X E N H A R M O N I K O N   X I I

An Informal Journal of Experimental Music

Contents:

cover design by Erv Wilson

Xenharmonikon is distributed through Frog Peak Music, P.O. Box 5036 Hanover, NH 03755

DEVELOPMENT OF A 53-TONE KEYBOARD LAYOUT

HISTORICAL BACKGROUND

My interest in music theory was sparked in early 1942 by my fiance, Evelyn Olmsted. I was finishing my senior year at the California Institute of Technology, and she was getting her Master's Degree in Music at the University of California at Los Angeles. She was a very capable pianist.

World War II had begun for the U.S. in December 1941. I volunteered and was commissioned in the US Naval Reserve in the spring of 1942. Evelyn and I both finished the school year and were married in June of 1942 before I reported for active duty. My duty assignments were ashore, and Evelyn was able to be with me.

In addition to her skills as a performing musician Evelyn had extensive schooling in musical theory. Because of my engineering training, the mathematical aspects of music theory attracted my interest. I did not then or now consider myself to be a musician.

From Evelyn I learned that harmony is explained in terms of just intonation intervals despite the fact that the 12-tone equally tempered scale has been employed for centuries. We discussed at length the problems of actually employing just intonation tunings in instrumental music.

Evelyn seriously doubted that a practical keyboard instrument could be devised which would accommodate even limited modulation in just intonation; wide modulations and the requirement to play in different keys made a solution practically impossible.

I was not so pessimistic. I had read that a 53-tone equally tempered scale offered an excellent approximation to just intonation, and decided to explore the possibility of its use. Although 53 tones is a large number, it is certainly a more manageable quantity than the almost limitless number of tones required by just intonation.

At that time I was unaware of Hermann Helmholtz's "On the Sensations of Tone" which contains a description of Mr. R.H.M. Bosanquet's harmonium employing the 53-tone tempered scale. In retrospect my ignorance was perhaps an advantage.

In the late summer of 1942 I decided to numerically compare the tones of the 53-tone scale to those of just intonation. Lacking a list of just intonation tones to use in such a comparison, I constructed what I now call the "Major-Minor Triadic Lattice" to obtain the 'just' fractions I needed. Erv Wilson has redrawn portions of my original work sheet for inclusion in this article. See Figure #1.

Using five place logarithm tables, I made the calculations and satisfied myself that the tones of the 53-tone system did in fact provide extremely close approximations over the entire range of the lattice. In fact the approximations were so close that the ear could probably not perceive the differences.

Although we had no instrumental means to produce tones tuned to just intonation, it helped enormously that Evelyn could mentally 'hear' these tones, intervals and chords when asked to think of them as if sung by an a cappella choir. Together we worked out a musical nomenclature for the 53 tones which was consistent with harmonic analysis.

I then abandoned further use of just intonation descriptions, with their awkward fractions and often trivial pitch distinctions. I replaced the unmanageable lot with a mere 53 tones, and devoted my efforts to finding an arrangement of their digitals which would permit a keyboard musician to utilize them.

From our discussions I learned that in the Western musical tradition the intervals of the major and minor thirds dominate the chordal structures. Movement to other scale degrees and modulations often proceed by intervals of fourths or fifths.

Fortunately, it eventually became clear that the digitals for only a limited subset of tones are required to be accessible to the fingers of the hand <u>at any given moment</u>. This was perhaps the crucial insight.

Another essential observation was that if the digitals for a limited subset of equally tempered tones are arranged to form a extendable pattern, then chords of the various scale degrees, modulations and key changes can be accommodated by movement of the performer's arms over that extended pattern, without change in the fingering.

The steps by which I implemented these fundamentals to derive a keyboard layout are given below in the section entitled "Details of the Keyboard Development".

By late 1942 I was stationed in the Navy Department in Washington, DC. In off hours I visited the Library of Congress where I searched both the literature and patent files to determine what others had done. Although I did find descriptions of other keyboards designed to extend tonal resources, I found no layouts with the same pattern as mine. Despite my extensive search, I did not discover Helmholtz's "The Sensations of Tone" with his description of Bosanquet's 53-cycle harmonium. Perhaps I was misled by the title or its cataloguing.

I then turned my attention to perfecting the size and shape of the digitals. At one point I felt strongly that I needed to confirm the workability of my design on a three dimensional model. I gathered some stiff common mud and fashioned a model of a portion of the proposed keyboard in a small cardboard box. After it dried Evelyn and I tried out the 'feel' of the fingering of various melodies, chords and arpeggios. Only minor modifications were made.

I seriously considered filing for a patent, but implementing the idea with digitals, switches, tone generators, etc. was far beyond my skills and resources. Furthermore, I had no assurance that there was a market for such a device.

On my way back to California after the war in early 1946 I passed through Chicago. I phoned Loren Hammond, the electric organ pioneer, and briefly discussed the matter with him. He was very kind, but indicated that he was concentrating all his efforts on developing a chord organ. The notes of my studies were filed and largely forgotten for over 30 years.

In November 1977 my daughter, Chris, returned to Southern California after several years in Germany. While she was job hunting, she helped straighten my files and papers. She ran across my 1942 work on the 53-tone keyboard, and we discussed it at some length.

In February 1978 Chris brought to my attention an article in the "Rolling Stones" magazine about Tom Stone's guitar with interchangeable fretboards which utilized alternate tunings. I phoned Tom Stone in Fairfield, Iowa. He indicated that he had little interest in keyboard instruments, but referred me to Erv Wilson in Highland Park.

I first met with Erv on Saturday afternoon February 25, 1978. He was intrigued with the keyboard layout I term 'Design A', and we had a second meeting Monday night the 27th. He seemed particularly impressed by the feature of the height of the digitals of successive minor thirds increasing regularly. My reaction was "So?!".

I had realized the final keyboard by a simple geometric extension of the pattern of tones along the diagonals and columns. Of course I knew, in a general sense, that there were many musically useful intervals which repeated themselves throughout the layout, but I had assigned no particular importance to the minor third.

Erv also expressed particular interest in a 19 tone subset that I had marked out on my keyboard layout. He pointed out that my layout <u>could</u> be considered to <u>have been</u> generated by the interval of a minor third, and that the 19 tone subset <u>did</u> have the properties of a Wilson MOS scale. The 19 tone subsets within the 53 tone layout are discussed below.

I keep reminding Erv that interesting features that he may discern in my layout are usually his insights, and not the result of my having consciously contrived to build each of them in.

Erv introduced me to Hermann Helmholtz's "On the Sensations of Tone", Joel Mandelbaum's dissertation on "Multiple Division of the Octave...", and Harry Partch's "Genesis of a Music". At his prompting I prepared several documents which described the work I had done so many years before, and sent them to Tom Stone, George Secor, Joel Mandelbaum, Ben Johnston, and John Chalmers.

As an extension of my Keyboard Design 'A', Erv came up with a layout for a 53 tones per octave set of his aluminum Tubulongs. Glen Prior fabricated two complete sets of two octaves for me in the summer of 1978, and some experiments and demonstrations were made with them. They attracted little interest.

At that time Erv urged me to write and publish an account of my keyboard and how I came to devise it. Obviously this has not been a high priority with me. That it is now being published is due to Erv's unfailing interest, encouragement, help, and especially persistence. He has also provided the full page figures for this paper, which are essential to its elucidation. With the publication of this article perhaps he will feel free to pursue and publish those insights he has had as a result of analyzing my keyboard layouts.

From the beginning my primary goal has been to devise and promote a keyboard layout that would permit a keyboard musician to play harmonic music in close approximation to 'just intonation'. Beyond that I hoped that if the tonal resources of 53 tones were made conveniently available to performing musicians, they would find other ways to exploit them.

The present low cost of electronic tone generation offers pioneering musicians the opportunity to free Western music from its 12-tone cul-de-sac.

DETAILS OF THE KEYBOARD DEVELOPMENT

In this section I detail the development of my 53-tone keyboard layout some 46 years ago. I kept few notes, so this exposition is largely dependent on my recall.

After establishing that the 53-tone tempered scale very closely approximates just intonation, I turned my attention to the problem of keyboard design.

From discussions with my wife, Evelyn, I concluded that a workable keyboard layout should meet two conditions:

1) The placement of the digitals must at the minimum accommodate the fingering of the tones and chords of the major and minor scales in a single key, and

2) The pattern of placement must be extendable, so that tonal regions more distant than the sub- and super-tonic could be utilized, and key changes accommodated. I later found that this second condition describes a 'generalized keyboard'.

Further discussions revealed that only ten tones in an octave are required as tonal material to accommodate the major-minor tonal system of Western Europe <u>provided no transposition is involved</u>. These tones and their relation to the corresponding tones of the tempered 53-tone scale are listed in Figure #2 below.

Figure #2

Musical Tone	Frequency Fraction	Decimal Equivalent	53-tone Equivalent	53-tone Number	Successive Intervals
C	1/1	1.000	1.000	0	
D	9/8	1.125	1.125	9	9
E^b	6/5	1.200	1.201	14	5
E	5/4	1.250	1.249	17	3
F	4/3	1.333	1.333	22	5
G	3/2	1.500	1.500	31	9
A^b	8/5	1.600	1.601	36	5
A	5/3	1.667	1.665	39	3
B^b	9/5	1.800	1.801	45	6
B	15/8	1.875	1.873	48	3
C	2/1	2.000	2.000	53	5

When I refer to 53-tone scale tones or intervals by number, I call that number a "Mercator", after Nicholaus Mercator who described this scale in the 17th century: e.g. the interval of a fifth, or the tone G when C is 0, is 31 Mercators or abbreviated to 31 M.

The interval of one (1) Mercator closely approximates and performs the functions of the small just intonation intervals known as 'commas'. Consequently I sometimes refer to it as a 'Mercator comma'. I allude to the syntonic (or Didymic) comma, the ditonic (or Pythagorean) comma, and the diaschisma.

For increased clarity I often label musical tones with their corresponding 'Mercator' number in this manner:

C:0, D:9, E^b:14, E:17, F:22, G:31, A^b:36, A:39, B^b:45, B:48.

From the last column of the Figure #2 above, it is apparent that the 10 successive tones are separated by intervals of two types:

1) those of 3 Mercators (or multiples thereof), and
2) those of 5 Mercators.

As the first step in the design of the layout I placed the E^b in a position to the left and below the E, thus:

E: 17
E^b:14

I then arranged the other 3 sets of digitals, separated by intervals of 3 Mercators (or multiples of thereof), on diagonals directed upward and to the right, inserting unidentified tones as required. See Figure #3 below.

Figure #3

B:48
D:9 G:31 B^b:45
*:6 *:28 *:42
*:3 E:17 *:25 A:39
C:0/53 E^b:14 F:22 A^b:36 C:53/0

Note: '*' indicates a musical function not yet identified

For fingering the major and minor triads, I deemed it desirable that the digitals for E^b:14, E:17, G:31 and should lie at approximately the same horizontal level as C:0/53 and C:53/0.

If the first digital of each of these 4 diagonal sets is positioned on the third line <u>below</u> the end of the previous set, the arrangement of Figure #4 results.

Figure #4

```
Columns:
 #0   #1   #2   #3   #4   #5   #6   #7   #8   #9   #10   #11


                D:9
           *:6   :                                      B:48
     *:3         :  E:17             G:31         Bb:45   :
C:0/53        Eb:14  :          *:28   :      *:42        :
                     :  *:25           :  A:39          C:53/0
                    F:22            Ab:36
```

Note: '*' indicates a musical function not yet identified

The reader should note the following salient points:

1) Each digital which is above and to the right of another handles a tone which is 3 Mercators higher in pitch.

2) Each digital which is directly below another handles a tone which is 5 M higher in pitch.

3) The digitals for C:0/53, E^b:14, E:17, G:31, and C:53/0 lie at <u>nearly</u> the same level.

4) There are 11 columns per octave.

When the pattern of Figure #4 is extended, stopping when a tone differing only 1 M from previous basic tones is reached, and the tones labeled, the result is Figure #5.

Figure #5

```
Columns:
 #0  #1  #2  #3  #4  #5  #6  #7  #8  #9  #10 #11/0

Tones:

            D:9          /F:23
       Db:6        E#:20       G#:34       B:48
   C#:3       E:17        G:31        Bb:45
C:0/53   Eb:14       Gb:28       A#:42
       Ebb:11      F#:25       A:39        C:53/0
   \D:8       F:22        Ab:36       Cb:50
```

The reader will note that the /F:23 tone, which is located at the top of column #6, has a pitch just 1 M higher than that of the F:22, located at the bottom of column #4. Similarly, the \D:8 tone at the bottom of column #1 has a pitch just 1 M lower than that of the D:9 located at the top of column #3. These tones delimit a natural feature on the extendable pattern, defining a subset of 19 basic tones.

Beginning with /F:23, a second group of 19 tones is generated above the basic group by further extension of the pattern; the tones of this group are pitched just 1 M higher than those of the basic 19 tone group. Similarly, there is a another group of 19 tones lying below the basic group pitched just 1 M lower beginning with the \D:8 tone in column #1. This is perhaps clearer in Figure #8, Keyboard Design A, which is discussed later.

I noted this feature with considerable interest in 1942, and recognized it as the 53 tone expression of the just intonation 'syntonic comma', 81/80.

There is an apparent disparity here: 3 groups of 19 amounts to 57, not 53 tones. The discrepancy is resolved if one permits each of 4 of the 53 tones to serve two different musical functions. This is analogous to an $F^{\#}$ also serving as G^{b} in the 12-tone tempered system.

Evelyn and I had no problem with notation. With 7 diatonic tones labeled "A" through "G", and 3 possibilities provided by the natural, sharp and flat designations, there are 7 x 3 or 21 unique musical labels. These signs were sufficient to uniquely describe each of the 19 tones within each group.

To distinguish between the 3 groups themselves, we initially labeled the tones of the upper group of 19 by placing a bar above the basic musical designation. We correspondingly labeled those tones of the lower group with a bar below, as in "$\underline{D^{\#}}$".

I credit Mr. R.H.M. Bosanquet with his most convenient slash "/" and back-slash "\" to designate tones pitched one Mercator above or below a basic set of tones. These signs I only adopted when I later learned of them.

Erv Wilson has informed me that in the system of musical notation that Evelyn and I developed in 1942, and which I continue to promote for this keyboard layout, certain tones are differently designated as compared to notation now in common use.

See Figure #6 for my system of labeling.

Figure #6

53 TONES LISTED IN THREE GROUPS OF 19

LOWER GROUP				BASIC GROUP				UPPER GROUP			
53-TONE	CENTS	NAME	TONAL FUNCTION	53-TONE	CENTS	NAME	TONAL FUNCTION	53-TONE	CENTS	NAME	TONAL FUNCTION
52	1177	\C	160 / 81	0/53	0	C	1 / 1	1	23	/C	81 / 80
2	45	\C#	250 / 243	3	68	C#	25 / 24	4	91	/C#	135 / 128
5	113	\Db	16 / 15	6	136	Db	27 / 25	7	158	/Db	2187 / 2000
8	181	\D	10 / 9	9	204	D	9 / 8	10	226	/D	729 / 640
10	226	\Ebb	256 / 225	11	249	Ebb	144 / 125	12	272	/Ebb	729 / 625
13	294	\Eb	32 / 27	14	317	Eb	6 / 5	15	340	/Eb	243 / 200
17	385	\E	100 / 81	17	385	E	5 / 4	18	408	/E	81 / 64
19	430	\E#	625 / 486	20	453	E#	125 / 96	21	475	/E#	675 / 512
21	475	\F	320 / 243	22	498	F	4 / 3	23	521	/F	27 / 20
24	543	\F#	1000 / 729	25	566	F#	25 / 18	26	589	/F#	45 / 32
27	611	\Gb	64 / 45	28	634	Gb	36 / 25	29	657	/Gb	729 / 500
30	679	\G	40 / 27	31	702	G	3 / 2	32	725	/G	243 / 160
33	747	\G#	125 / 81	34	770	G#	25 / 16	35	792	/G#	405 / 256
35	792	\Ab	128 / 81	36	815	Ab	8 / 5	37	838	/Ab	81 / 50
38	860	\A	400 / 243	39	883	A	5 / 3	40	906	/A	27 / 16
41	928	\A#	1250 / 729	42	951	A#	125 / 72	43	974	/A#	225 / 128
44	996	\Bb	16 / 9	45	1019	Bb	9 / 5	45	1019	/Bb	729 / 400
47	1064	\B	50 / 27	48	1087	B	15 / 8	49	1109	/B	243 / 128
49	1109	\Cb	256 / 135	50	1132	Cb	256 / 135	51	1155	/Cb	243 / 125
52	1177	\C	160 / 81	53/0	1200	C	2 / 1	1	23	/C	81 / 80

To clarify how my notation of the 53 tones compares to their harmonic relations, Erv Wilson has prepared a special "Major-Minor Triadic Lattice for 53", Figure #7.

Referring back to tentative layout in Figure #4, the **C:53/0** digital in column #11/0 is seen to lie one row <u>below</u> that of the initial **C:0/53** digital in column #0. I deemed it desirable that it lie <u>exactly</u> on the same horizontal row. Raising each successive column by 1/11th of the height of a digital easily accomplished this.

Making this adjustment, further extending the pattern, and labeling all 53 tones results in my Keyboard Design A, illustrated in Figure #8. For the reader's convenience Erv Wilson has noted on the digitals the just intonation fractions which I had associated with the tones at the beginning of my quest.

Considering the Keyboard Design A in three dimensions, it is noted that the front portion of each digital, the stem, is raised above the 3 adjacent digitals; the mid section has a higher digital on its right, and the back portion has higher digitals on 3 sides.

I was concerned that a musician might have a problem accessing and depressing that rearward portion of the digital. The solution was to widen the digital in that section and, if necessary, narrow the width of the stem portion.

Since this is a generalized keyboard, the digitals must all be identical in shape and fit together to fill the plane. The shape which meets the requirements is that of a 'block 7', with its horizontal top exactly twice as wide as its vertical stem. This shape is illustrated in Figure #9A below.

Figure #9A

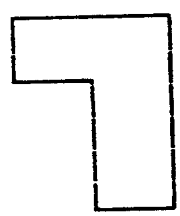
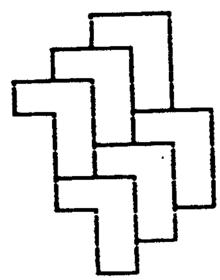


Figure #9B



The 'block 7' digitals can be arranged to form nested chains of digitals directed upward and to the right as illustrated in Figure #9B above.

A greater area for the musician's finger tip is offered if the corners at the right rear of each digital are rounded. When appropriately extended, my Keyboard Design B is the result. See Figure #10.

Although there are now 15 vertical columns to the octave, the digitals which formed the original 11 vertical columns are still found in 11 columns, but those columns are now tilted slightly to the left of vertical.

I hope this description of my keyboard design process makes it seem very straightforward, logical and obvious. It was not so at the time. I have recounted only the path that lead to the solution. I have forgotten the details of the many frustrating hours that were spent on fruitless leads.

APPENDIX #1

THE APPLICATION OF MY KEYBOARD DESIGN TO OTHER SCALES

Erv Wilson has noted that my keyboard Designs A and B can accommodate any tonal system that can be explicitly represented as a chain of minor thirds. Of special harmonic interest are those systems where the octave plus a fifth, totaling 3/1, is the multiple of 6 minor thirds. For example in the 53-tone system the number of tones in an octave and a fifth is 53 + 31, or 84. The interval 14 M expresses the minor third, and 84 is exactly 6 times 14.

Equally tempered scales of this type include those of 19, 34, 53, 72, and 87 tones per octave. Erv has shown in Figure #11 how their tones would be patterned on my Keyboard Design B.

Figure # 1 Major-Minor Triadic Lattice for 53
(redrawn from 1942 original)

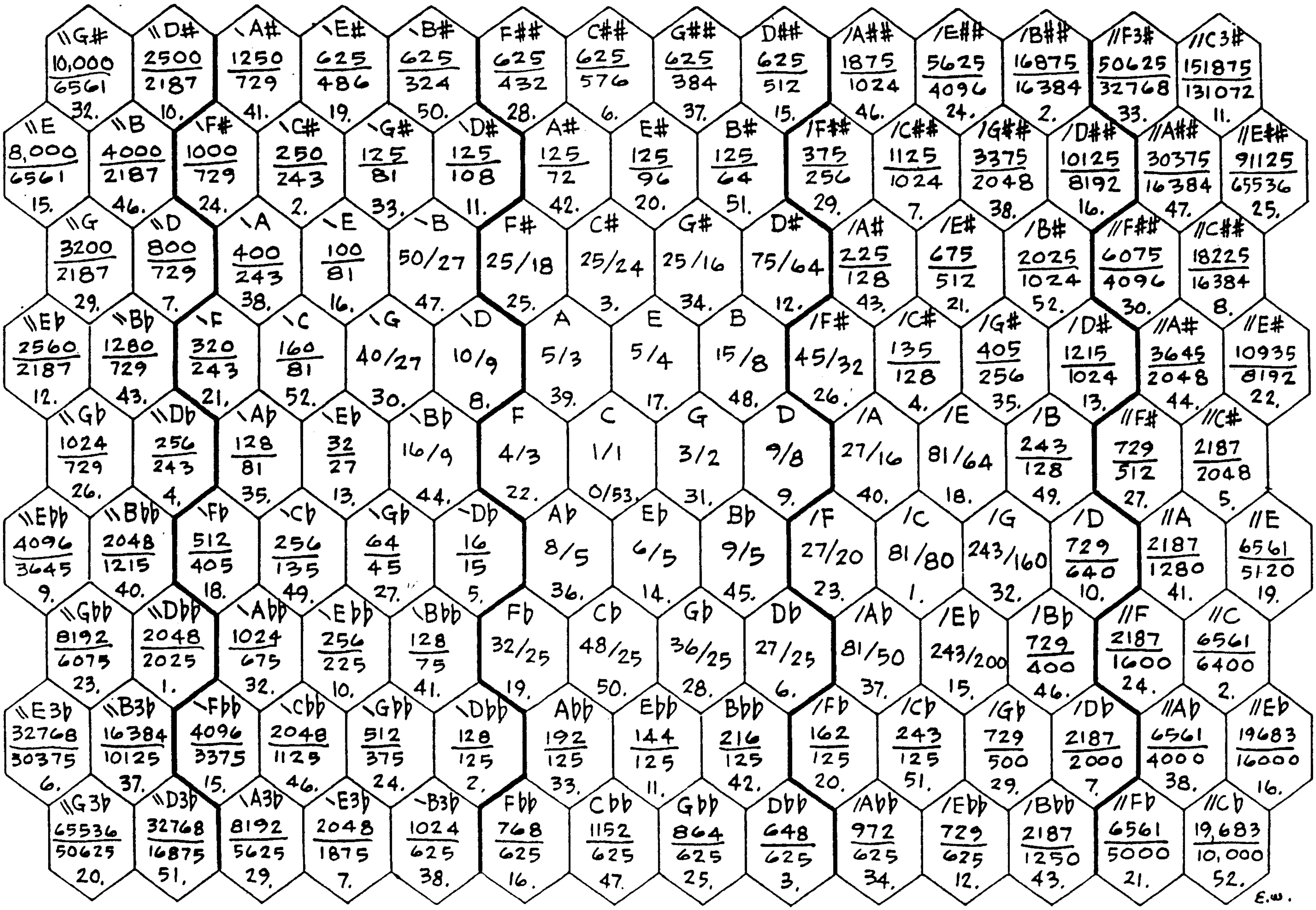


Figure # 7
Major-Minor Triadic Lattice for 53
(19 defined by minor thirds, 1978)

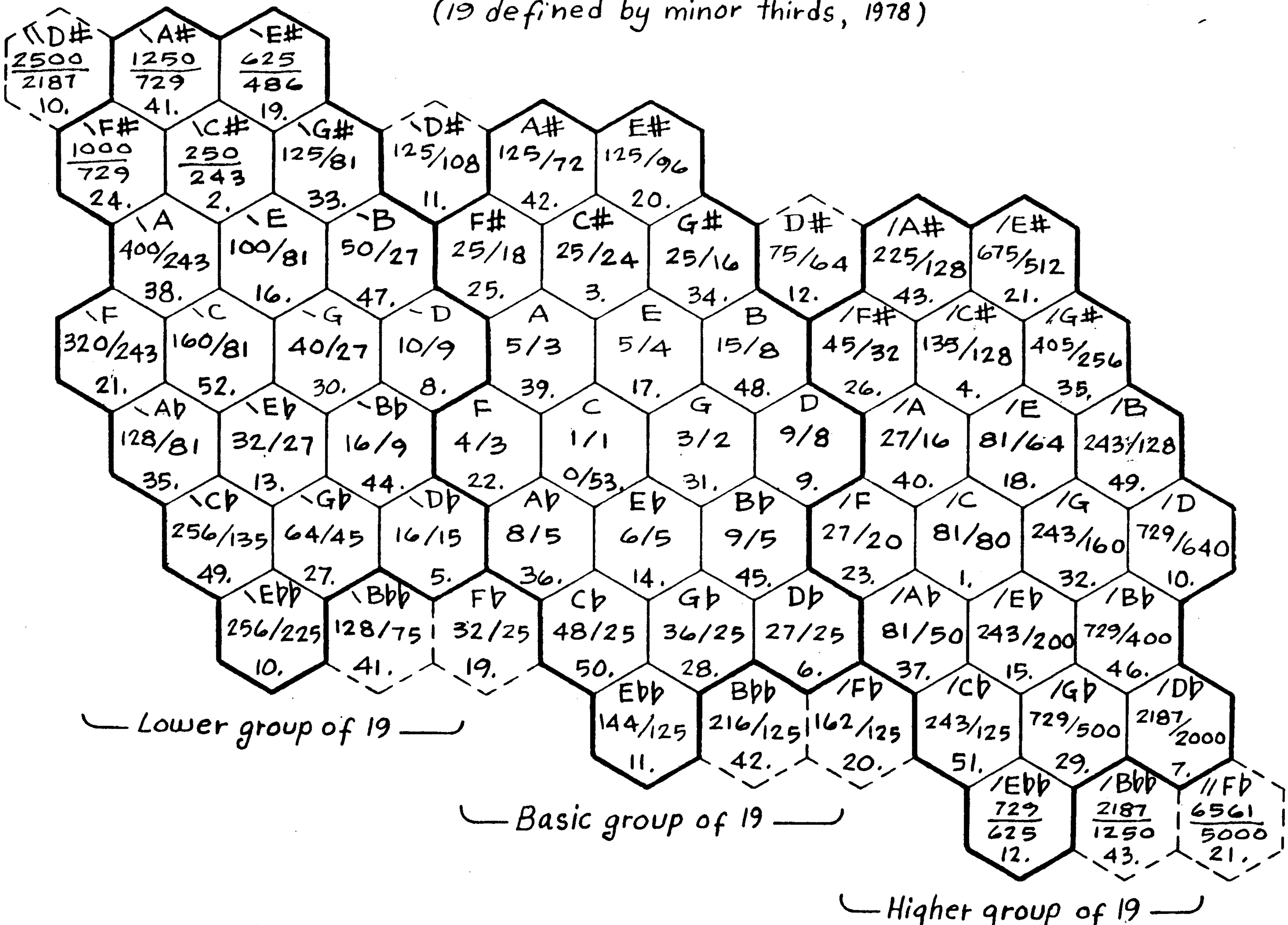


Figure # 8
Keyboard Design "A"

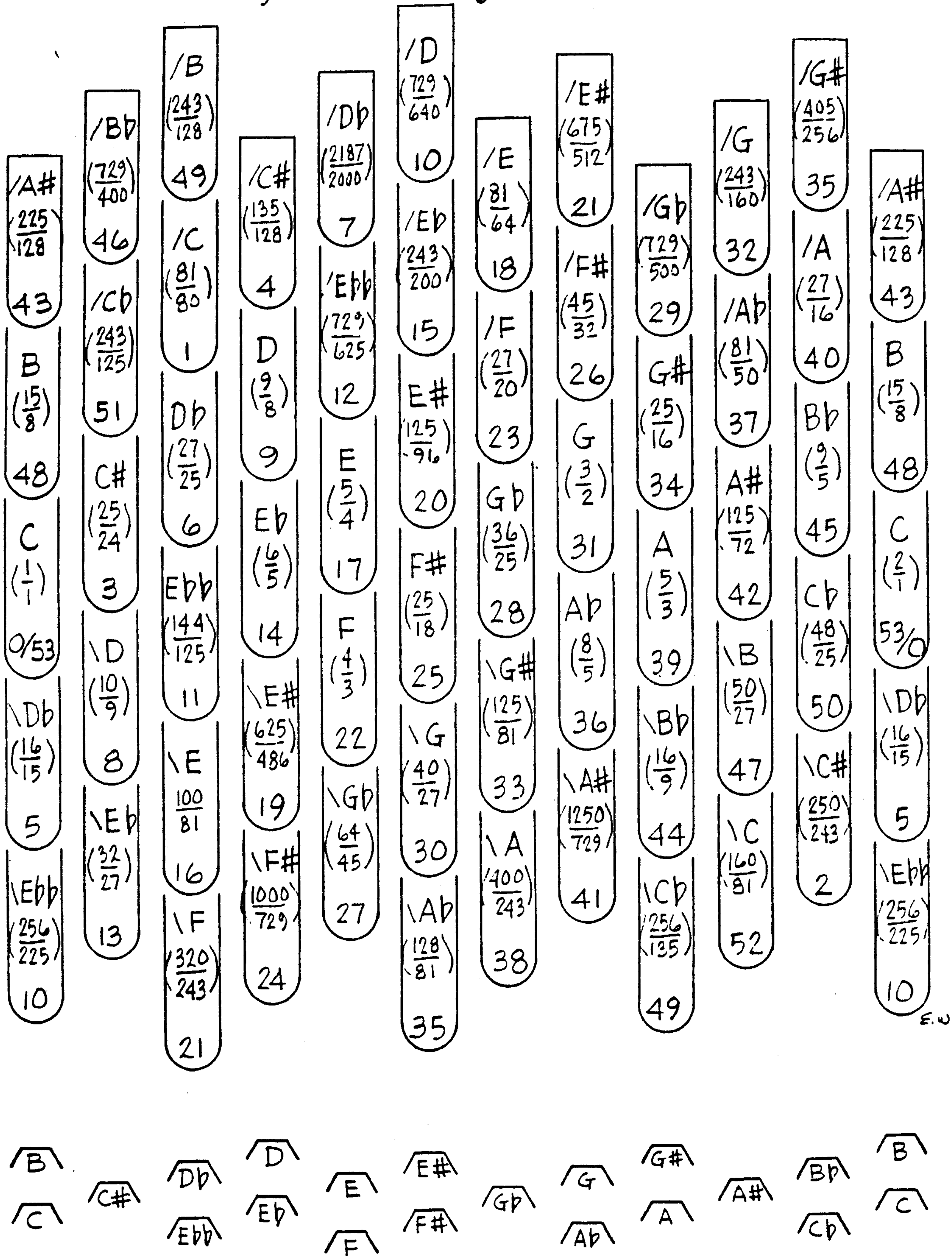


Figure #10
Keyboard Design "B"

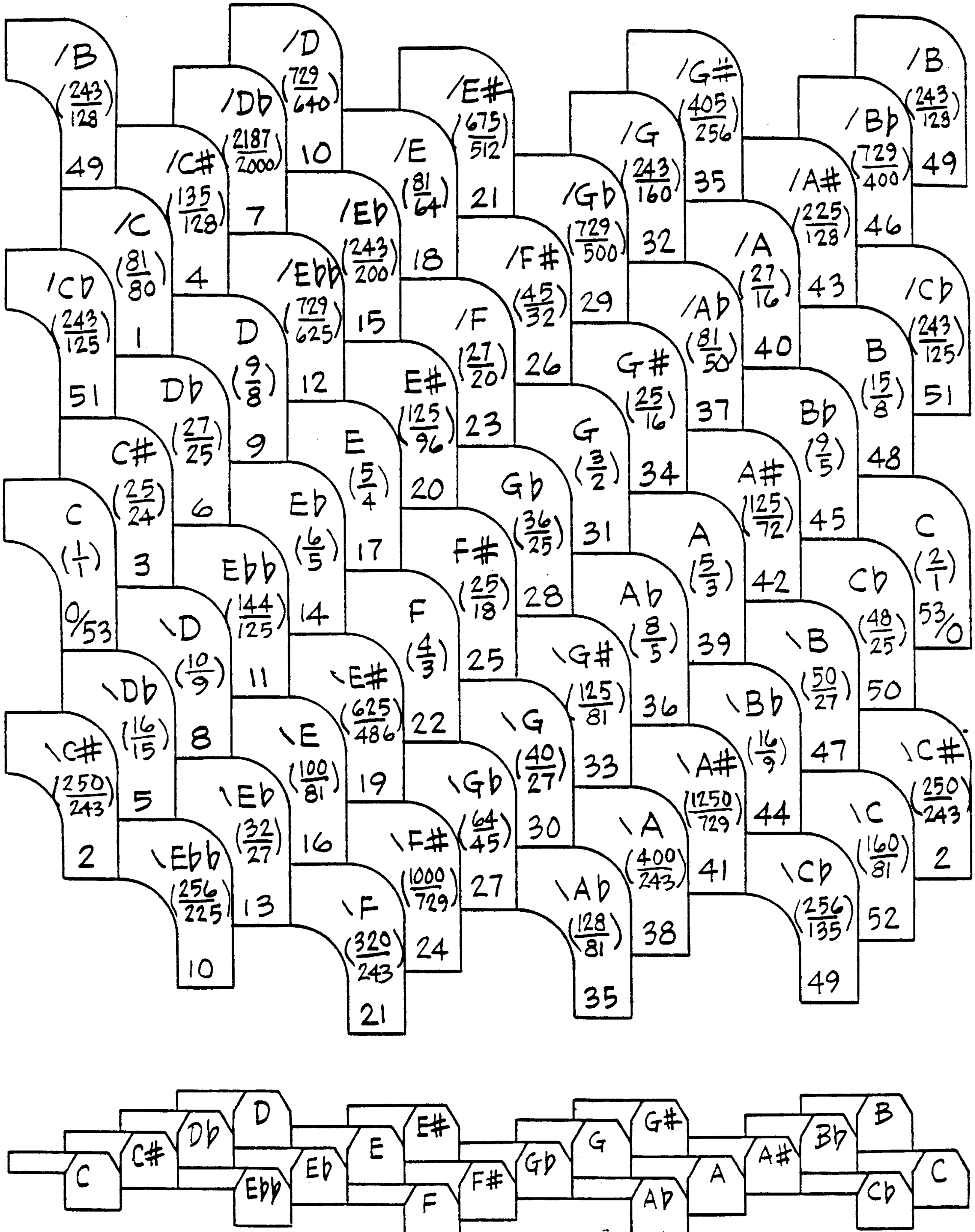
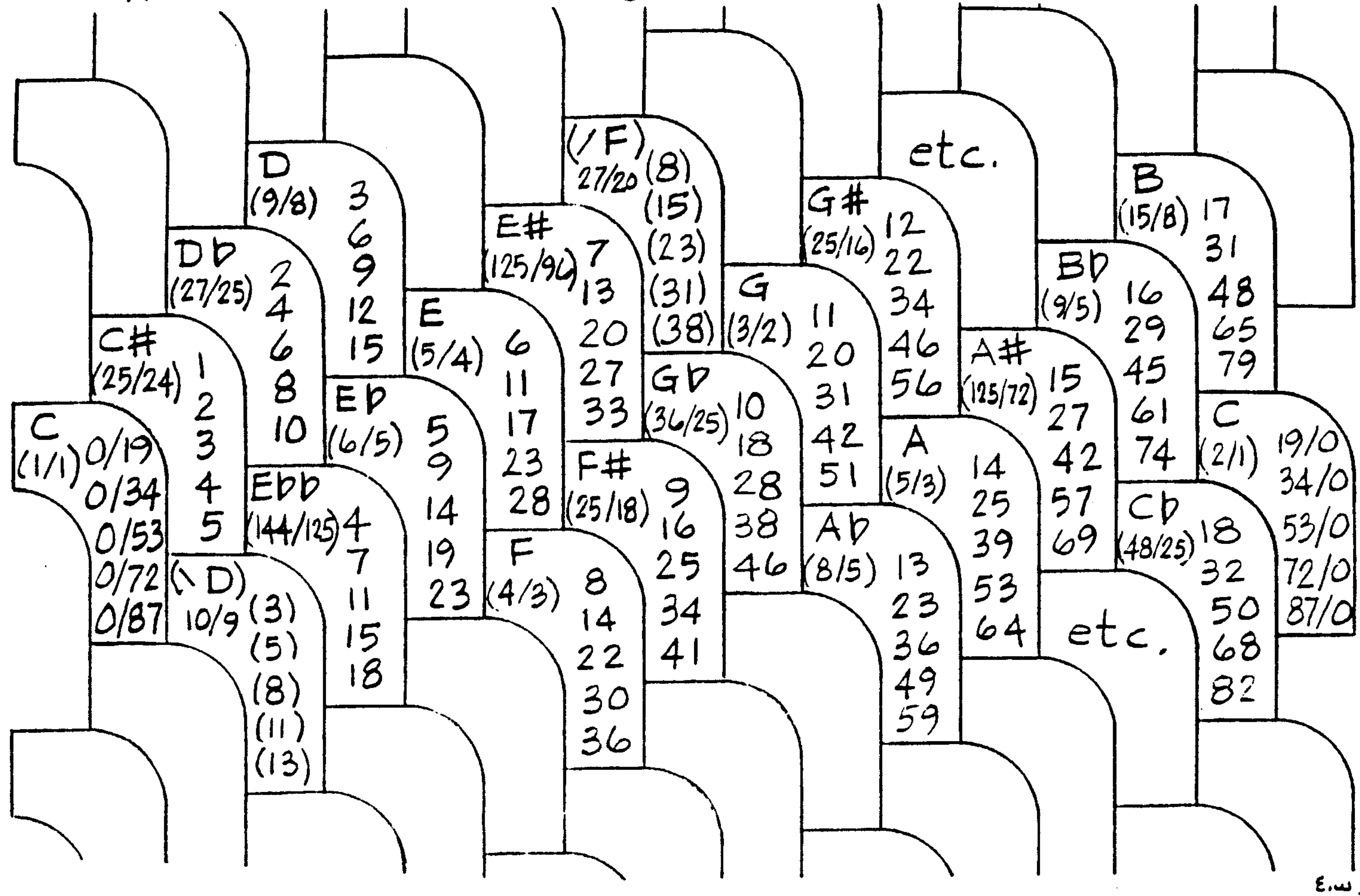
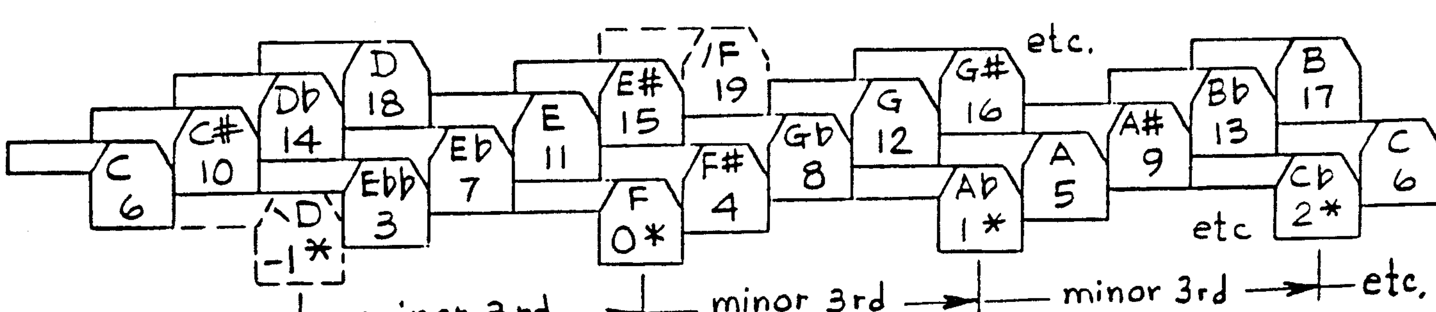


Figure # 11

The Basic Group of 19, as derived by a chain (liner series) of minor Thirds in systems of 19, 34, 53, 72, & 87 tones, applied to keyboard Design "B"





* numerals indicate sequence of minor thirds.

APPENDIX #2

UPPER HARMONIC RATIOS

In view of the interest of contemporary composers in harmonic ratios involving partials above the classic 3 and 5, Erv Wilson has prepared two Figures which indicate which digitals of my keyboard Design B would be employed to play the tones best approximating some of those ratios.

Figure #12 shows the placement of tones of the Ogdoadic Diamond on the 1/1 mapped to modulus 53.

Figure #13 shows the placement of the tones of 3 Ogdoadic Diamonds on the 1/1, 4/3 and 3/2 mapped to modulus 72.

Figure 12

Ogdoadic Diamond mapped to modulus 53

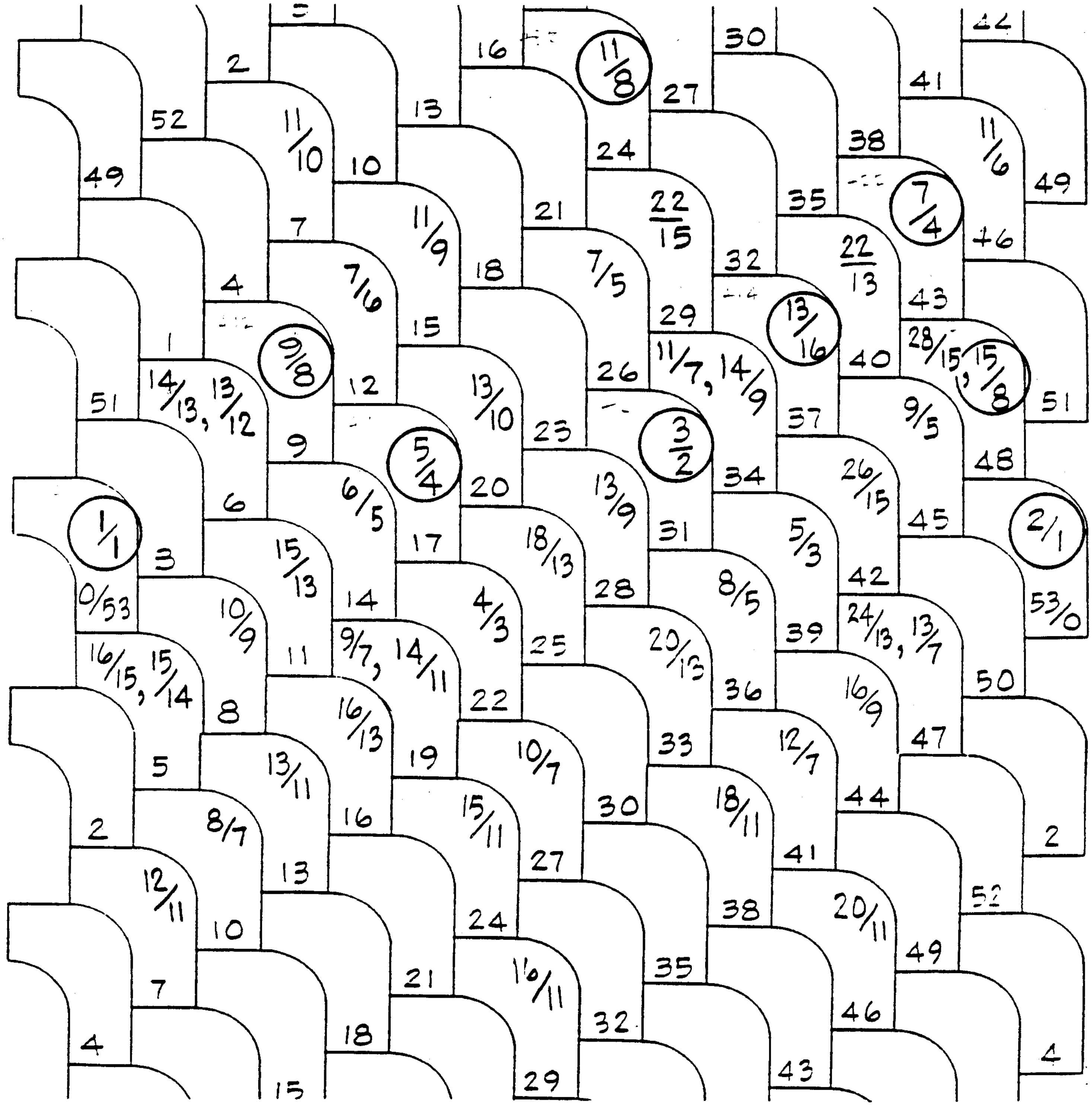


Figure 13. 3 Ogdoadic Diamonds (at 1/1, 4/3 & 3/2) mapped to modulus 72