perform the aforementioned modulation I sacrifice up to 16 complexity from the 48-complexity tables; or I am guaranteed a minimum 32-complexity.

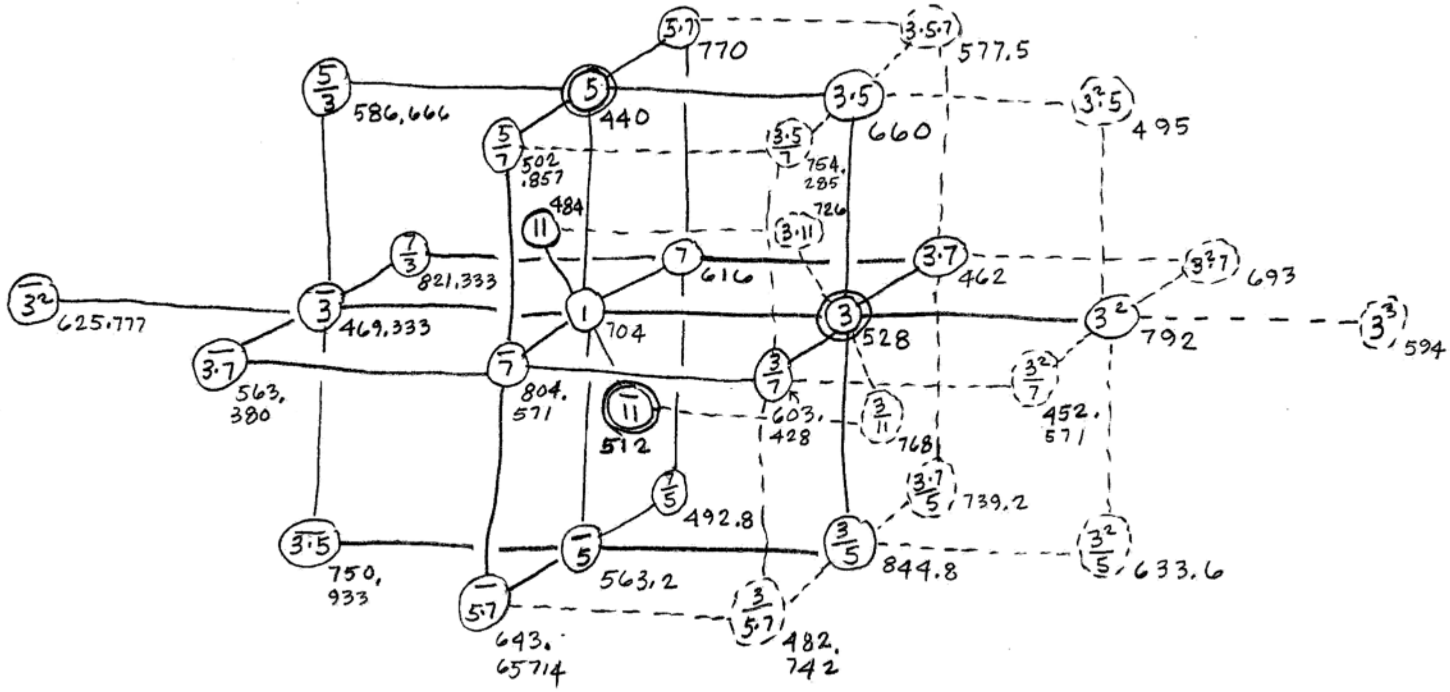
I can alleviate this situation by doin 2 things; 1. I can provide several skillfully related pitch bases. 2. I can restrict my tonal-gestalten to a lesser complexity and modulate within the remaining complexity, from the 48 limit. Let me illustrate the 2nd point, first;
Any concievable 36-complexity limit tonal aggregate may be modulated to any member of a 12-complexity limit tonal nucleus, (this, in Tables carrying factorads to 31 and complexity to 48), specifically this nucleus:



Now, regarding the 1st point, a core of integrated primal pitches; I discover, on meditation, that I can integrate the pitches 256 and 440 by making them ancillary to 352. This makes them unexpectedly useful in a relationship I doubt has been seriously entertained by many theorists.

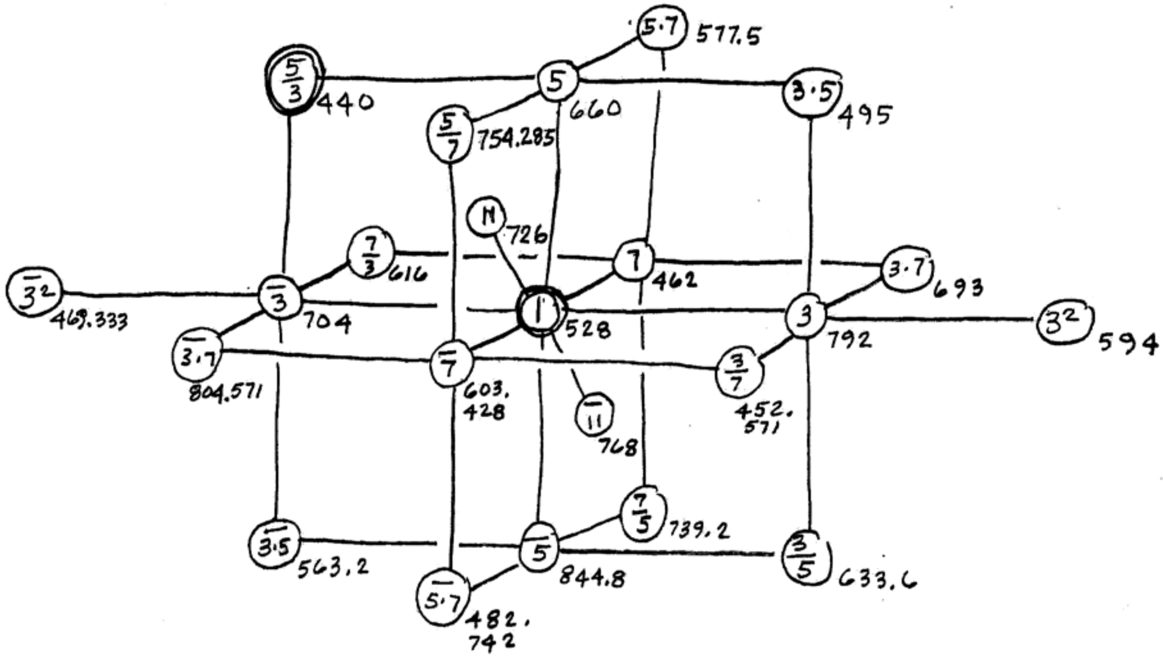


This establishes a viable organization which can be developed as the need and opportunity arise. Further closely related pitches would develop in this manner;

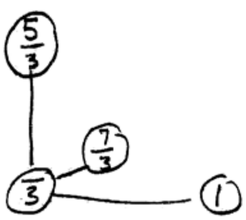


Its obvious, at this point that 528, not 704, is going to make a far more credible reference point as far as whole-numbered primal pitches are concerned. To illustrate this I will extend the solid lattice, above, by a factor of 3 to the right, in dotted lines.

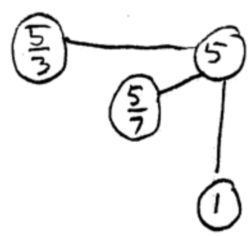
Reconstructing the pitch-core lattice at the focal point 528 we see there is still ample justification for a rapport with 440, but 512 is getting slightly lost.



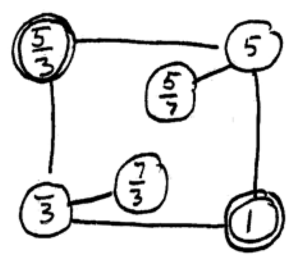
$\frac{5}{3}$ & 1 become the double-common-tone axis for the modulation of the harmonic Tetrad; $\overline{3}$, $\frac{5}{3}$, 1, $\frac{7}{3}$ to the subharmonic tetrad; $\frac{5}{3}$, 1, 5, $\frac{5}{7}$: thus



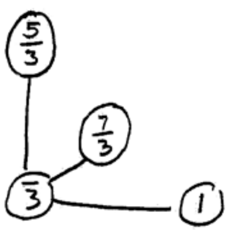
to



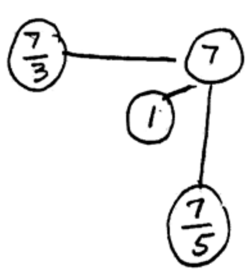
equals



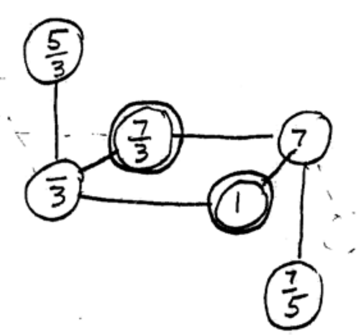
* a similar, powerful, harmonic tie exists between 1 & $\frac{7}{3}$;



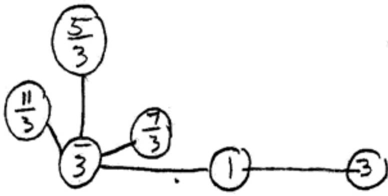
to



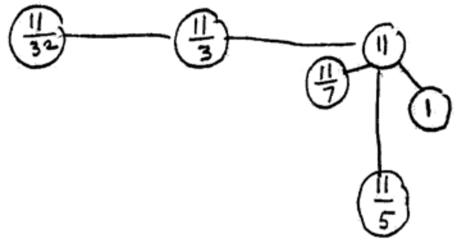
equals



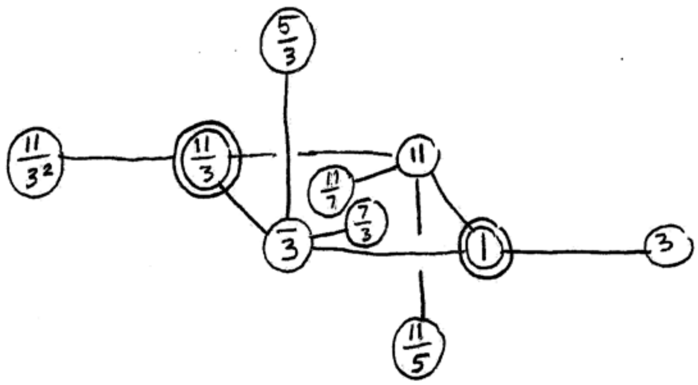
and on this basis certainly a further migration to 616 (or its 3 multiple, 462) would be coherent & resourceful.



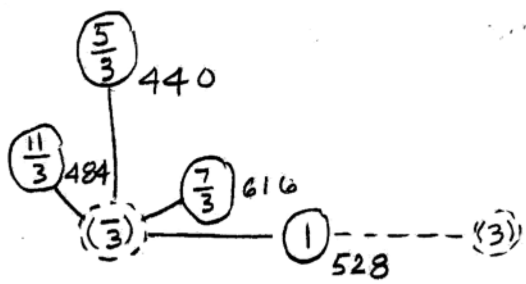
Plus



equals



In the context of $\underline{1}$ 528, $\frac{5}{3}$ 440, $\frac{7}{3}$ 616 it is $\frac{11}{3}$ 484 (or it's 3 modulation, 11 726) which will provide the combination of variable resource with coherence, which is highly desireable, (and not the $\overline{3{:}11}$ 512). At this point I have recommended (and justified, I believe) 4 pitch centers related in this manner To the Harmonic Hexad on 3



providing, from $\underline{1}$, migration to the 5, 7, & 11 areas. via double-common-tone modulation
(a set of pitches migrating by a 3 factor is easily derived from the $\underline{1}$ pitches). Expressing These same pitches a a harmonic hexad from 704 we get:

(1)	3	5	7	(9)	11
	528	440	616		484

(Before I proceed in this line of thinking; under what condition can 4 sets of pitches be introduced into the tables. If we contract the ten intervals between Sum 3 5 7 11 13 17 19 23 29 31 by 2 spaces each we acquire 20 spaces, and there are 18 spaces between 31 and LOGS2 column, equalling 38 available spaces. This is enough for 3 additional columns of 9 spaces plus 2 spaces in between (33 spaces) and 5 spaces separating the factorad 31 from the LOGS2 column.)

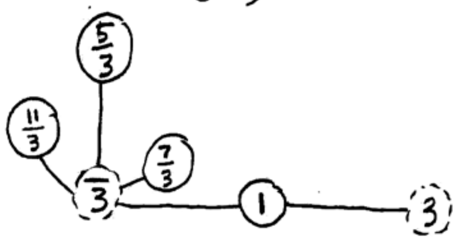
The average student can, I think, be given the 4 tables related by the factorads 1, 5/3, 7/3, 11/3 and by merely making modulations by factorads of 3, up to a complexity of 12, derive 36 keys, (as shown in the plotting on page 6) in which unconditional 36-complexity — limit can be realized. The clever student can, of course, do much, *much* more.

THIS IS AN ASYMETRIC RELATION OF 4 KEYS.

NEW TOPIC

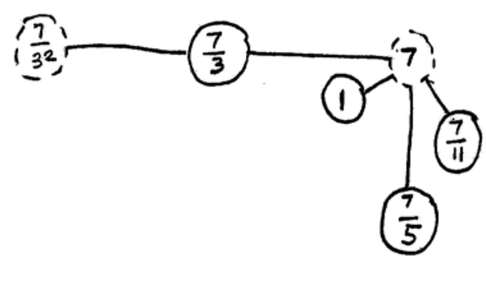
A future generation of thought might wish to symetricize the core. As long as 440 is retained as one of the defining points, it is further likely that a migration in the 7 direction will occur, 1. to avoid the fractional pitches of 7 and 2. because that is often what happens musically, that is, it's an effective modulation. Symetry and Sevenward migration can be achieved, embodying Harmonic & Subharmonic Tetradic insinuations as shown in the last lattice page 3. The tables would be related symetrically about $\frac{7}{3}$ and 1 and would be related as $\frac{5}{3}$, $\frac{7}{3}$, 1, $\frac{7}{5}$ the Hexadic equivalent of this relation would appear in lattice:

Harmonic Hexad on 3

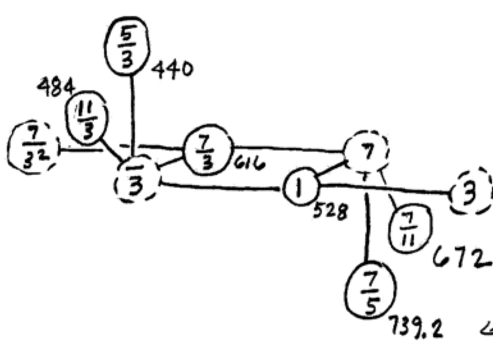


PLUS

Subharmonic Hexad on 7



equals



$\frac{3\cdot 7}{5\cdot 11}$ = pitch 672

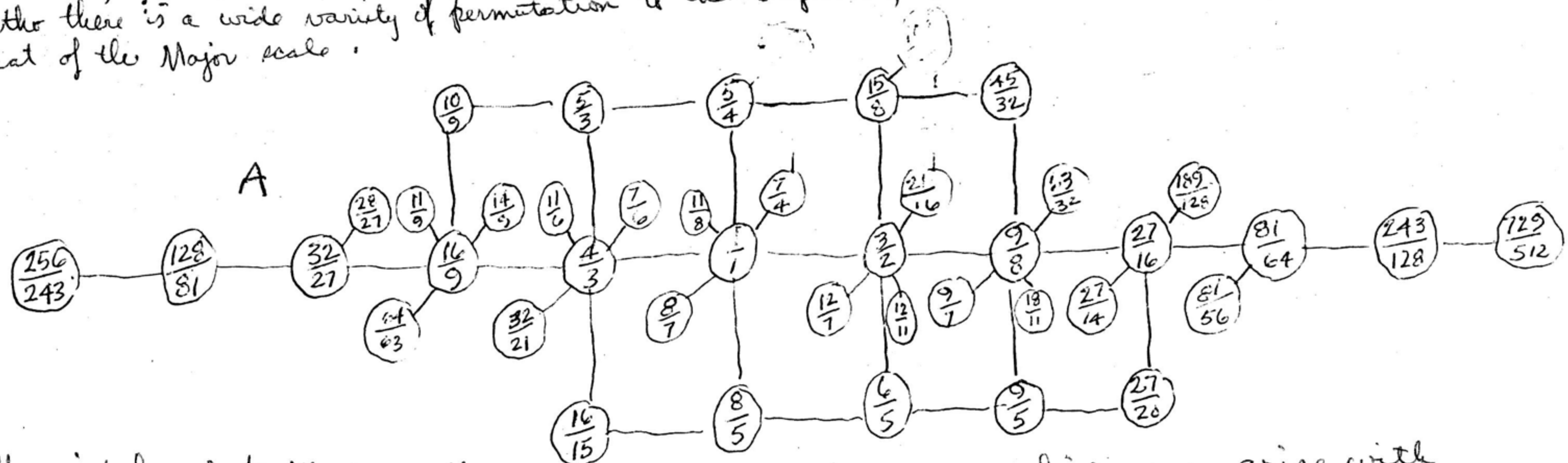
← The 2 additional pitches

With or without the elevens, this is a a beautiful and powerful relationship! and is symetrical about $\frac{7}{3}$ and 1.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

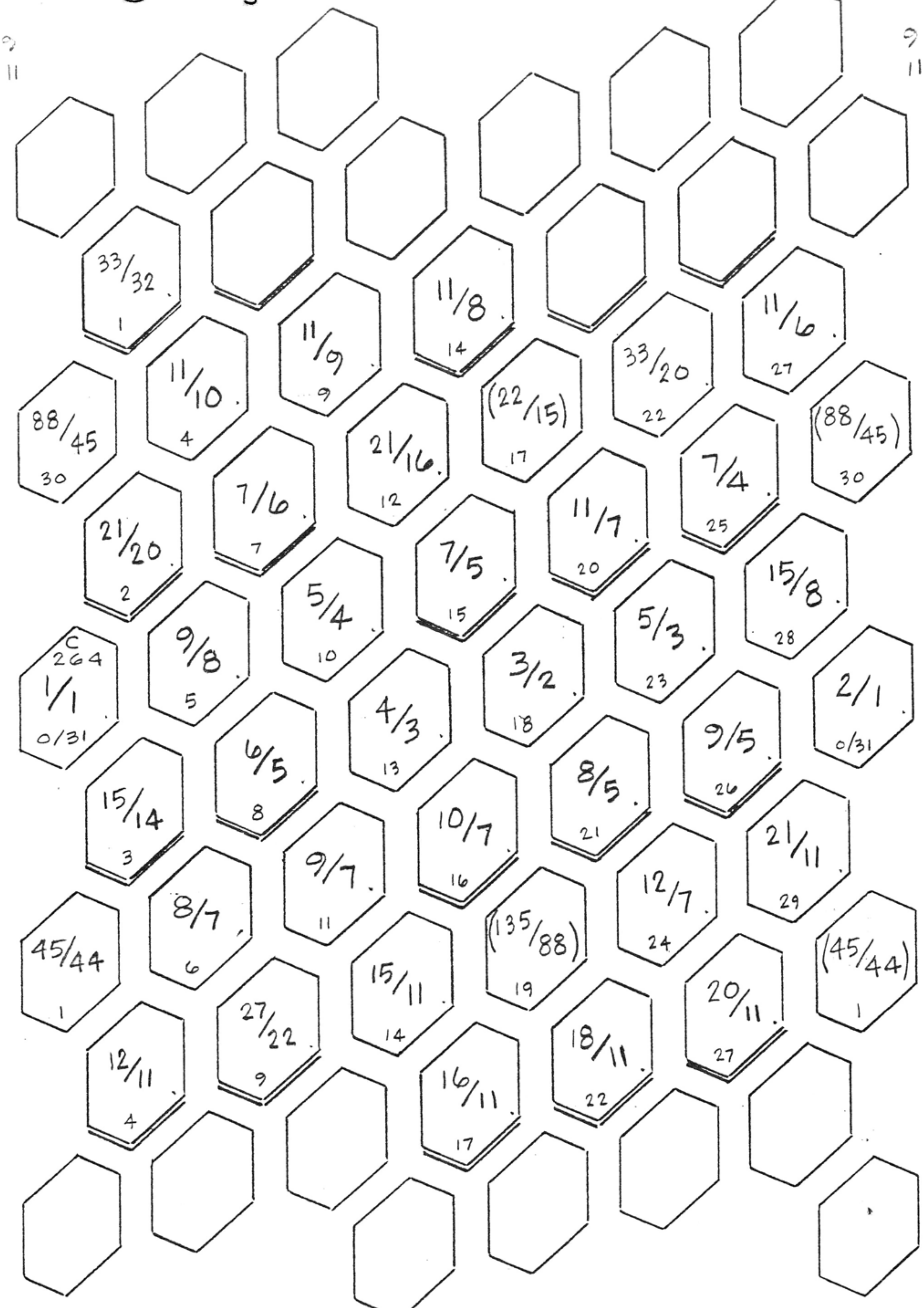
1/1 64/63 28/27 256/243 16/15 35/32 10/9 9/8 8/7 7/6 32/27 6/5 5/4 81/64 9/7 21/16 4/3 27/20 45/32 729/512 81/56 189/128 3/2 32/21 14/9 128/81 8/5 5/3 27/16 12/7 7/4 16/9 9/5 15/8 243/128 27/14 63/32 2/1

A is an 11-limit 41-tone constant scale structure embodying a neo-hindu 5-limit 22-tone constant scale structure. B is an 11-limit 41-tone constant scale structure embodying an inversion of my 7-limit 22-tone constant scale structure. These are, essentially, 11-limit moments-of-symetry culminating on the no. 41. In both A & B the large interval 45/44 occurs 5 times indicating the possibility of a further family of constant scale structures on the number 46. (41+5 = 46) Altho there is a wide variety of permutation to these 2 species, their structure is as strict and basic as that of the Major scale.



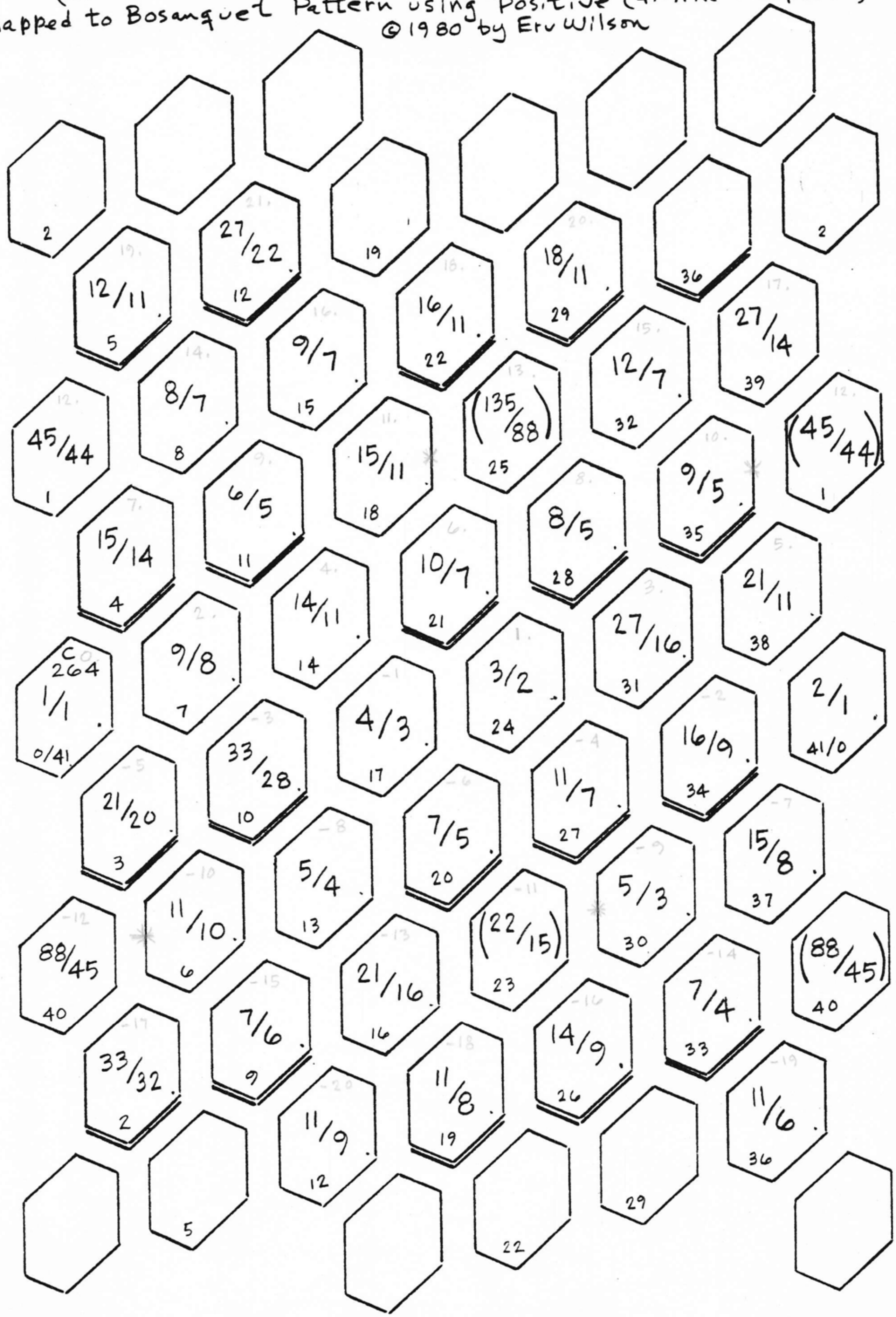
My point here is to illustrate the complexity of the intervals which may arise with almost no effort at all.

Complementary, Negative, Quasi-Diamonds,
Mapped to Bosanquet Pattern using Negative (31-like) Template



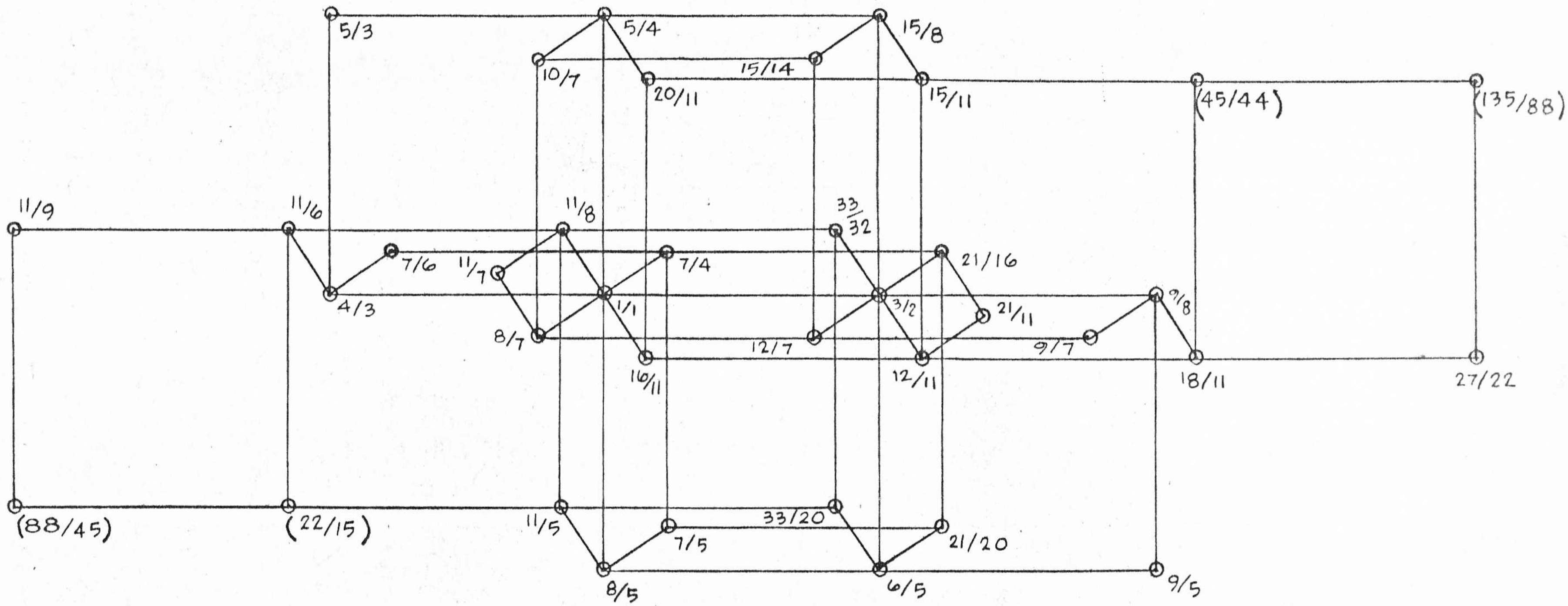
"ECKLAND" Basic Layer
of a multi-layer Tuning.

Complementary, Positive, Quasi-Diamonds
(with added 15-limit materials)
Mapped to Bosanquet Pattern using Positive (41-like Template)



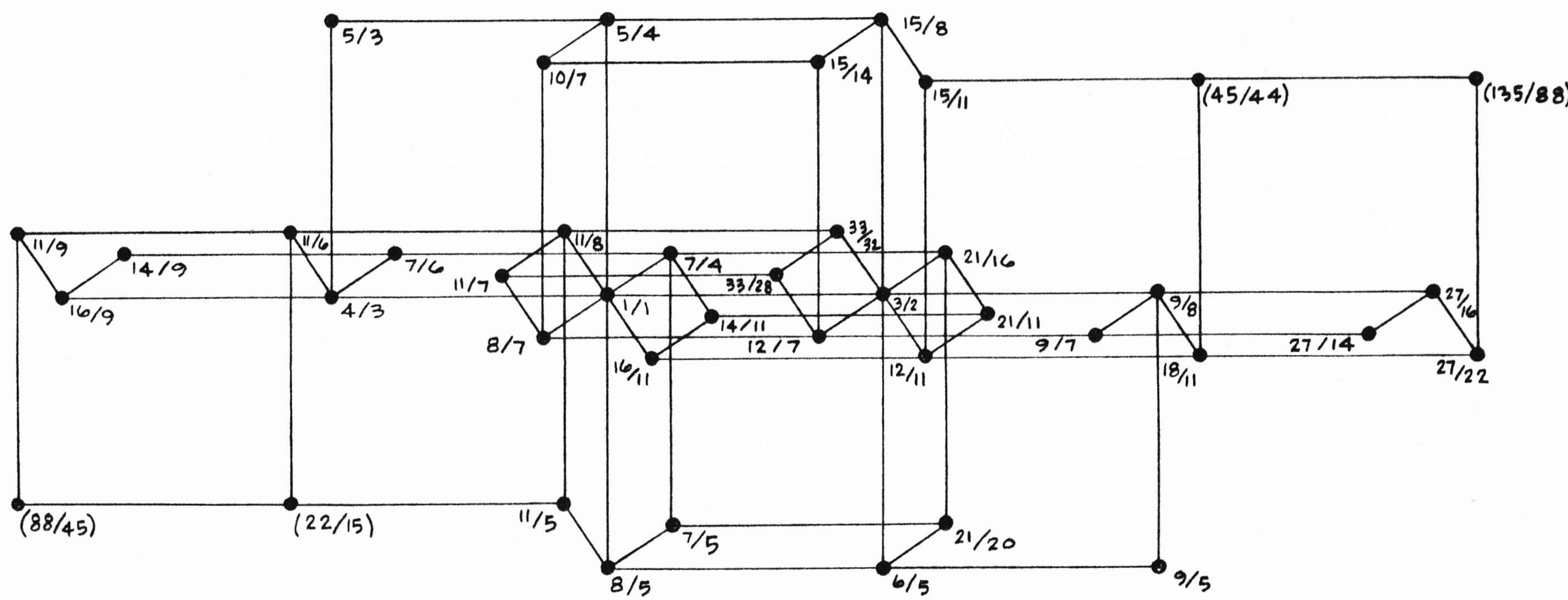
"ANDRUS" basic layer at C (1/1)

Lattice for Complementary, Negative Quasi-Diamonds
© 1980 by Erv Wilson

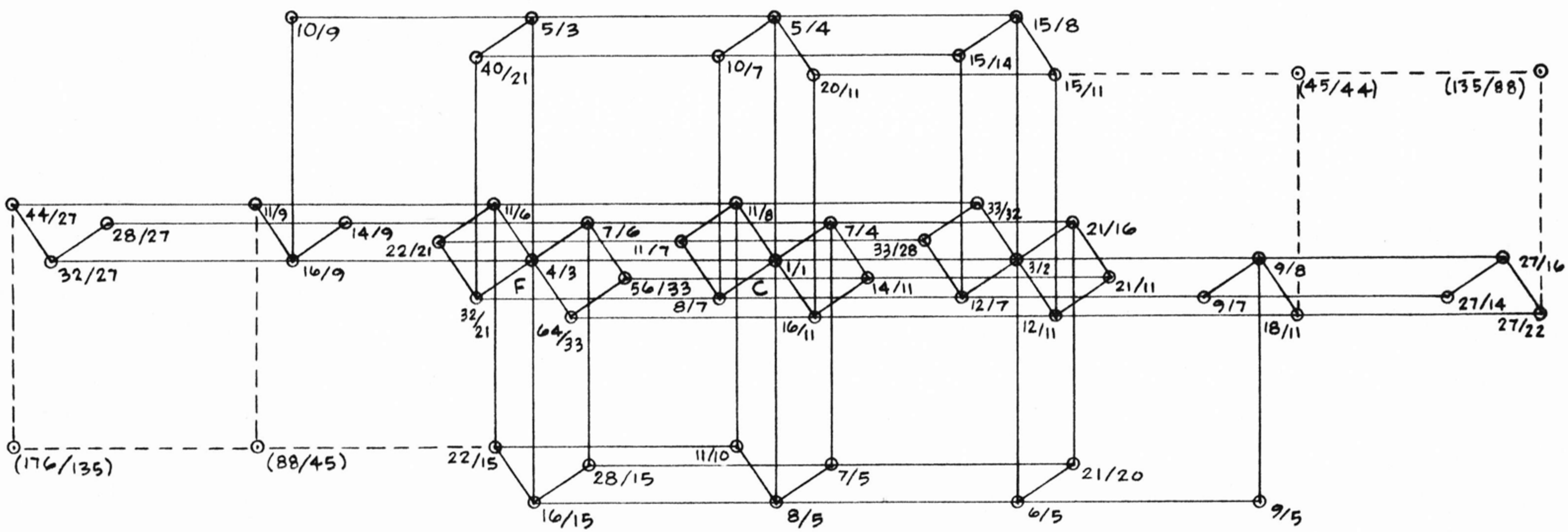


"ECKLAND"
Basic Layer

Lattice for Complementary, Positive Quasi-Diamonds
[4 "fillers" in parentheses]



"ANDRUS"
Basic layer of a multiple-layer tone-structure



"ANDRUS"

The superimposition of 2 layers; at Key 1/1 (C) and at Key 4/3 (F)

× 1/1

×	6	7	8	9	10	11
$\overline{6}$	1/1	7/6	4/3	3/2	5/3	11/6
$\overline{7}$	12/7	1/1	8/7	9/7	10/7	11/7
$\overline{8}$	3/2	7/4	1/1	9/8	5/4	11/8
$\overline{9}$	4/3	14/9	16/9	1/1	10/9	11/9
$\overline{10}$	6/5	7/5	8/5	9/5	1/1	11/10
$\overline{11}$	12/11	14/11	16/11	18/11	20/11	1/1

× 3/2

×	6	7	8	9	10	11
$\overline{6}$	3/2	7/4	1/1	9/8	5/4	11/8
$\overline{7}$	9/7	3/2	12/7	27/14	15/14	33/28
$\overline{8}$	9/8	21/16	3/2	27/16	15/8	33/32
$\overline{9}$	1/1	7/6	4/3	3/2	5/3	11/6
$\overline{10}$	9/5	21/20	6/5	27/20	3/2	33/20
$\overline{11}$	18/11	21/11	12/11	27/22	15/11	3/2

ECKLAND

31-like

Complementary, Negative Quasi-Diamonds

Complementary, Positive Quasi-Diamonds © 1980 by Erv Wilson

× 1/1

×	6	7	8	9	10	11
$\overline{6}$	1/1	7/6	4/3	3/2	5/3	11/6
$\overline{7}$	12/7	1/1	8/7	9/7	10/7	11/7
$\overline{8}$	3/2	7/4	1/1	9/8	5/4	11/8
$\overline{9}$	4/3	14/9	16/9	1/1	10/9	11/9
$\overline{10}$	6/5	7/5	8/5	9/5	1/1	11/10
$\overline{11}$	12/11	14/11	16/11	18/11	20/11	1/1

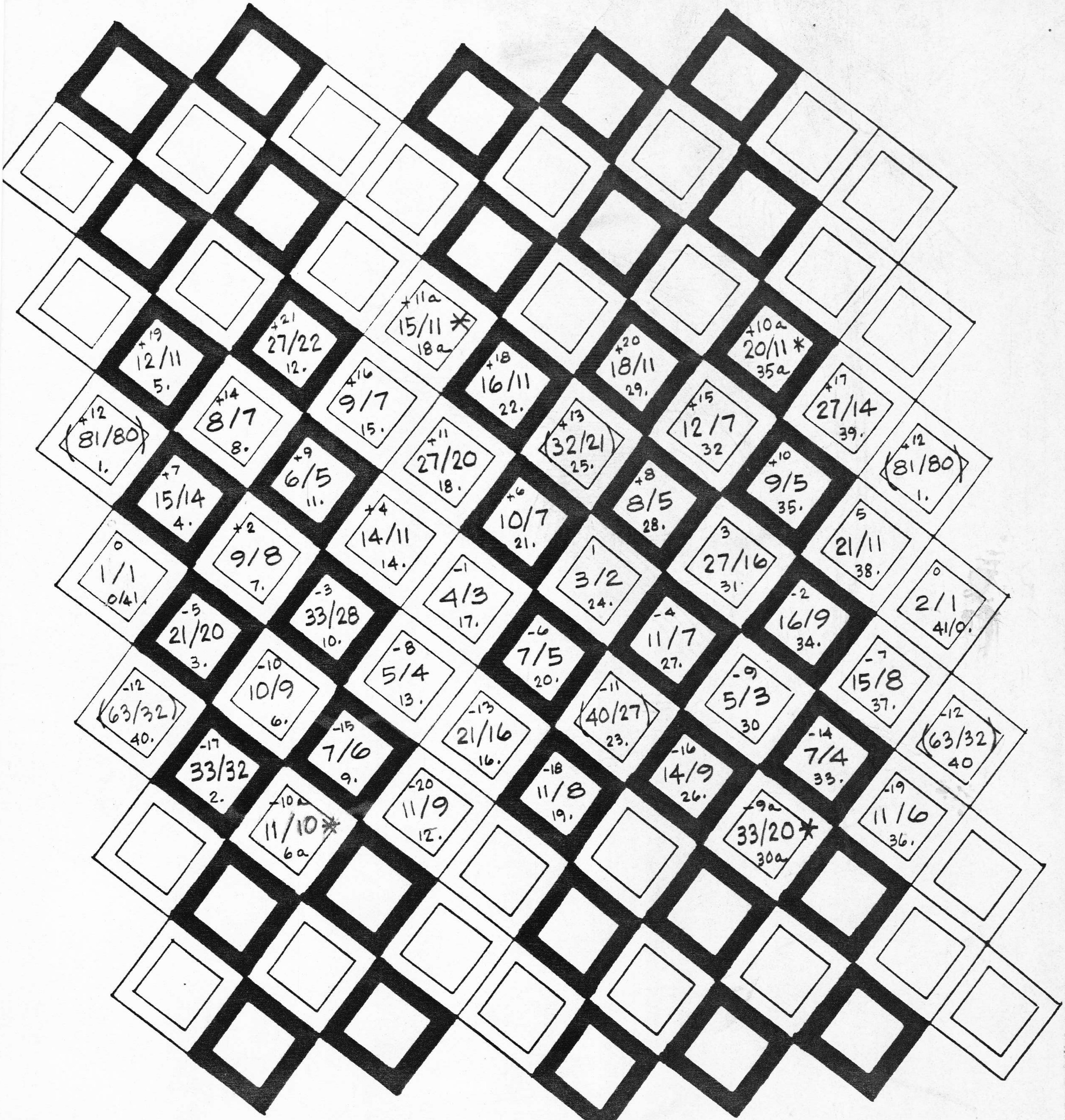
× 3/2

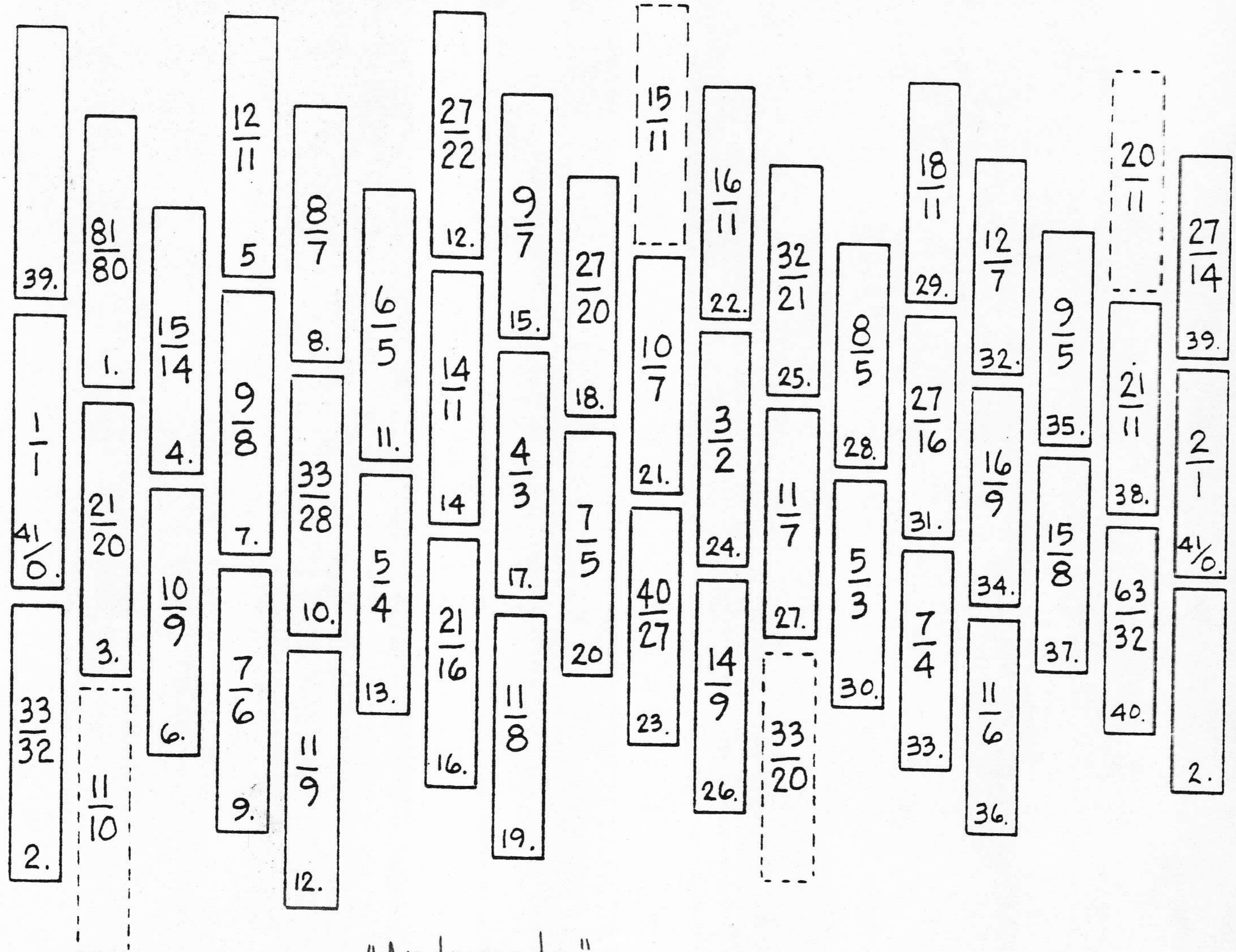
×	6	7	8	9	10	11
$\overline{6}$	3/2	7/4	1/1	9/8	5/4	11/8
$\overline{7}$	9/7	3/2	12/7	27/14	15/14	33/28
$\overline{8}$	9/8	21/16	3/2	27/16	15/8	33/32
$\overline{9}$	1/1	7/6	4/3	3/2	5/3	11/6
$\overline{10}$	9/5	21/20	6/5	27/20	3/2	33/20
$\overline{11}$	18/11	21/11	12/11	27/22	15/11	3/2

"ANDRUS"

"Andromeda"

A Keyboard Tuning for Two Diamonds, on 1/1 and on 3/2



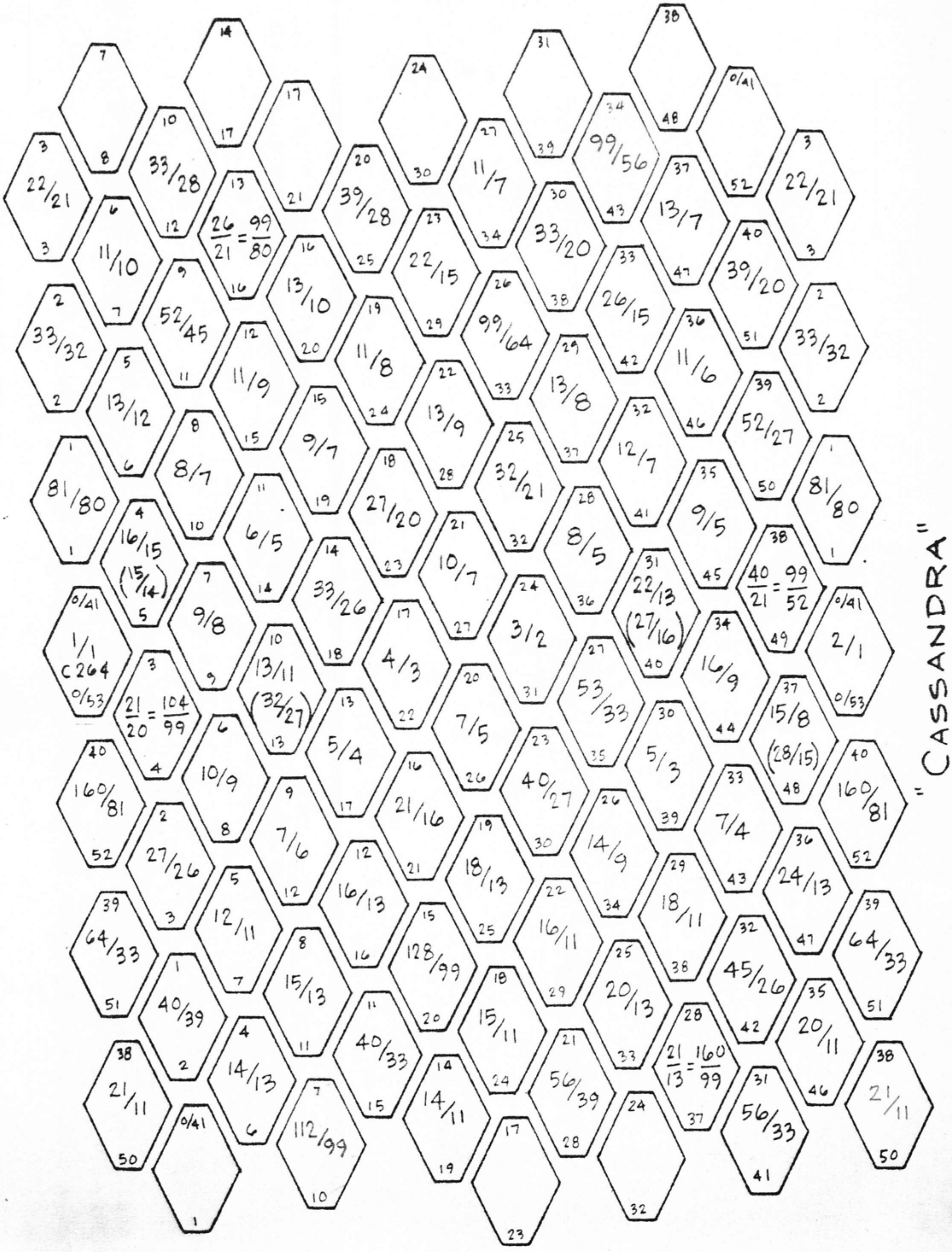


"Andromeda"

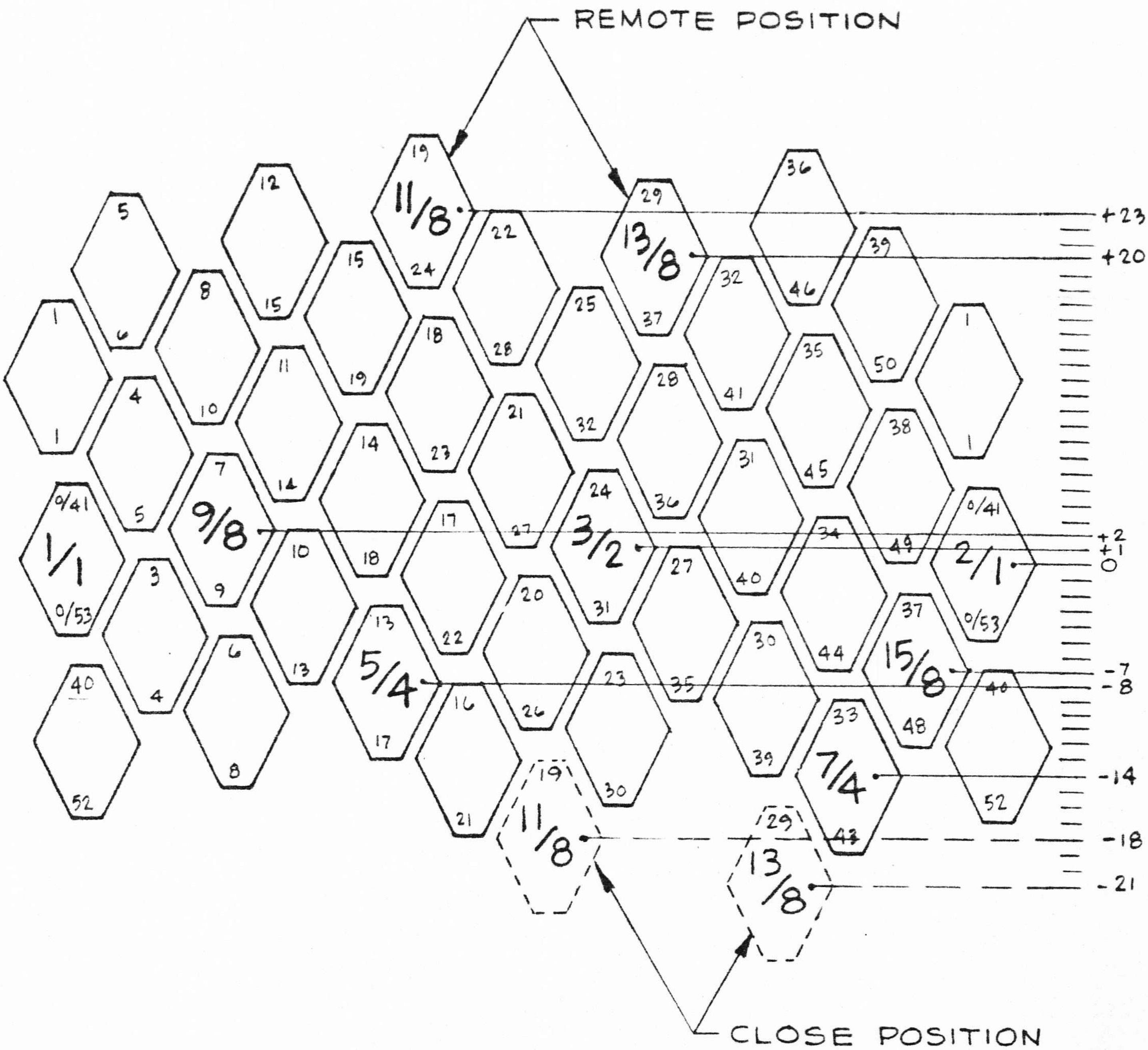
Fig 6b. A 17-Rank Generalized Marimba Keyboard, showing 2 Diamonds, Key C and Key G.

Comments; Homogeneous malletting except, again, the $\frac{11}{10}$ & $\frac{20}{11}$ of each Diamond, in dotted lines, require special malletting. The special malletting is consistent, however, and derived from modulus 53.

Partch's Scale, Ogdoadic Diamond, & Fillers — Mapped to the Bosanquet Pattern using Remote-Positive (41- & 53-like) Template



POSITIVE (41-LIKE) TEMPLATE II
SHOWING 11/8 & 13/8 IN REMOTE POSITION



"CASSANDRA" Template

Feininger Cactus series

a/b	c/d	e/f	c/d dec.
1/6	2/11	1/5	.181818

	1	3	5	7	9	11	13	15
/1	1/1	3/1	5/1	7/1	9/1	11/1	13/1 50, +38	15/1 65.
/3	1/3	3/3	5/3	7/3	9/3	11/3	13/3 8.	15/3
/5	1/5	3/5	5/5	7/5	9/5	11/5	13/5 27.	15/5
/7	1/7	3/7	5/7	7/7	9/7	11/7	13/7 64.	15/7
/9	1/9	3/9	5/9	7/9	9/9	11/9	13/9 38.	15/9
/11	1/11	3/11	5/11	7/11	9/11	11/11	13/11 17.	15/11 32.
/13	1/13 22.	3/13 64.	5/13 45.	7/13 8.	9/13 34.	11/13 55.	13/13	15/13 15.
/15	1/15	3/15	5/15	7/15 65.	9/15	11/15 46.	13/15 57.	15/15

Cross-Set of Reciprocal Ogdoads (reduced to 1 8ve)

×	8	9	10	11	12	13	14	15
$\overline{8}$	1/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8
$\overline{9}$	16/9	1/1	10/9	11/9	4/3	13/9	14/9	5/3
$\overline{10}$	8/5	9/5	1/1	11/10	6/5	13/10	7/5	3/2
$\overline{11}$	16/11	18/11	20/11	1/1	12/11	(13/11)	14/11	15/11
$\overline{12}$	4/3	3/2	5/3	11/6	1/1	13/12	7/6	5/4
$\overline{13}$	16/13	18/13	20/13	(22/13)	24/13	1/1	14/13	15/13
$\overline{14}$	8/7	9/7	10/7	11/7	12/7	13/7	1/1	~~15/14~~
$\overline{15}$	16/15	6/5	4/3	22/15	8/5	26/15	~~28/15~~	1/1

"CASSANDRA"

x 4/3

	8	9	10	11	12	13	14	15
$\overline{8}$	4/3	3/2	5/3	11/6	1/1	13/12	7/6	5/4
$\overline{9}$	(32/27)	4/3	40/27	44/27	16/9	52/27	28/27	10/9
$\overline{10}$	16/15	6/5	4/3	22/15	8/5	26/15	28/15	1/1
$\overline{11}$	64/33	12/11	40/33	4/3	16/11	52/33	56/33	20/11
$\overline{12}$	16/9	1/1	10/9	11/9	4/3	13/9	14/9	5/3
$\overline{13}$	[illegible]/39	24/13	40/39	44/39	16/13	4/3	56/39	20/13
$\overline{14}$	32/21	12/7	40/21	22/21	8/7	26/21	4/3	10/7
$\overline{15}$	[illegible]/45	8/5	16/9	44/45	16/15	52/45	[illegible]	4/3

x 3/2

	8	9	10	11	12	13	14	15
$\overline{8}$	3/2	(27/16)	15/8	33/32	9/8	39/32	21/16	[illegible]
$\overline{9}$	4/3	3/2	5/3	11/6	1/1	13/12	7/6	5/4
$\overline{10}$	6/5	27/20	3/2	33/20	9/5	39/20	21/20	9/8
$\overline{11}$	12/11	27/22	15/11	3/2	18/11	39/22	21/11	45/22
$\overline{12}$	1/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8
$\overline{13}$	24/13	27/26	15/13	33/26	18/13	3/2	21/13	45/26
$\overline{14}$	12/7	27/14	15/14	33/28	9/7	39/28	3/2	45/28
$\overline{15}$	8/5	9/5	1/1	11/10	6/5	13/10	7/5	3/2

"CASSANDRA"

× 9/8

	8	9	10	11	12	13	14	15
$\overline{8}$	9/8	81/64	45/32	99/64	(27/16)	117/64	63/32	135/128
$\overline{9}$	1/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8
$\overline{10}$	9/5	81/80	9/8	99/80	27/20	117/80	63/40	(27/16)
$\overline{11}$	18/11	81/88	45/44	9/8	27/22	117/88	63/44	135/88
$\overline{12}$	3/2	(27/16)	15/8	33/32	9/8	39/32	21/16	45/32
$\overline{13}$	18/13	81/52	45/26	99/52	27/26	9/8	63/52	135/104
$\overline{14}$	9/7	81/56	45/28	99/56	27/14	117/112	9/8	135/112
$\overline{15}$	6/5	27/20	3/2	33/20	9/5	39/20	21/20	9/8

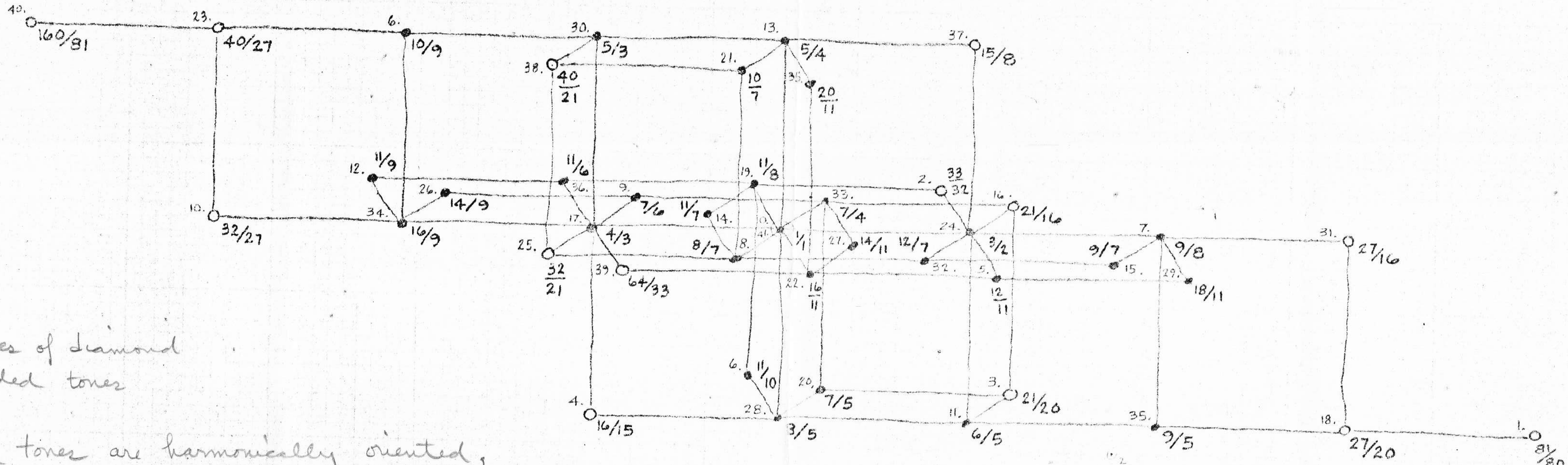
× 16/9

×	8	9	10	11	12	13	14	15
$\overline{8}$	16/9	1/1	10/9	11/9	4/3	13/9	14/9	5/3
$\overline{9}$	128/81	16/9	160/81	88/81	(32/27)	104/81	112/81	40/27
$\overline{10}$	64/45	8/5	16/9	88/45	16/15	52/45	56/45	4/3
$\overline{11}$	128/99	16/11	160/99	16/9	64/33	104/99	112/99	40/33
$\overline{12}$	(32/27)	4/3	40/27	44/27	16/9	52/27	28/27	10/9
$\overline{13}$	128/117	16/13	160/117	176/117	64/39	16/9	224/117	40/39
$\overline{14}$	64/63	8/7	80/63	88/63	32/21	104/63	16/9	40/21
$\overline{15}$	256/135	16/15	(32/27)	176/135	64/45	208/135	224/135	16/9

"CASSANDRA"

Letter to John Chalmers from Erv Wilson 22 Dec 67

Re Harry's tonal-aggregate



• = Tones of diamond
o = added tones

His added tones are harmonically oriented, that is; a hexad from the $\frac{3}{2}$, a pentad from the $\frac{6}{5}$, etc.

Nevertheless, each of the added tones (except $\frac{27}{16}$, $\frac{81}{80}$, $\frac{32}{27}$ & $\frac{160}{81}$) relates to 5 other tones, already existing in the Diamond, in such a way that it completes a Hexany.

⊙ = $\frac{1}{1}$

• = diamond, Fig I

O = tones per Fig II (which are not simultaneous with the tones of Fig I)

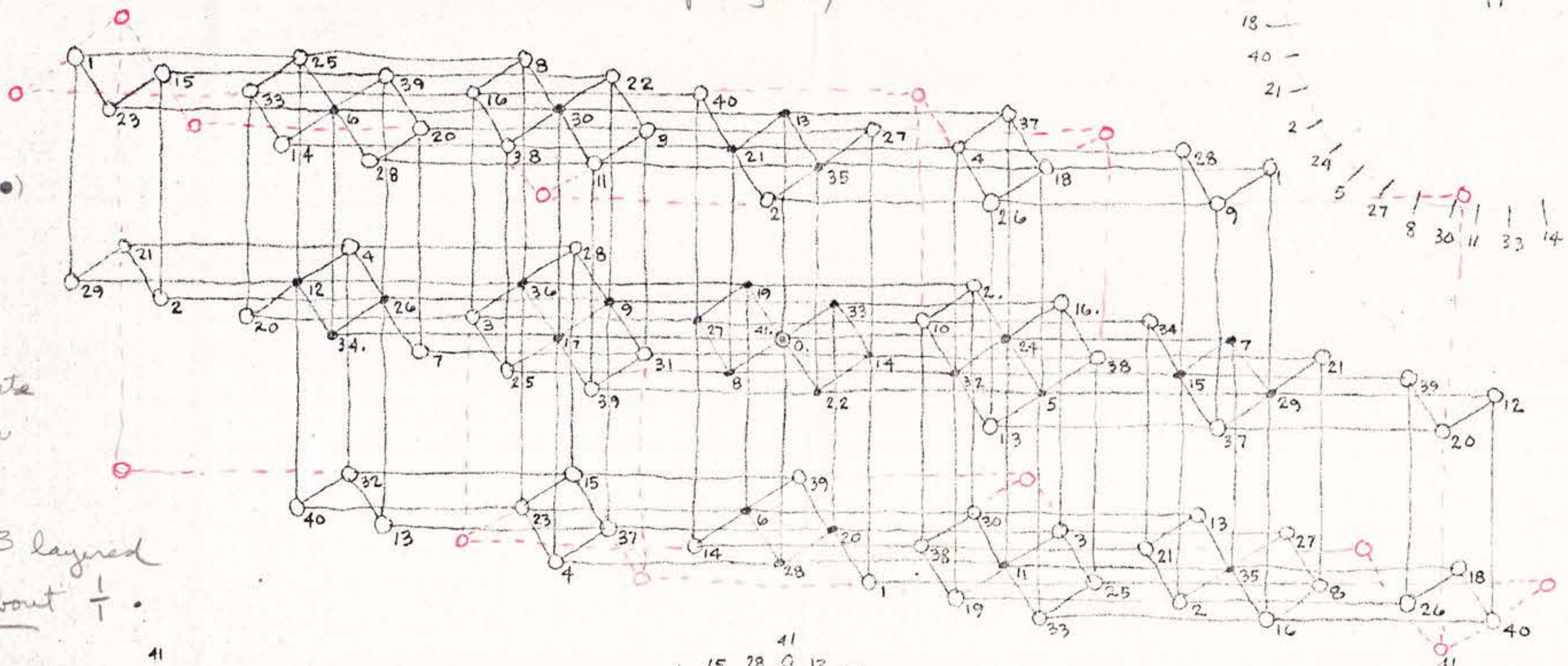
O = Tones per Fig III (which are not simultaneous with the tones of Fig II)

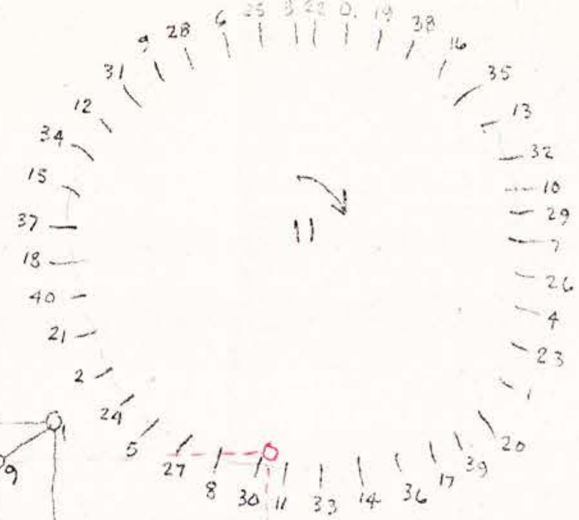
Each of the 90 tones (O) per figure II will complete a Hexany in relation to the tones (•) In the Diamond ($\frac{1}{1}$ and fig I).

Each of the subsequent 20 tones (O) will complete one of the inversions of the Eikosany.

The aggregate is a 3 layered monadic structure about $\frac{1}{1}$.

But still we have not accounted for $\frac{27}{16}$, $\frac{81}{80}$ & their inversions, $\frac{32}{27}$ & $\frac{160}{81}$. $\frac{27}{16}$ can only be explained by a modulation of diamond to the $\frac{3}{2}$. $\frac{81}{80}$ they can be explained as a 2nd layer (Fig II) tone of that key.





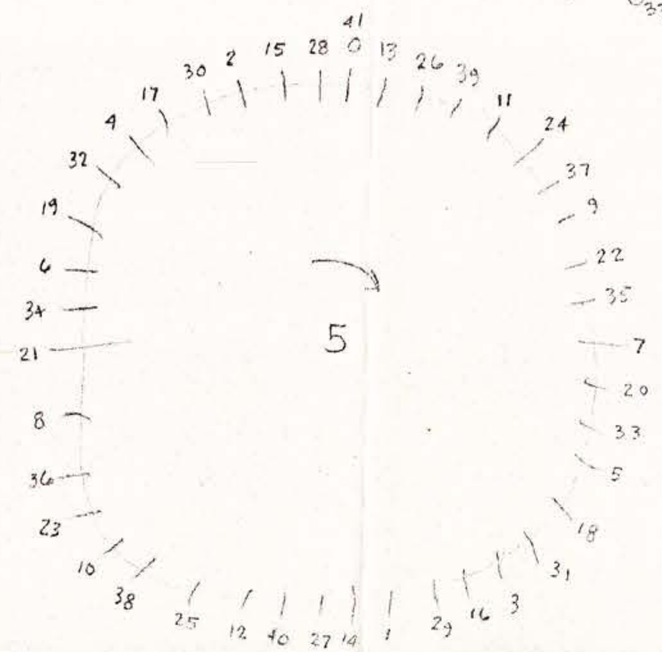


Fig. II

1 3 5 7 9 11

1 3 5 7	$\frac{1\cdot3}{5\cdot7}$	$\frac{1\cdot5}{3\cdot7}$	$\frac{1\cdot7}{3\cdot5}$	$\frac{3\cdot5}{1\cdot7}$	$\frac{3\cdot7}{1\cdot5}$	$\frac{5\cdot7}{1\cdot3}$
1 3 5 9	$\frac{1\cdot3}{5\cdot9}$ $\left(\frac{1}{3\cdot5}\right)$	$\frac{1\cdot5}{3\cdot9}$	$\frac{1\cdot9}{3\cdot5}$ $\left(\frac{3}{5}\right)$	$\frac{3\cdot5}{1\cdot9}$ $\left(\frac{5}{3}\right)$	$\frac{3\cdot9}{1\cdot5}$	$\frac{5\cdot9}{1\cdot3}$ $\left(\frac{3\cdot5}{1}\right)$
1 3 5 11	$\frac{1\cdot3}{5\cdot11}$	$\frac{1\cdot5}{3\cdot11}$	$\frac{1\cdot11}{3\cdot5}$	$\frac{3\cdot5}{1\cdot11}$	$\frac{3\cdot11}{1\cdot5}$	$\frac{5\cdot11}{1\cdot3}$
1 3 7 9	$\frac{1\cdot3}{7\cdot9}$ $\left(\frac{1}{3\cdot7}\right)$	$\frac{1\cdot7}{3\cdot9}$	$\frac{1\cdot9}{3\cdot7}$ $\left(\frac{3}{7}\right)$	$\frac{3\cdot7}{1\cdot9}$ $\left(\frac{7}{3}\right)$	$\frac{3\cdot9}{1\cdot7}$	$\frac{7\cdot9}{1\cdot3}$ $\left(\frac{3\cdot7}{1}\right)$
1 3 7 11	$\frac{1\cdot3}{7\cdot11}$	$\frac{1\cdot7}{3\cdot11}$	$\frac{1\cdot11}{3\cdot7}$	$\frac{3\cdot7}{1\cdot11}$	$\frac{3\cdot11}{1\cdot7}$	$\frac{7\cdot11}{1\cdot3}$
1 3 9 11	$\frac{1\cdot3}{9\cdot11}$ $\left(\frac{1}{3\cdot11}\right)$	$\frac{1\cdot9}{3\cdot11}$ $\left(\frac{3}{11}\right)$	$\frac{1\cdot11}{3\cdot9}$	$\frac{3\cdot9}{1\cdot11}$	$\frac{3\cdot11}{1\cdot9}$ $\left(\frac{11}{3}\right)$	$\frac{9\cdot11}{1\cdot3}$ $\left(\frac{3\cdot11}{1}\right)$
1 5 7 9	$\frac{1\cdot5}{7\cdot9}$	$\frac{1\cdot7}{5\cdot9}$	$\frac{1\cdot9}{5\cdot7}$	$\frac{5\cdot7}{1\cdot9}$	$\frac{5\cdot9}{1\cdot7}$	$\frac{7\cdot9}{1\cdot5}$
1 5 7 11	$\frac{1\cdot5}{7\cdot11}$	$\frac{1\cdot7}{5\cdot11}$	$\frac{1\cdot11}{5\cdot7}$	$\frac{5\cdot7}{1\cdot11}$	$\frac{5\cdot11}{1\cdot7}$	$\frac{7\cdot11}{1\cdot5}$
1 5 9 11	$\frac{1\cdot5}{9\cdot11}$	$\frac{1\cdot9}{5\cdot11}$	$\frac{1\cdot11}{5\cdot9}$	$\frac{5\cdot9}{1\cdot11}$	$\frac{5\cdot11}{1\cdot9}$	$\frac{9\cdot11}{1\cdot5}$
1 7 9 11	$\frac{1\cdot7}{9\cdot11}$	$\frac{1\cdot9}{7\cdot11}$	$\frac{1\cdot11}{7\cdot9}$	$\frac{7\cdot9}{1\cdot11}$	$\frac{7\cdot11}{1\cdot9}$	$\frac{9\cdot11}{1\cdot7}$
3 5 7 9	$\frac{3\cdot5}{7\cdot9}$ $\left(\frac{5}{3\cdot7}\right)$	$\frac{3\cdot7}{5\cdot9}$ $\left(\frac{7}{3\cdot5}\right)$	$\frac{3\cdot9}{5\cdot7}$	$\frac{5\cdot7}{3\cdot9}$	$\frac{5\cdot9}{3\cdot7}$ $\left(\frac{3\cdot5}{7}\right)$	$\frac{7\cdot9}{3\cdot5}$ $\left(\frac{3\cdot7}{5}\right)$
3 5 7 11	$\frac{3\cdot5}{7\cdot11}$	$\frac{3\cdot7}{5\cdot11}$	$\frac{3\cdot11}{5\cdot7}$	$\frac{5\cdot7}{3\cdot11}$	$\frac{5\cdot11}{3\cdot7}$	$\frac{7\cdot11}{3\cdot5}$
3 5 9 11	$\frac{3\cdot5}{9\cdot11}$ $\left(\frac{5}{3\cdot11}\right)$	$\frac{3\cdot9}{5\cdot11}$	$\frac{3\cdot11}{5\cdot9}$ $\left(\frac{11}{3\cdot5}\right)$	$\frac{5\cdot9}{3\cdot11}$ $\left(\frac{3\cdot5}{11}\right)$	$\frac{5\cdot11}{3\cdot9}$	$\frac{9\cdot11}{3\cdot5}$ $\left(\frac{3\cdot11}{5}\right)$
3 7 9 11	$\frac{3\cdot7}{9\cdot11}$ $\left(\frac{7}{3\cdot11}\right)$	$\frac{3\cdot9}{7\cdot11}$	$\frac{3\cdot11}{7\cdot9}$ $\left(\frac{11}{3\cdot7}\right)$	$\frac{7\cdot9}{3\cdot11}$ $\left(\frac{3\cdot7}{11}\right)$	$\frac{7\cdot11}{3\cdot9}$	$\frac{9\cdot11}{3\cdot7}$ $\left(\frac{3\cdot11}{7}\right)$
5 7 9 11	$\frac{5\cdot7}{9\cdot11}$	$\frac{5\cdot9}{7\cdot11}$	$\frac{5\cdot11}{7\cdot9}$	$\frac{7\cdot9}{5\cdot11}$	$\frac{7\cdot11}{5\cdot9}$	$\frac{9\cdot11}{5\cdot7}$

Fig III

1	3	5	7	9	11	
1	3	5				$\frac{1\cdot3\cdot5}{7\cdot9\cdot11}$
1	3		7			$\frac{1\cdot3\cdot7}{5\cdot9\cdot11}$
1	3			9		$\frac{1\cdot3\cdot9}{5\cdot7\cdot11}$
1	3				11	$\frac{1\cdot3\cdot11}{5\cdot7\cdot9}$
1		5	7			$\frac{1\cdot5\cdot7}{3\cdot9\cdot11}$
1		5		9		$\frac{1\cdot5\cdot9}{3\cdot7\cdot11}$
1		5			11	$\frac{1\cdot5\cdot11}{3\cdot7\cdot9}$
1			7	9		$\frac{1\cdot7\cdot9}{3\cdot5\cdot11}$
1			7		11	$\frac{1\cdot7\cdot11}{3\cdot5\cdot9}$
1				9	11	$\frac{1\cdot9\cdot11}{3\cdot5\cdot7}$
	3	5	7			$\frac{3\cdot5\cdot7}{1\cdot9\cdot11}$
	3	5		9		$\frac{3\cdot5\cdot9}{1\cdot7\cdot11}$
	3	5			11	$\frac{3\cdot5\cdot11}{1\cdot7\cdot9}$
	3		7	9		$\frac{3\cdot7\cdot9}{1\cdot5\cdot11}$
	3		7		11	$\frac{3\cdot7\cdot11}{1\cdot5\cdot9}$
	3			9	11	$\frac{3\cdot9\cdot11}{1\cdot5\cdot7}$
		5	7	9		$\frac{5\cdot7\cdot9}{1\cdot3\cdot11}$
		5	7		11	$\frac{5\cdot7\cdot11}{1\cdot3\cdot9}$
		5		9	11	$\frac{5\cdot9\cdot11}{1\cdot3\cdot7}$
			7	9	11	$\frac{7\cdot9\cdot11}{1\cdot3\cdot5}$

Fig. I

1	3	5	7	9	11		
1	3					$\frac{1}{3}$	$\frac{3}{1}$
1		5				$\frac{1}{5}$	$\frac{5}{1}$
1			7			$\frac{1}{7}$	$\frac{7}{1}$
1				9		$\frac{1}{9}$	$\frac{9}{1}$
1					11	$\frac{1}{11}$	$\frac{11}{1}$
	3	5				$\frac{3}{5}$	$\frac{5}{3}$
	3		7			$\frac{3}{7}$	$\frac{7}{3}$
	3			9		$\frac{3}{9}$	$\frac{9}{3}$
	3				11	$\frac{3}{11}$	$\frac{11}{3}$
		5	7			$\frac{5}{7}$	$\frac{7}{5}$
		5		9		$\frac{5}{9}$	$\frac{9}{5}$
		5			11	$\frac{5}{11}$	$\frac{11}{5}$
			7	9		$\frac{7}{9}$	$\frac{9}{7}$
			7		11	$\frac{7}{11}$	$\frac{11}{7}$
				9	11	$\frac{9}{11}$	$\frac{11}{9}$

PITCHES OF THE 41-TONE EQUAL DIVISION OF THE OCTAVE
TO THE 41ST ROOT OF 2 (1.017049744) BASE C˥ 512

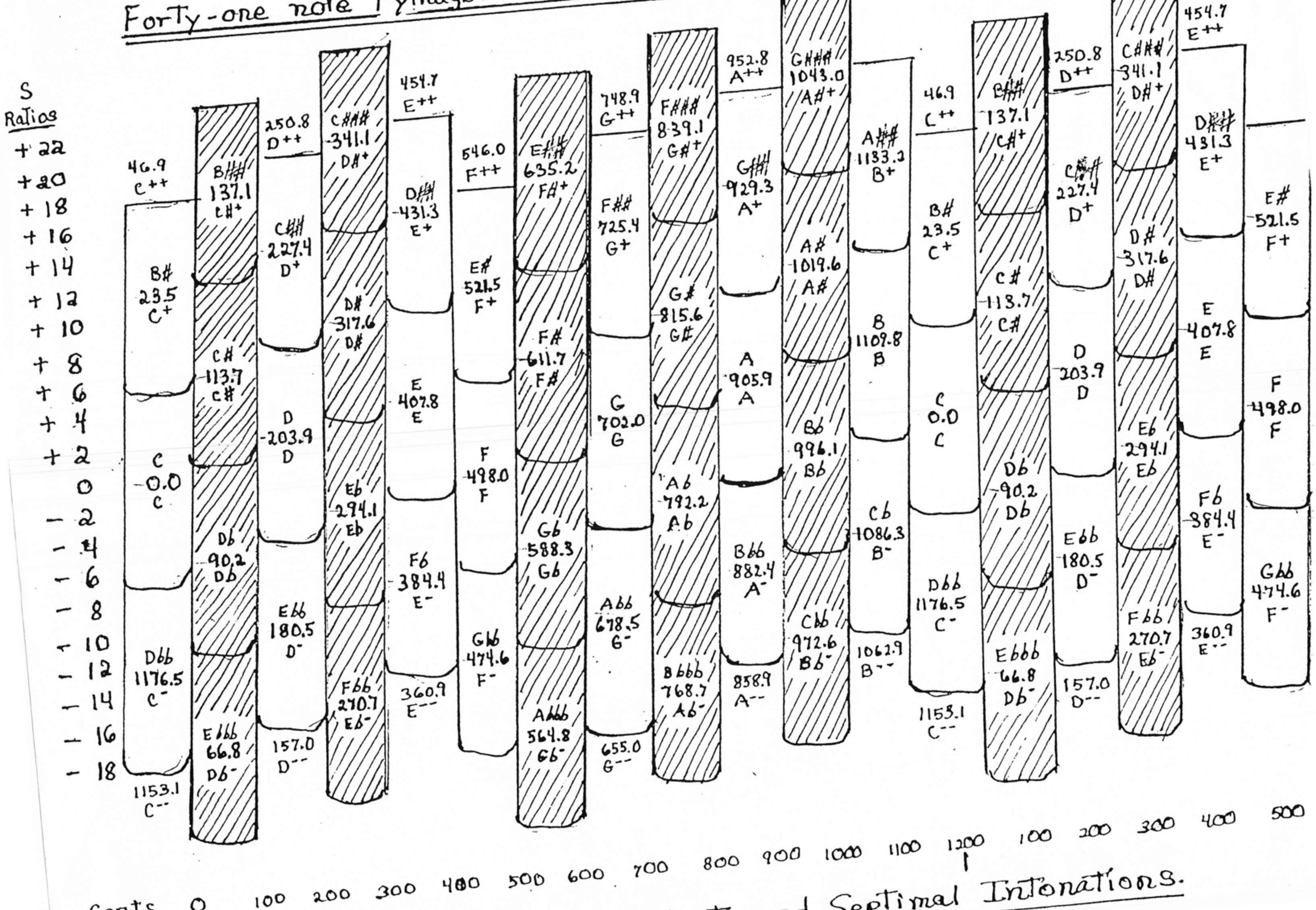
Issued by Ervin M. Wilson Jan. 27, 1965

(BASE	C˥	32.000	64.000	128.00	256.00	512.00	1024.0	2048.0	4096.0)
0.	C	32.546	65.091	130.18	260.36	520.73	1041.5	2082.9	4165.8
1.	Cψ Dh	33.100	66.201	132.40	264.80	529.61	1059.2	2118.4	4236.9
2.	C‡ Dƀ	33.665	67.330	134.66	269.32	538.64	1077.3	2154.5	4309.1
3.	C¥ D♭	34.239	68.548	136.96	273.91	547.82	1095.6	2191.3	4382.6
4.	C# Dɫ	34.823	69.645	139.29	278.58	557.16	1114.3	2228.6	4457.3
5.	C⋕ D⊦	35.416	70.833	141.67	283.33	566.66	1133.3	2266.6	4533.3
6.	C⩩ D˥	36.020	72.040	144.08	288.16	576.32	1152.6	2305.3	4610.6
7.	D	36.634	73.269	146.54	293.07	586.15	1172.3	2344.6	4689.2
8.	Dψ Eh	37.259	74.518	149.04	298.07	596.14	1192.3	2384.6	4769.1
9.	D‡ Eƀ	37.894	75.788	151.58	303.15	606.31	1212.6	2425.2	4850.5
10.	D¥ E♭	38.540	77.080	154.16	308.32	616.64	1233.3	2466.6	4933.1
11.	D# Eɫ	39.197	78.395	156.79	313.58	627.16	1254.3	2508.6	5017.3
12.	D⋕ E⊦	39.865	79.731	159.46	318.92	637.85	1275.7	2551.4	5102.8
13.	D⩩ E˥	40.545	81.091	162.18	324.36	648.72	1297.4	2594.9	5189.8
14.	E	41.237	82.473	164.95	329.89	659.79	1319.6	2639.1	5278.3
15.	Eψ F⊦	41.940	83.879	167.76	335.52	671.03	1342.1	2684.1	5368.3
16.	E‡ F˥	42.655	85.309	170.62	341.24	682.48	1365.0	2729.9	5459.8
17.	F	43.382	86.764	173.53	347.06	694.11	1388.2	2776.4	5552.9
18.	Fψ Gh	44.122	88.243	176.49	352.97	705.95	1411.9	2823.8	5647.6
19.	F‡ Gƀ	44.874	89.748	179.50	358.99	717.98	1436.0	2871.9	5743.9
20.	F¥ G♭	45.639	91.278	182.56	365.11	730.22	1460.4	2920.9	5841.8
21.	F# Gɫ	46.417	92.834	185.67	371.34	742.61	1485.3	2970.7	5941.4
22.	F⋕ G⊦	47.209	94.417	188.83	377.67	755.34	1510.7	3021.3	6042.7
23.	F⩩ G˥	48.013	96.026	192.05	384.11	768.21	1536.4	3073.0	6145.7
24.	G	48.832	97.664	195.33	390.66	781.31	1562.6	3125.3	6250.5
25.	Gψ Ah	49.665	99.329	198.66	397.32	794.63	1589.3	3178.5	6357.1
26.	G‡ Aƀ	50.511	101.02	202.05	404.09	808.18	1616.4	3232.7	6465.5
27.	G¥ A♭	51.373	102.75	205.49	410.98	821.96	1643.9	3287.8	6575.7
28.	G# Aɫ	52.248	104.50	208.99	417.99	835.98	1672.0	3343.9	6687.8
29.	G⋕ A⊦	53.139	106.28	212.56	425.11	850.23	1700.5	3400.9	6801.8
30.	G⩩ A˥	54.045	108.09	216.18	432.36	864.72	1729.4	3458.9	6917.8
31.	A	54.967	109.93	219.87	439.73	879.47	1758.9	3517.9	7035.7
32.	Aψ Bh	55.904	111.81	223.62	447.23	894.46	1788.9	3577.9	7155.7
33.	A‡ Bƀ	56.857	113.71	227.43	454.86	909.71	1819.4	3638.9	7277.7
34.	A¥ B♭	57.826	115.65	231.31	462.61	925.22	1850.4	3700.9	7401.8
35.	A# Bɫ	58.812	117.62	235.25	470.50	941.00	1882.0	3764.0	7528.0
36.	A⋕ B⊦	59.815	119.63	239.26	478.52	957.04	1914.1	3828.2	7656.3
37.	A⩩ B˥	60.835	121.67	243.34	486.68	973.36	1946.7	3893.4	7786.9
38.	B	61.872	123.74	247.49	494.98	989.96	1979.9	3959.8	7919.6
39.	Bψ C⊦	62.927	125.85	251.71	503.42	1006.8	2013.7	4027.3	8054.7
40.	B‡ C˥	64.000	128.00	256.00	512.00	1024.0	2048.0	4096.0	8192.0
41.	C	65.091	130.18	260.36	520.73	1041.5	2082.9	4165.8	8331.7

SIGNATURE:

-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8
♭♭	ƕ	h	ƀ	♭	ɫ	⊦	˥	♮	ψ	‡	¥	#	⋕	⩩	⧻	𝄪

Forty-one note Pythagorean scale in Perfect Fifths: Abbb through G###

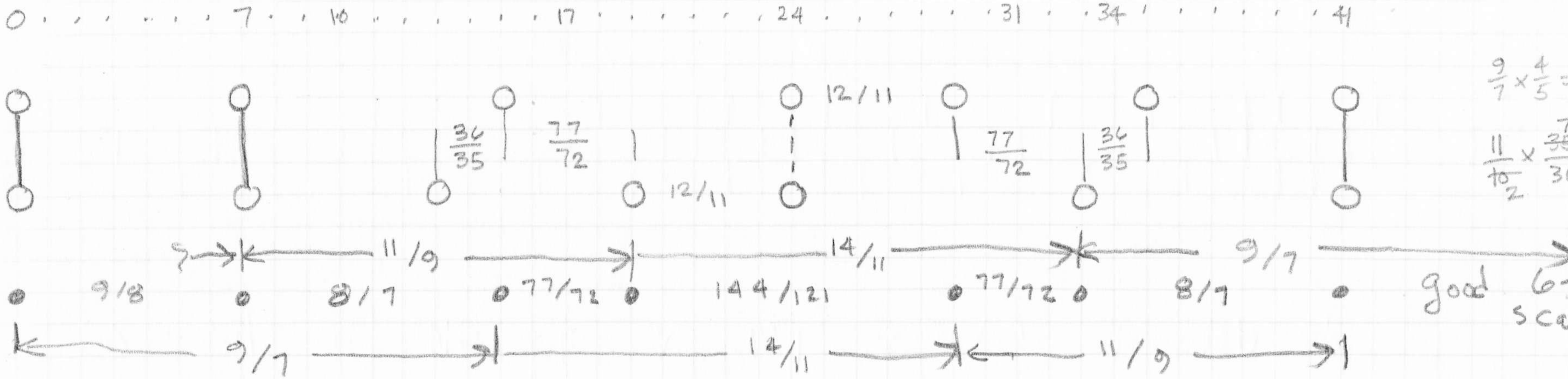


... in Pythagorean, Just and Septimal Intonations.

Tubulong-41

Design © 1976 by Erv Wilson

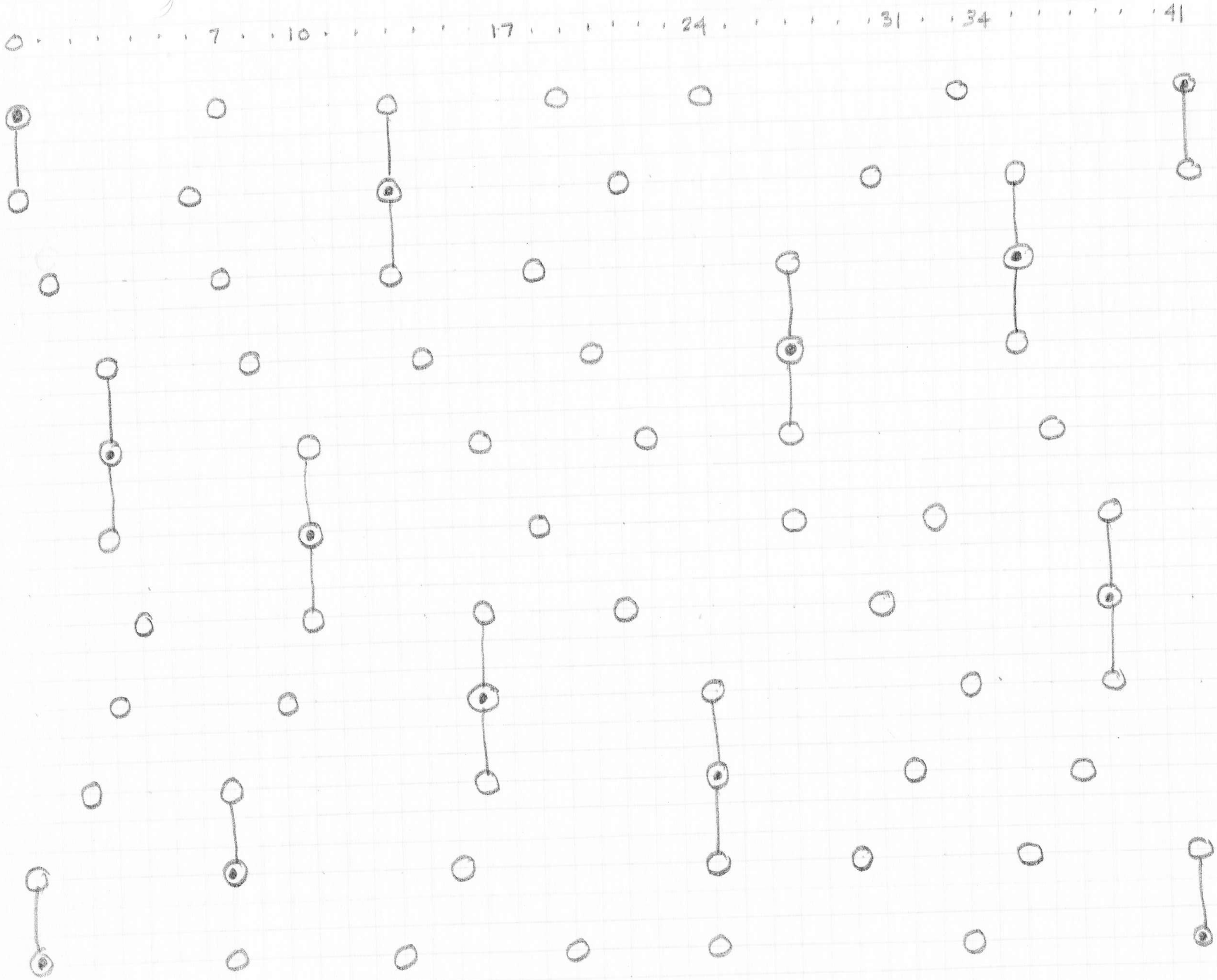
No.	Frequency	Symbol
41/0	528.00	u
39.	510.44	b+
40.	519.15	u/
38.	501.89	b
36.	485.20	B+
37.	493.47	b/
35.	477.07	B
34.	469.07	β
32.	453.48	a+
33.	461.21	β/
31.	445.87	a
29.	431.05	A+
30.	438.40	a/
28.	423.82	A
27.	416.72	α
25.	402.86	g+
26.	409.73	α/
24.	396.11	g
22.	382.94	F+
23.	389.47	g/
21.	376.52	F
20.	370.21	γ
18.	357.90	f+
19.	364.00	γ/
17.	351.90	f
15.	340.20	e+
16.	346.00	f/
14.	334.50	e
12.	323.38	E+
13.	328.89	e/
11.	317.96	E
10.	312.63	ε
8.	302.23	d+
9.	307.38	ε/
7.	297.17	d
5.	287.29	Δ+
6.	292.18	d/
4.	282.47	Δ
3.	277.73	δ
1.	268.50	u+
2.	273.08	δ/
0/41	264.00	u

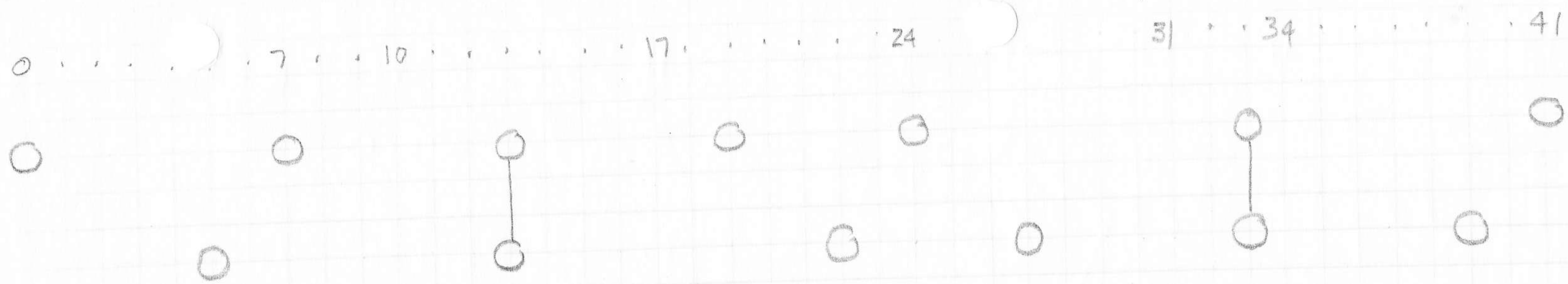


$\frac{9}{7}\times\frac{4}{5}=\frac{36}{35}$

$\frac{11}{\cancel{10}_{2}}\times\frac{\cancel{35}^{7}}{36}=\frac{77}{72}$

good 6-tone scale


0
7
10
17
24
31
34
41

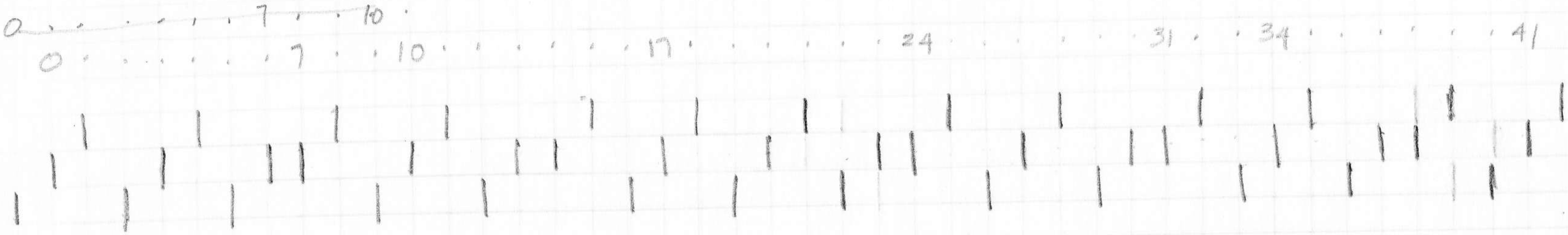


0
7
10
17
24
31
34
41

0 . . . . . . 7 . . 10 . . . . . . 17 . . . . . . 24 . . . . . . 31 . . 34 41

11 12 14 16 33 18 21 22

$\frac{96}{4}$ 4



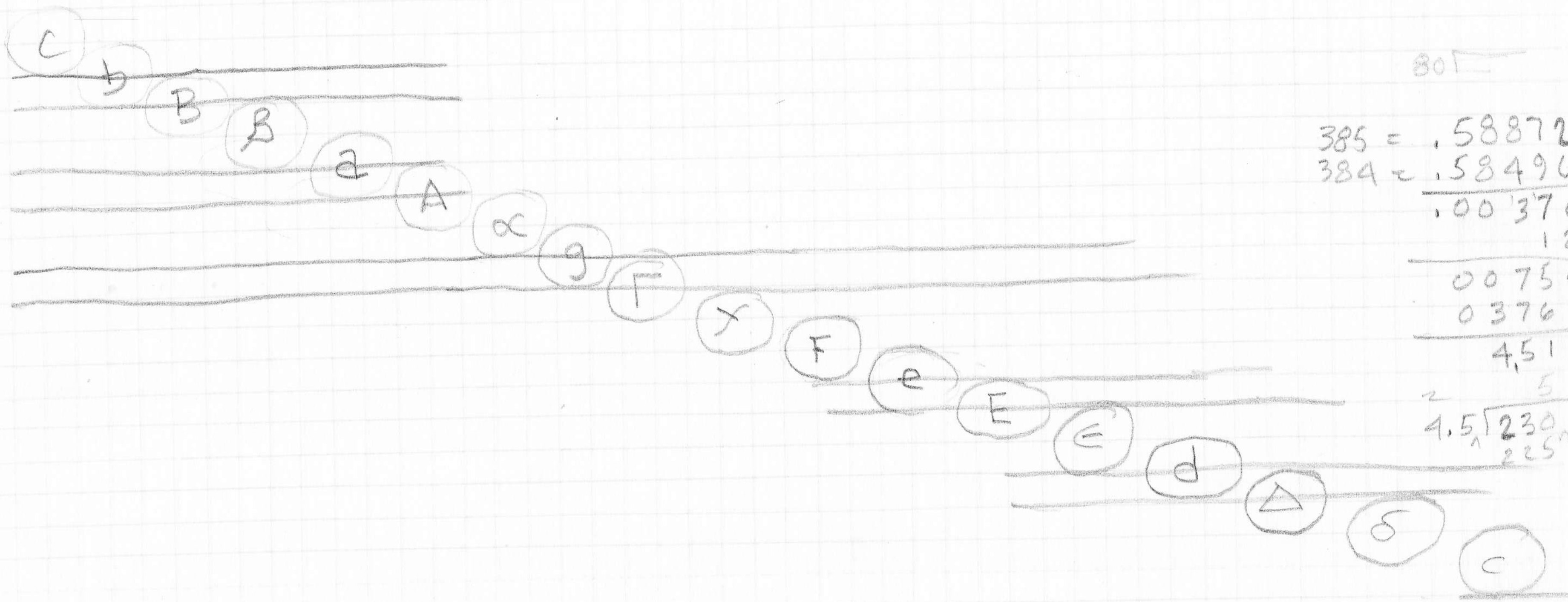
$\frac{\cancel{8}^{4}}{7} \times \frac{11}{\cancel{10}_{5}} =$

$\frac{8}{7} \times \frac{10}{11} = \frac{80}{77}$

c δ Δd ε Ee f γ Γg α Aa β Bb c

$\frac{35}{33}$

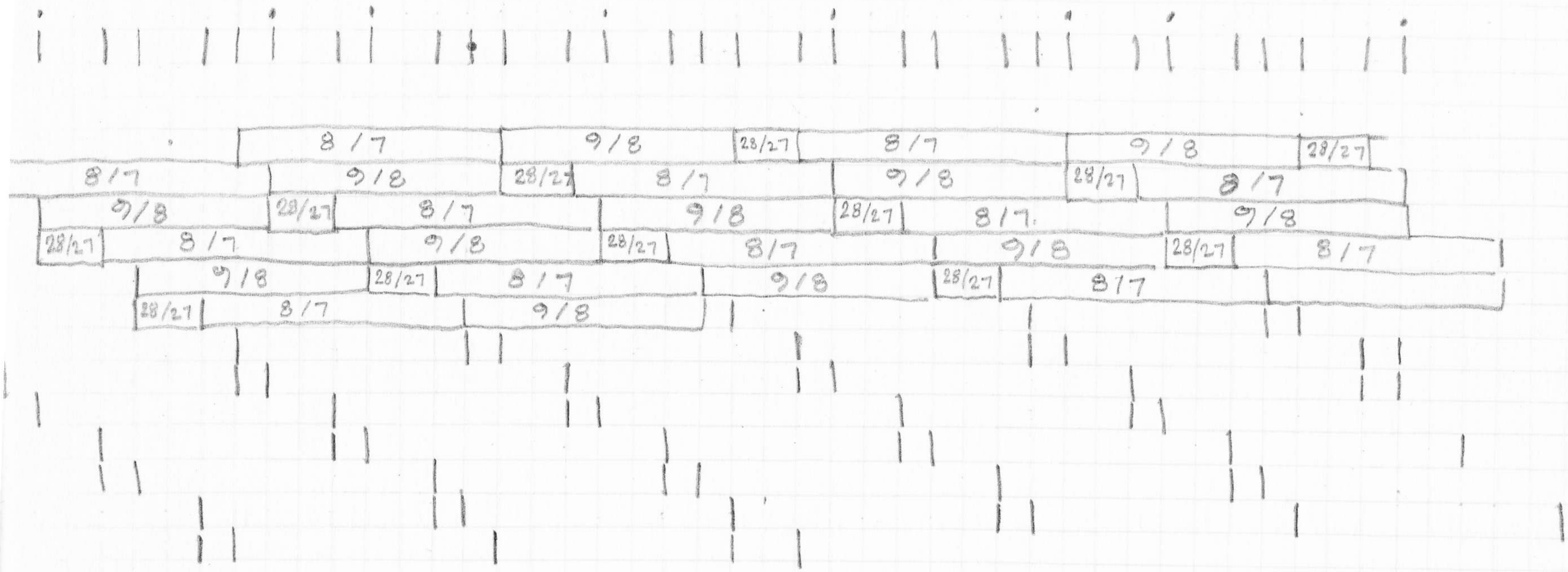
$\frac{5 \cdot 7 \cdot 11}{3} = \frac{385}{384}$



80

385 = .58872
384 = .58496
.00376
12
00752
0376
4.512
5
4.5) 230
225

$\frac{7^2}{5 \cdot 7 \cdot 9} = \frac{7}{59}$



8 11 14 19

7 12 17 22

11 16 21 26

13 18 23 28

~~14~~ 19 24 29 34

29

12 17 22

good scale 12 14 17 18 22 24