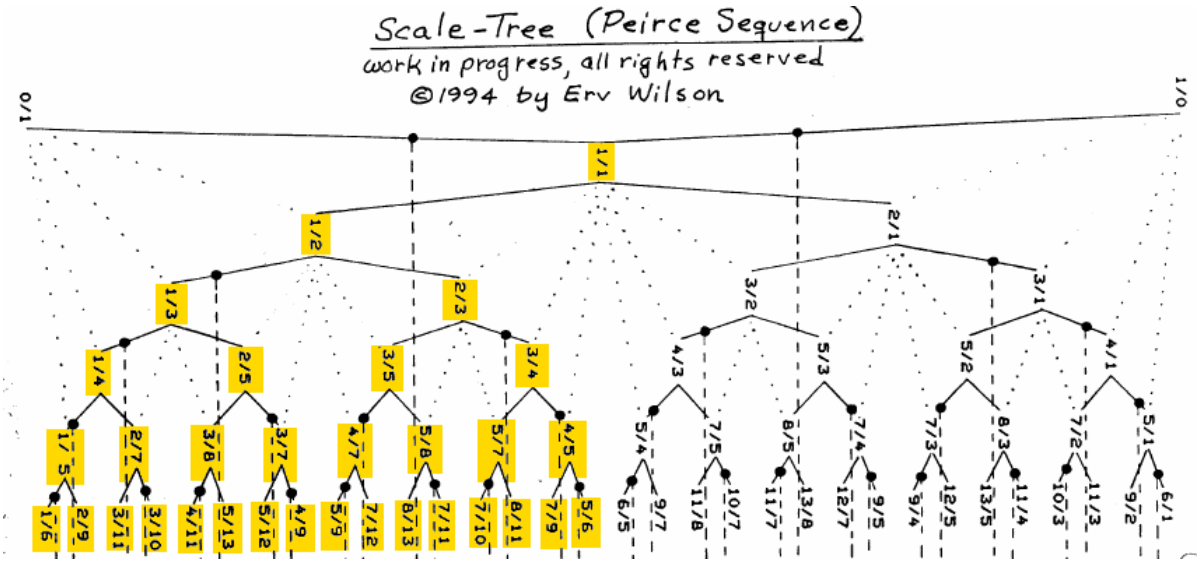
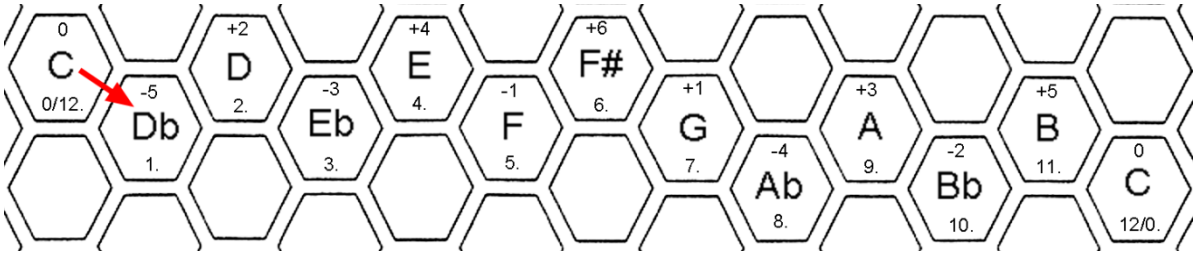
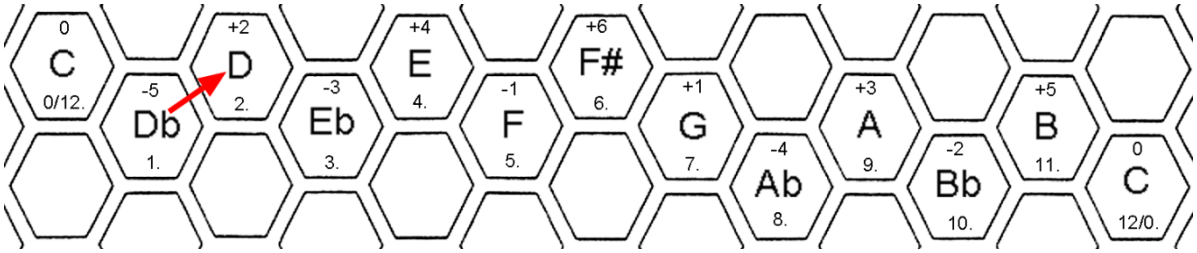
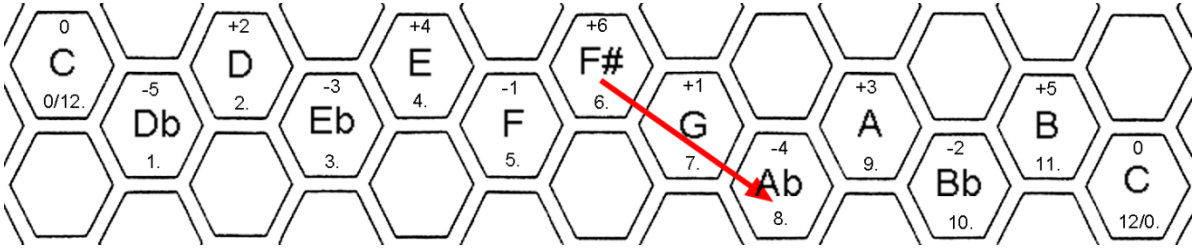
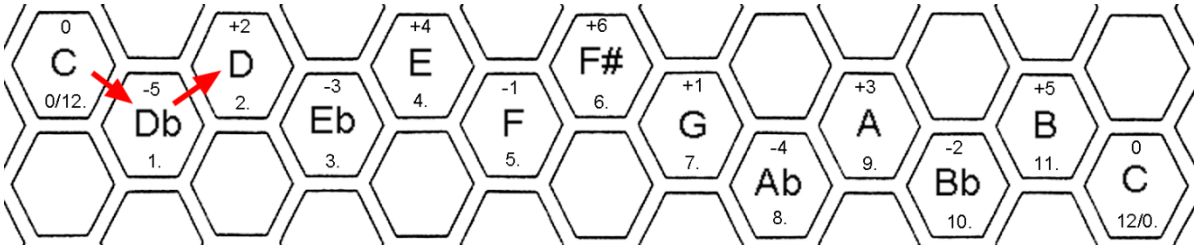
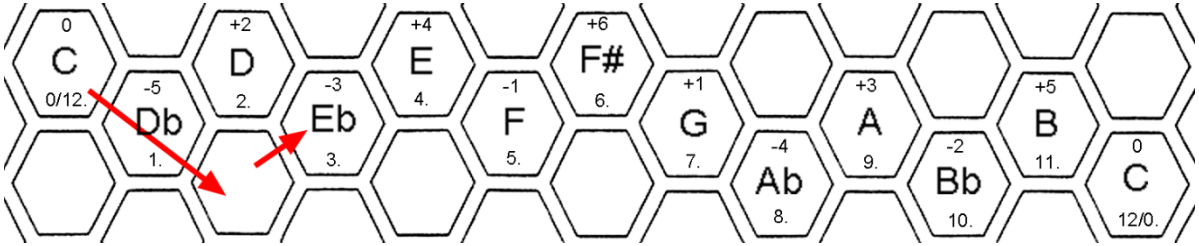
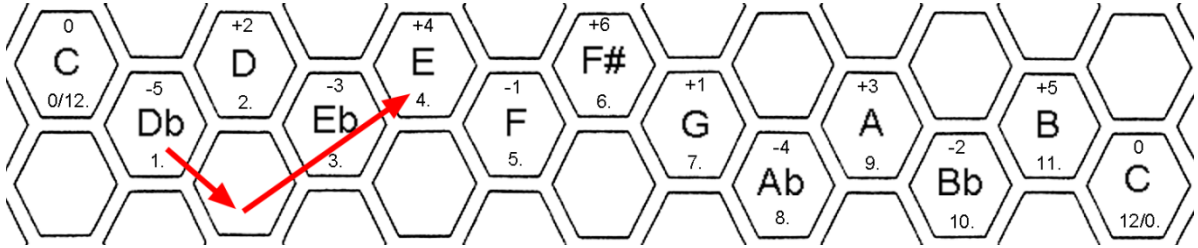


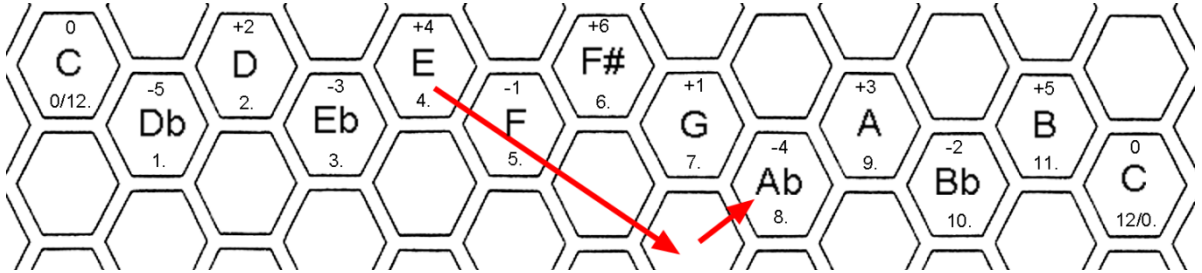
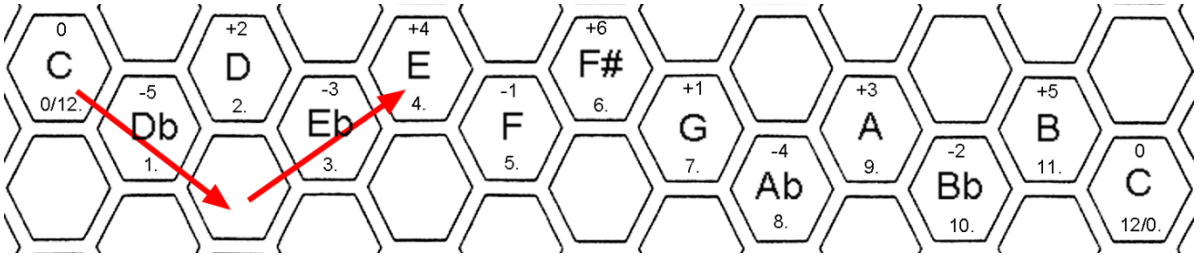
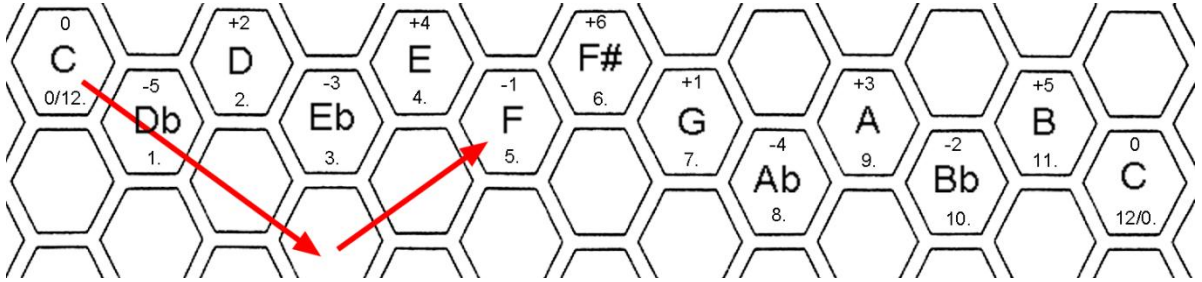
Web Figure 3.1. The first 32 keyboards from the Scale Tree, starting with the 1/1 keyboard and covering all keyboards down to the sixth row, are highlighted in yellow. The layouts of these keyboards are included in "A Spectrum of Keyboards from the Gral Keyboard Guide".

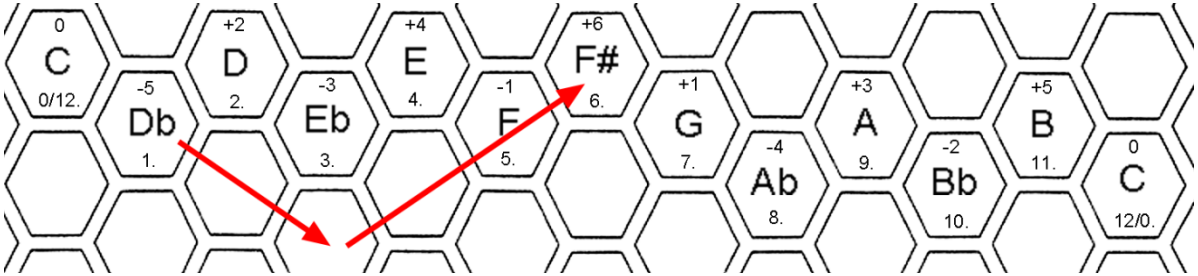
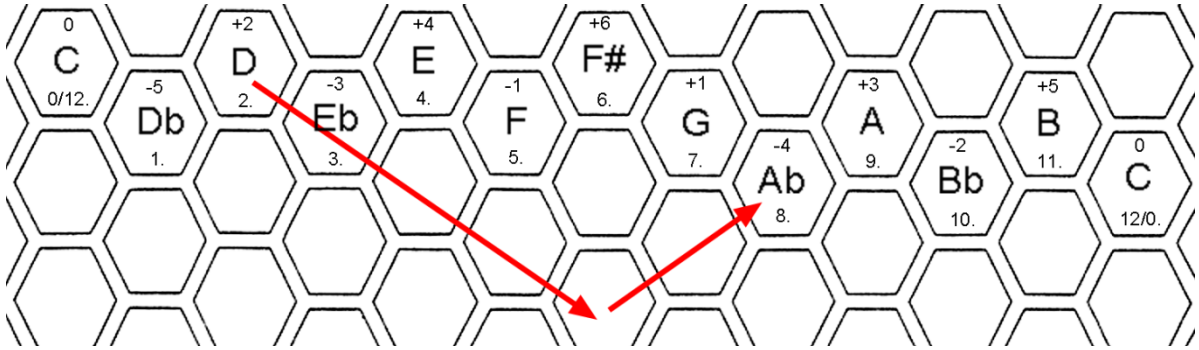
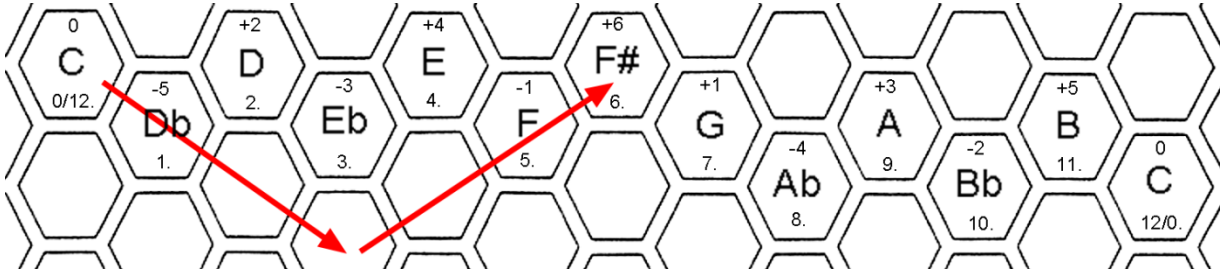
From \ To	1/1 C	256/243 Db	9/8 D	32/27 Eb	81/64 E	4/3 F	729/512 F#	3/2 G	128/81 Ab	27/16 A	16/9 Bb	243/128 B
1/1 C	1/1	256/243	9/8	32/27	81/64	4/3	729/512	3/2	128/81	27/16	16/9	243/128
256/243 Db	243/128	1/1	2187/2048	9/8	19683/16384	81/64	177147/131072	729/512	3/2	6561/4096	27/16	59049/32768
9/8 D	16/9	4096/2187	1/1	256/243	9/8	32/27	81/64	4/3	1024/729	3/2	128/81	27/16
32/27 Eb	27/16	16/9	243/128	1/1	2187/2048	9/8	19683/16384	81/64	4/3	729/512	3/2	6561/4096
81/64 E	128/81	32768/19683	16/9	4096/2187	1/1	256/243	9/8	32/27	8192/6561	4/3	1024/729	3/2
4/3 F	3/2	128/81	27/16	16/9	243/128	1/1	2187/2048	9/8	32/27	81/64	4/3	729/512
729/512 F#	1024/729	262144/177147	128/81	32768/19683	16/9	4096/2187	1/1	256/243	65536/59049	32/27	8192/6561	4/3
3/2 G	4/3	1024/729	3/2	128/81	27/16	16/9	243/128	1/1	256/243	9/8	32/27	81/64
128/81 Ab	81/64	4/3	729/512	3/2	6561/4096	27/16	59049/32768	243/128	1/1	2187/2048	9/8	19683/16384
27/16 A	32/27	8192/6561	4/3	1024/729	3/2	128/81	27/16	16/9	4096/2187	1/1	256/243	9/8
16/9 Bb	9/8	32/27	81/64	4/3	729/512	3/2	6561/4096	27/16	16/9	243/128	1/1	2187/2048
243/128 B	256/243	65536/59049	32/27	8192/6561	4/3	1024/729	3/2	128/81	32768/19683	16/9	4096/2187	1/1

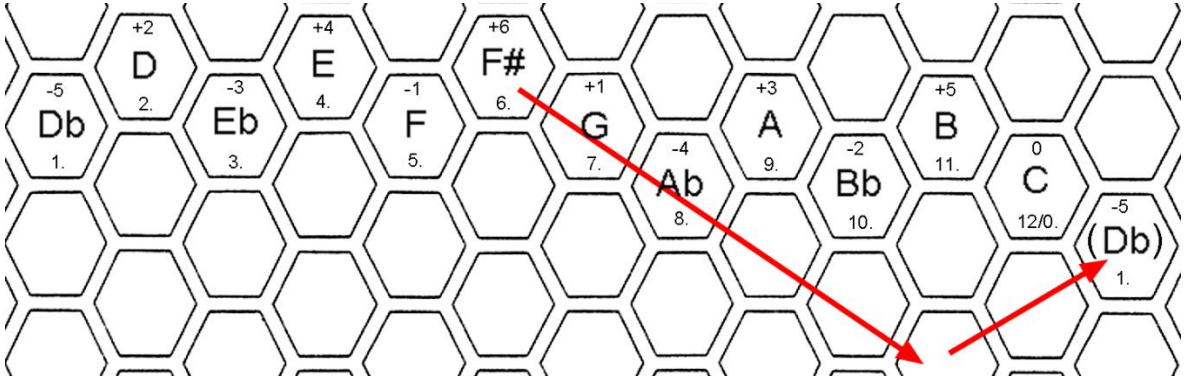
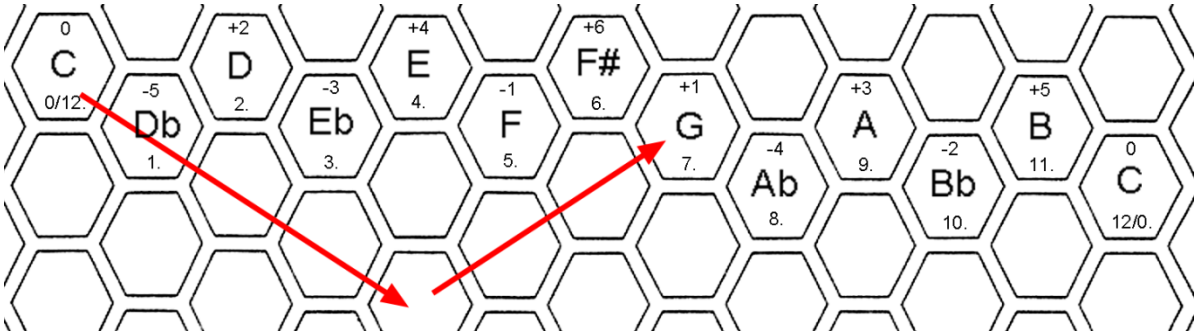
Web Figure 3.2. All intervals found between pitches of the 12-tone Pythagorean scale: intervals are shown between the lower pitch in the leftmost column and the higher pitch in the top row.

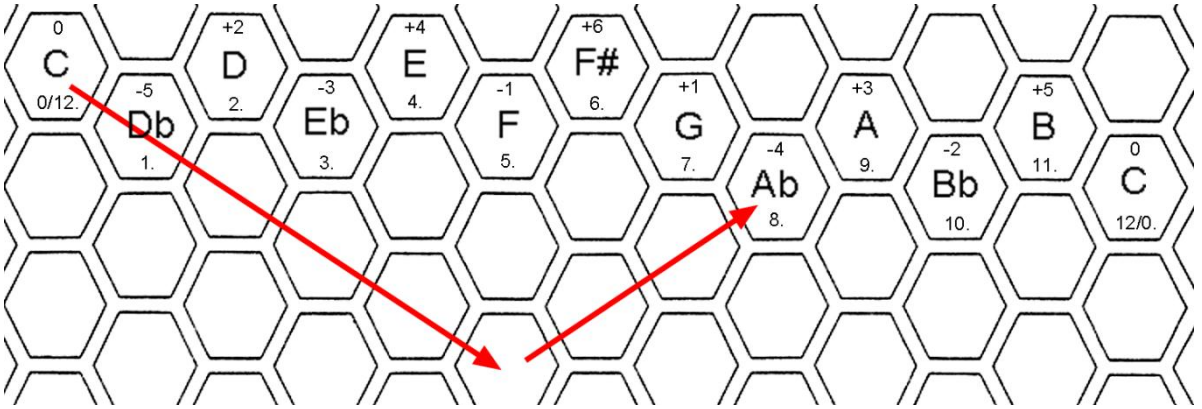
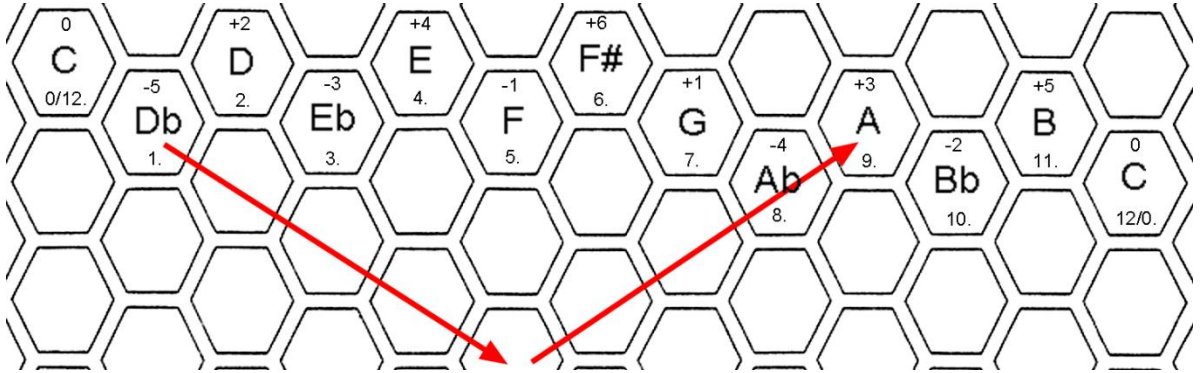
No. of scale steps	Ratio	Letter names between which interval is found	Keyboard coordinates	Example on keyboard
1	256/243	C-Db, D-Eb, E-F, F#-G, G-Ab, A-Bb, B-C	(1*x*, 0*y*)	
	2187/2048	Db-D, Eb-E, F-F#, Ab-A, Bb-B	(0*x*, 1*y*)	
2	65536/59049	F#-Ab, B-Db	(2*x*, 0*y*)	

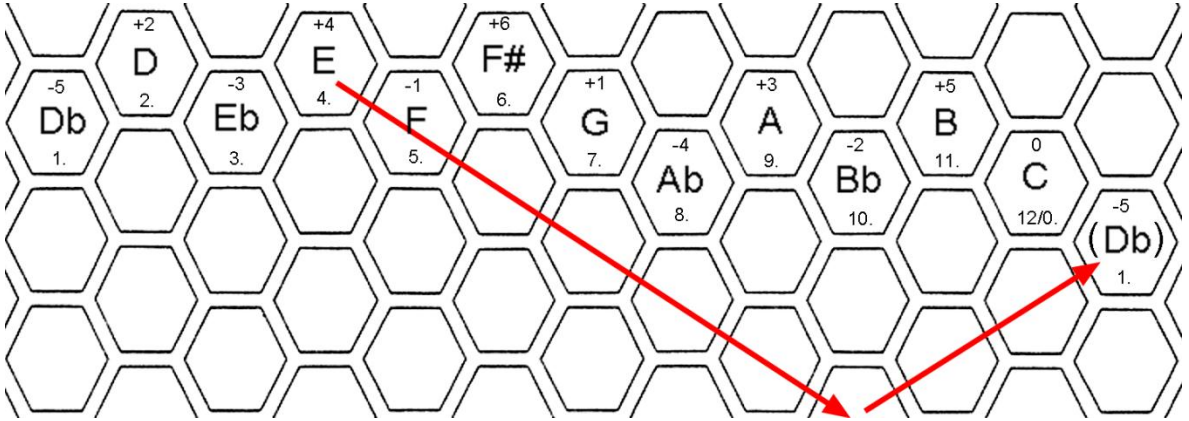
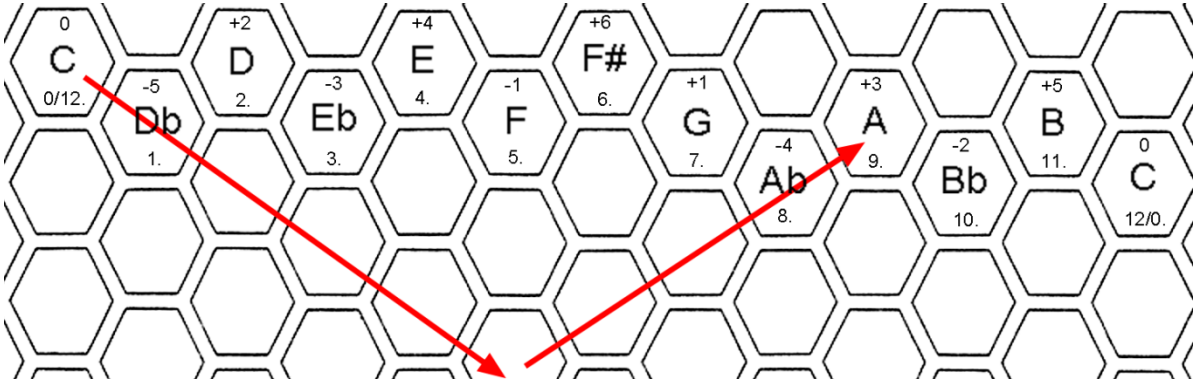
	9/8	C-D, Db-Eb, D-E, Eb-F, E-F#, F-G, G-A, Ab-Bb, A-B, Bb-C	(1*x*, 1*y*)	
3	32/27	C-Eb, D-F, E-G, F-Ab, F#-A, G-Bb, A-C, Bb-Db, B-D	(2*x*, 1*y*)	
	19683/16384	Db-E, Eb-F#, Ab-B	(1*x*, 2*y*)	

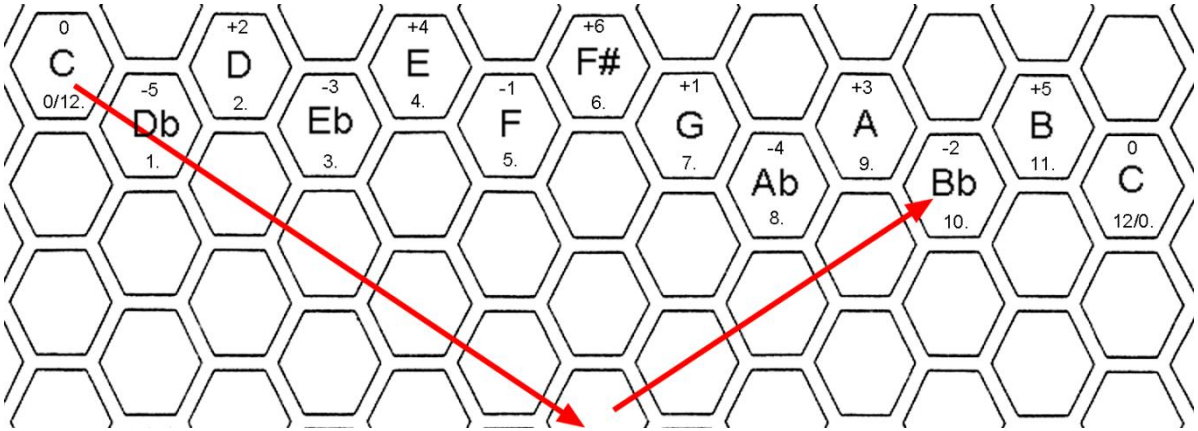
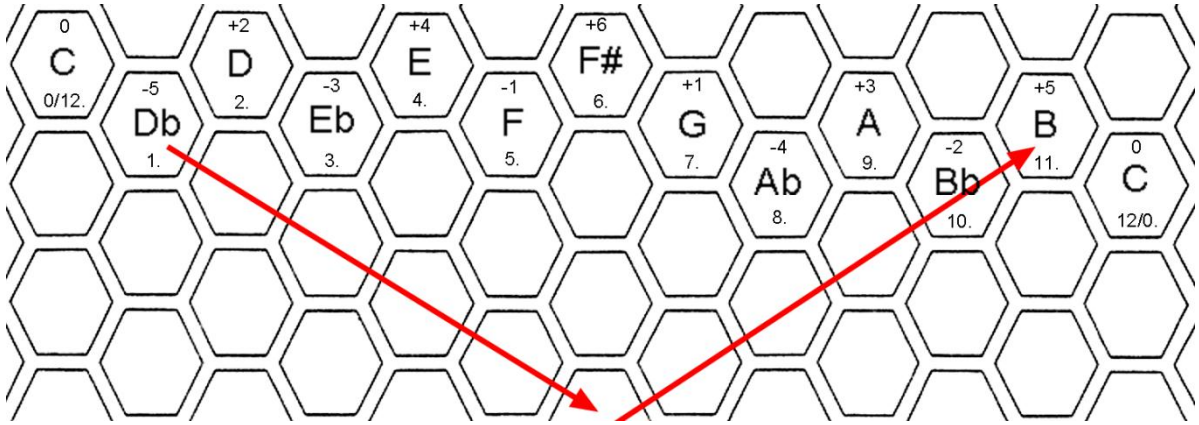
4	8192/6561	E-Ab, F#-Bb, A-Db, B-Eb	(3*x*, 1*y*)	
	81/64	C-E, Db-F, D-F#, Eb-G, F-A, G-B, Ab-C, Bb-D	(2*x*, 2*y*)	
5	4/3	C-F, D-G, Eb-Ab, E-A, F-Bb, F#-B, G-C, Ab-Db, A-D, Bb-Eb, B-E	(3*x*, 2*y*)	

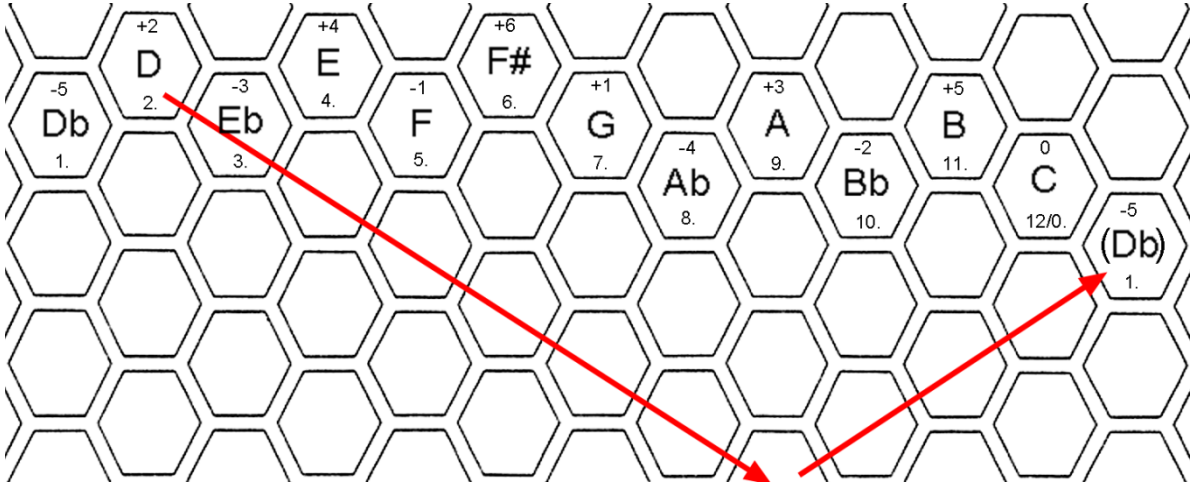
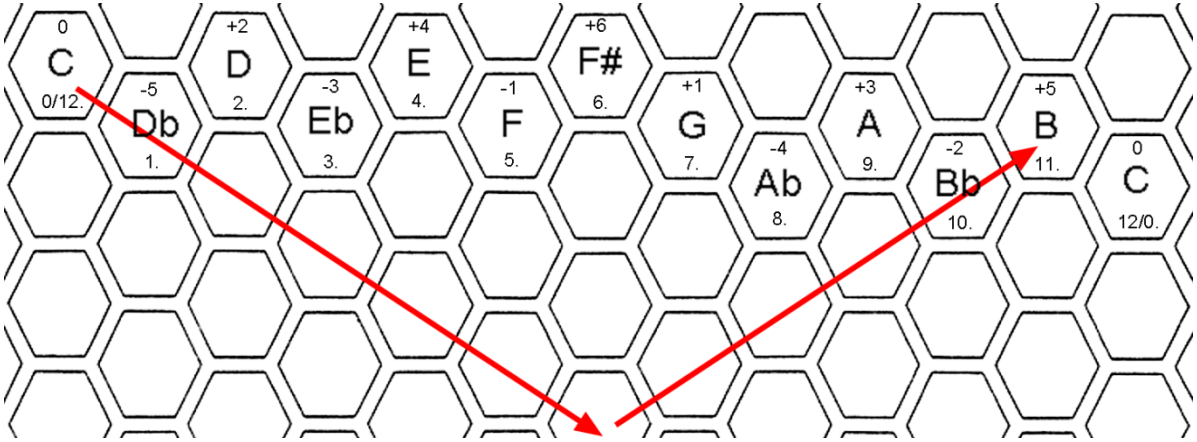
	177147/131072	Db-F#	(2*x*, 3*y*)	
6	1024/729	D-Ab, E-Bb, F#-C, G-Db, A-Eb, B-F	(4*x*, 2*y*)	
	729/512	C-F#, DbG, Eb-A, F-B, Ab-D, Bb-E	(3*x*, 3*y*)	

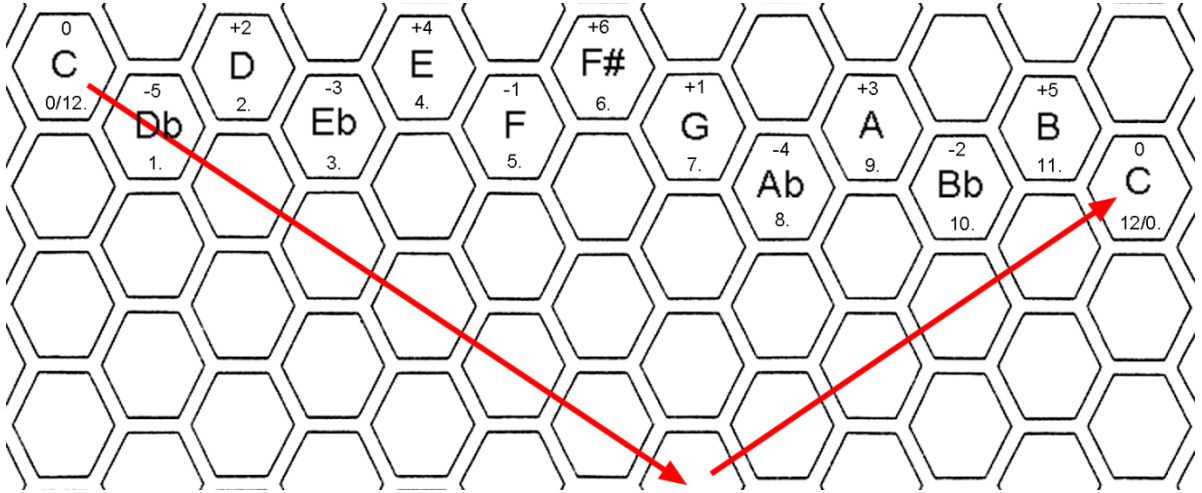
7	262144/177147	F#-Db	(5*x*, 2*y*)	
	3/2	C-G, Db-Ab, D-A, Eb-Bb, E-B, F-C, G-D, Ab-Eb, A-E, Bb-F, B-F#	(4*x*, 3*y*)	

8	128/81	C-Ab, D-Bb, E-C, F-Db, F#-D, G-Eb, A-F, B-G	(5*x*, 3*y*)	
	6561/4096	Db-A, Eb-B, Ab-E, Bb-F#	(4*x*, 4*y*)	

9	32768/19683	E-Db, F#-Eb, B-Ab	(6*x*, 3*y*)	
	27/16	C-A, Db-Bb, D-B, Eb-C, F-D, G-E, Ab-F, A-F#, Bb-G	(5*x*, 4*y*)	

10	16/9	C-Bb, D-C, Eb-D b, E-D, F-Eb, F#-E, G-F, Bb-Ab, A-G, B-A	(6*x*, 4*y*)	
	59049/32768	Db-B, Ab-F#	(5*x*, 5*y*)	

11	4096/2187	D-Db, E-Eb, F#-F, A-Ab, B-Bb	(7*x*, 4*y*)	
	243/128	C-B, Db-C, Eb-D, F-E, G-F#, Ab-G, Bb-A	(6*x*, 5*y*)	

12	2/1	C-C, Db-Db, D-D, Eb-Eb, E-E, F-F, F#-F#, G-G, Ab-Ab, A-A, Bb-Bb, B-B	(7*x*, 5*y*)	

Web Figure 3.3. All intervals found between notes in the 12-tone Pythagorean scale: the first column shows the number of scale steps, the second column shows the just intonation ratio, the third column shows the pitches between which the ratio is found, the fourth column shows the keyboard coordinates for the interval, and the fifth column shows an example of the interval on the keyboard.