JUST INTONATION

N.V. VAN DE GARDE & CO'S DRUKKERIJ, ZALTBOMMEL

JUST INTONATION

AND THE COMBINATION OF HARMONIC DIATONIC MELODIC GROUPS

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CHAPTER I

SIMPLE MELODIC GROUPS

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1. *Aim.*

The ordinary equal temperament of twelve semitones to the octave has rendered very important services in music, especially with regard to instruments producing notes with fixed pitch, like the organ and the pianoforte. It offers complete freedom of modulation. This advantage is so great that after the general reluctance in the seventeenth century to adopt it and thereby to sacrifice the purity and beauty of perfect concords the musicians of the eighteenth century gradually accepted and acquiesced in the deficiencies and restrictions of the compromise embodied in that equal temperament. Nowadays it rules unchallenged. Still, few will deny that much has been lost in the musical consciousness of the educated public. Real values are disregarded by and inaccessible to people using the pianoforte for their instrument. Therefore, stress must be laid on a special training to acquire a just intonation of perfect intervals and chords.

This treatise aims at planning a series of exercises which will both assist the ear to a keen judgment of accuracy of intonation, and give the voice or the fingers of a player of stringed instruments a training in obtaining the right pitch, and in correcting a less just intonation at once. These two results are complementary. The method here proposed will at first have to be used consciously, but practice and the power of habit should finally enable the subconscious reflexes to operate automatically without the special attention of the executant.

The method will involve the intonation of the seventh harmonic. The use of this seventh harmonic is of real musical value. Far back, in the middle of the seventeenth century, CHRISTIAAN HUYGENS urged its importance. In the middle of the eighteenth century SORGE, and especially GIUSEPPE TARTINI, indicated the place of the seventh harmonic in the scale. Without knowing about TARTINI the mathematician LEONHARD EULER at the same time offered a theory which was able to cope with the new possibilities. Equal temperament, however, which in those days was beginning its triumphant conquest of the European musical world, excluded the seventh harmonic. Occasionally, in more modern times, it reappears. Towards the end of the BRAHMS horn trio and in BRITTEN'S *Serenade* for tenor voice and horn it appears significantly; even the eleventh harmonic is stressed in the prologue of BRITTEN'S serenade. But the harmonic seventh remains foreign to conventional music. Sir DONALD FRANCIS TOVEY, in his article on Harmony in the Encyclopaedia Brittanica, dealing with theoretical possibilities for the future says : 'Harmony has not yet found a place for so simple a natural phenomenon as the seventh note of the harmonic series'. I am attempting to find that place.

This first chapter will establish a melodic subscale consisting of the harmonics six to ten. The second and third chapters will show the various ways in which the combination of two and three melodic subscales gives rise to classical and to new scales. The structure as a whole might well be strong, simple and sufficiently sound to open the way to a new musical development.

2. *Method.*

The method of acquiring a just intonation consists in supplying an aid by adding an accompanying sustained supporting note. This can be either a low-pitched note or a high-pitched one.

Having chosen, say, a low-pitched note, one adjusts the intonation of one own's voice, or of one's instrument, in such a way that there is a complete concord with this fundamental note. One should sing or play without any strain whatever, the sound being utterly plain and smooth, without harshness or beats. Under these conditions one produces *harmonics* of the low-pitched note, which is the *fundamental.*

If one chooses a high-pitched supporting note, and again adjusts oneself with voice or instrument to a perfectly smooth concord free from beats, one is producing one of the *subharmonics* of the high-pitched note which, serving as a guide for just intonation, will be called hereafter the *guiding note.* In this case the sound sung or played may be heard as the fundamental and the guiding note as a harmonic.

3. *Classical chords. Harmonics from three to five. The eight basic notes and their vocables.*

In the use of vocables I adopt for the basic notes those suggested by CURWEN in the tonic sol-fah system. Deviating from the conventional series:

doh : *ray* : *me* : *fah* : *sol* : *lah* : *se* : *doh,*

CURWEN proposed to sing with *soh* instead of *sol* and *te* instead of *se*:

doh : *ray* : *me* : *fah* : *soh* : *lah* : *te* : *doh.*

I shall presently have occasion to introduce a new vocable, *way,* for a note between *fah* and *soh.* The vocables are not tied to special notes. They may be sung at any pitch. It is only their relative pitch which matters and which is invariable.

Choosing a low-pitched fundamental one can practise the harmonics three, four and five. CURWEN's system affords three possibilities for vocalizing *)

$$\begin{aligned} g : c' : e' = 3 : 4 : 5 &= soh : doh : me \\ &= doh : fah : lah \\ &= ray : soh : te \end{aligned}$$

The harmonics three to five constitute the major common chord in what is frequently called its twice inverted position.

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*) I shall adopt the rule that harmonics will be put down on the stave as notes with tails below, pointing downward to their fundamental.

The harmonics four, five and six provide the major common chord in its root position. There are three vocalizing possibilities :

$$c : e : g = 4 : 5 : 6 = \textit{doh} : \textit{me} : \textit{soh}$$
$$= \textit{fah} : \textit{lah} : \textit{doh}$$
$$= \textit{soh} : \textit{te} : \textit{ray}$$

In all the examples given one might change the order of notes *ad libitum,* transposing the exercise to all pitches.

Choosing a high-pitched guiding note, and practising the subharmonics three, four, five, one finds minor common chords. Curwen's system affords only two possibilities for vocalizing.

$$e'' : b' : g' = \frac{1}{3} : \frac{1}{4} : \frac{1}{5} = \textit{me} : \textit{te} : \textit{soh}$$
$$= \textit{lah} : \textit{me} : \textit{doh}$$

For reasons of symmetry I propose to add a third possibility *):

$$b : f\sharp : d = \frac{1}{3} : \frac{1}{4} : \frac{1}{5} = \textit{te} : \textit{way} : \textit{ray}$$

Taking the subharmonics four to six one hears the minor common chords :

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*) In order to indicate that we are handling subharmonics of a guiding note, I put the inverted numbers as fractions $1/3 : 1/4 : 1/5$. On the stave the notation adopted for subharmonics is with tails pointing upward, toward the high-pitched guiding note.

$$\begin{aligned} b : g : e = \frac{1}{4} : \frac{1}{5} : \frac{1}{6} &= \textit{te} \;:\; \textit{soh} \;:\; \textit{me} \\ &= \textit{me} \;:\; \textit{doh} \;:\; \textit{lah} \\ &= \textit{way} \;:\; \textit{ray} \;:\; \textit{teh} \end{aligned}$$

Again the order of notes in the minor common chords may be varied at will, and the chords should be sung at all pitches.

Each of the major common chords has now found a companion minor common chord. These are the pairs:

c : *e* : *g* and *e* : *g* : *b* or *doh* : *me* : *soh* and *me* : *soh* : *te*,
f : *a* : *c* and *a* : *c* : *e* or *fah* : *lah* : *doh* and *lah* : *doh* : *me*,
g : *b* : *d* and *b* : *d* : *f*♯ or *soh* : *te* : *ray* and *te* : *ray* : *way*

The pairs are very closely related indeed, because each pair has both a fundamental and a guiding note in common. For the first pair these are *C''* and *b'''*, *doh* and *te*, respectively. The pair thus looks in a certain sense like a simple example of what EULER had in mind as a *complete chord*. Therefore these pairs of common chords exhibit the most intimate relation.

Another close relation exists between two other pairs of chords having in common two notes, which make a major third. Here they are:

a : *c'* : *e'* and *c'* : *e'* : *g'*, or *lah* : *doh* : *me* and *doh* : *me* : *soh*,
e : *g* : *b* and *g* : *b* : *d'*, or *me* : *soh* : *te* and *soh* : *te* : *ray*.

Looking for a common fundamental for the first pair of chords, one finds, *f*, *fah*, and the common guiding note is found to be *b*, *te*. These now do not belong to the chords themselves. The mutual relation in these pairs of chords is not so close therefore.

These relations come out clearly if one arranges the eight vocables sung thus far in a table with rectangular rows and columns. There will be perfect fifths between adjacent notes written in the same row, and there will be perfect major thirds between those in the same column, thus:

$$\left.\begin{array}{cccc} \textit{lah} & \textit{me} & \textit{te} & \textit{way} \\ \textit{fah} & \textit{doh} & \textit{soh} & \textit{ray} \end{array}\right\} \frac{5}{4}$$

$$2 : 3 \quad 2 : 3 \quad 2 : 3$$

or

$$\left.\begin{array}{cccc} \textit{a} & \textit{e} & \textit{b} & \textit{f}\sharp \\ \textit{f}\natural & \textit{c} & \textit{g} & \textit{d} \end{array}\right\} \text{perfect major thirds}$$

$$\text{perfect fifths}$$

A major and a minor common chord sharing a minor third are represented in a square, the simplest possible arrangement — in a closed circuit, so to speak:

e *b* or *me* *te*

c *g* *doh* *soh*

A major and a minor common chord sharing a major third are represented by a broken line:

e *b* *me* *te*

or

g *d* *soh* *ray*

Looking at the eight notes as a whole one sees that the fundamental which they possess in common is a low-pitched *F* or *fah*. There is a common guiding note too. This is a high pitched *f*♯ or *way*. The collection contains representatives of all the harmonics of the low F which at the same time are subharmonics of the high-pitched *f*♯. If it is repeated through all the octaves one gets what LEONHARD EULER called a *genus musicum*. The genus in question is the complete *genus diatonicum*.

It contains two modes, or keys. Leaving out the guiding note *f* ♯ one is left with the key of *Doh* major, or *C* major:

doh : *ray* : *me* : *fah* : *soh* : *lah* : *te* : *doh*
c : *d* : *e* : *f* : *g* : *a* : *b* : *c′*

Leaving out the fundamental *f* one has the key of *me* minor or *e* minor:

me : *way* : *soh* : *lah* : *te* : *doh* : *ray* : *me*
e : *f* ♯ : *g* : *a* : *b* : *c′* : *d′* : *e′*

The eight notes of the genus diatonicum will serve as *basic notes*. Other notes will receive names and notations with relation to these basic notes.

It must be borne in mind that the basic notes *ray* and *lah* in the genus diatonicum do *not* make a perfect fifth; the interval is one comma flat.

In this genus the most distantly related note from the fundamental *f*, the representative of the guiding note, has been called *f*♯. This is not an accidental. It is not a chromatic semitone from *f*, but a minor limma from *f* (not 25/24, but 135/128). It makes a perfect major third with *d*, but instead of a perfect minor third it makes a pythagorean minor third with *a*. The vocable for this guiding note has been chosen with a labial consonant *w*, labial like the consonant of the vocable *fah* for the fundamental, and a vowel *ay*, reminiscent of *ay* in *ray*, the partner of *way* in a perfect major third, in the same way as the partners *fah* and *lah* share a vowel in common. It is seen that in the centre of the genus, as tabulated on page 6, partners in perfect fifths are sharing their vowels.

4. *Derivation of classical chromatic semitones. Genus chromaticum, genus diatonico-chromaticum and genus diatonico-hyperchromaticum.*

Apart from major and minor common chords sharing two notes which make a minor or a major third, there is another way in which each major chord can have a conjugate minor chord, and vice versa. The conjugate chords may share their root and the note which makes the perfect fifth. The other notes will then show a difference of pitch which is known as the chromatic semitone. Taking the major common chord

$$c : e : g = 4 : 5 : 6 = 20 : 25 : 30$$

$$c : e\flat : g = \tfrac{1}{6} : \tfrac{1}{5} : \tfrac{1}{4} = 20 : 24 : 30,$$

we can make the minor common chord between the same *c* and *g*, and the difference of the chromatic semitone is

$$e\flat : e = 24 : 25.$$

In order to represent by vocables the minor chord containing *doh* and *soh* a new vocable is required for *me*, flattened by a chromatic semitone. I propose to choose the vocable by changing the vowel *e* of *me* into *oo*. Thus one gets:

doh : *me* : *soh* and *doh* : *moo* : *soh*
fah : *lah* : *doh* and *fah* : *loo* : *doh*
soh : *te* : *ray* and *soh* : *too* : *ray*

for these conjugate common chords.

When arranged in columns and rows these pairs of common chords show the following arrangements

e			*me*		*lah*			*a*	
c	*g*	or	*doh*	*soh*	*fah*	*doh*	or	*f*	*c*
	e♭			*moo*		*loo*			*a* ♭

Taking a minor common chord, and making the conjugate major chords with coinciding root and fifth, one finds the third note sharpened by a chromatic semitone. In the vocable this sharpening may be expressed by changing the vowel into *üh* (as in the French *pur*). Thus one gets :

a : *c* : *e* and *a* : *c*♯ : *e*
lah : *doh* : *me* and *lah* : *düh* : *me,*
me : *soh* : *te* and *me* : *süh* : *te,*
te : *ray* : *way* and *te* : *rüh* : *way.*

Altogether six new notes have been added to the basic notes. Arranging them as before (by perfect fifths and major thirds) in an orderly frame of notes, one finds a square with opposite corners missing. The notes corresponding to these places happen to be the representatives of the common fundamental and the common guiding note to all the other notes. We must therefore include them in the table to obtain once more a genus musicum, this time the *genus diatonico-hyperchromaticum.*

düh	*süh*	*rüh*	*pay*	$\frac{5}{4}$	
lah	*me*	*te*	*way*		$\frac{5}{4}$
fah	*doh*	*soh*	*ray*	$\frac{5}{4}$	
bah	*loo*	*moo*	*too*		

2 : 3 2 : 3 2 : 3

c♯	*g*♯	*d*♯	*a*♯	perfect
a	*e*	*b*	*f*♯	major
f	*c*	*g*	*d*	thirds
d♭	*a*♭	*e*♭	*b*♭	

perfect fifths

In choosing vocables for the representatives of the fundamental and the guiding note I propose to attach to them the labial explosive consonants *b* and *p* and the vowels of their neighbours among the basic notes.

From the large square one can select a rectangle of three rows and four columns in two ways, by leaving out either the top row or the bottom row. These two sets of twelve notes constitute each a *genus diatonico-chromaticum.* Again, by leaving out a row and a column which meet in a corner point, one can get a square of nine notes constituting a *genus chromaticum* (See Ch. III, § 5). This can be done in four ways.

5. *The primary four-note chord with harmonic seventh. Harmonics from four to eight.*

When singing with a low-pitched fundamental

$$\mathit{doh} : \mathit{me} : \mathit{soh} : \mathit{doh} = 4 : 5 : 6 : 8$$
$$= c : e : g : c'$$

one has a gap in the interval *soh* : *doh* = 6 : 8 which is naturally filled by the seventh harmonic between the sixth and eighth.

In vocalising I propose to give this note a vocable by taking *d* from its fundamental *Doh,* and adding a new vowel, viz. *dey* pronounced as in the French *pareil* or the Dutch *dijk.* Thus the primary chord of four notes will be sung as

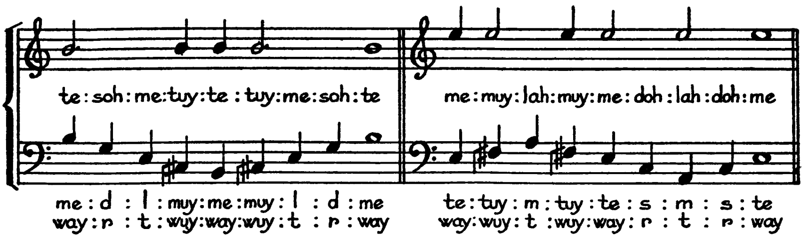
doh : *me* : *soh* : *dey.*

For a fundamental *C* the seventh harmonic is slightly flatter than *b*-flat. It will therefore be called *b*-flat-minus. For the notation on the stave one can use a special sign invented by GIUSEPPE TARTINI as long ago as 1756, viz. the sign ∠♭ *). For the fundamental *G* the harmonic seventh, slightly flatter than *f*, will be called *f*-minus, with a sign ∠. This was also invented by TARTINI. The vocable for the harmonic seventh belonging to *soh* will be *sey*, and similarly for other fundamentals. Thus we have

4 : 5 : 6 : 7 : 8 = *c* : *e* : *g* : *b*∠♭ : *c′*
= *f* : *a* : *c′* : *e′*∠♭ : *f′*
= *g* : *b* : *d′* : *f* ∠*′* : *g′*
= *doh* : *me* : *soh* : *dey* : *doh*, or
= *fah* : *la* : *doh* : *fey* : *fah*, or
= *soh* : *te* : *ray* : *sey* : *soh*

The examples give various inversions and permutations of this chord. One may invent and practise them at will, at all pitches.

Again, singing with a high-pitched guiding note, one finds the reflected images of these chords, being the minor common chords extended by the seventh subharmonic. As regards vocables, I propose to take the consonant of the guiding note, and attach to it the diphthong *uy*, which sounds as in the French *oeuil* or in the Dutch *luit*. The seventh subharmonic of *b‴* will be slightly sharper than *c*-sharp. It will be called "*c*-sharp-plus". One needs signs which can be made in a similar way to the signs of TARTINI. The sign for "plus" will be ⦣, and for "sharp-plus" ⦣♯ *).



Thus after singing the subharmonic primary four-note chords one can write them down as follows :

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*) This is an approximation in printing type of TARTINI's sign which is shown in the examples on the staves.

$$\frac{1}{4} : \frac{1}{5} : \frac{1}{6} : \frac{1}{7} : \frac{1}{8} = b : g : e : c\backslash\sharp : B$$

= *te* : *soh* : *me* : *tuy* : *te*, or
= *me* : *doh* : *lah* : *muy* : *me*, or
= *way* : *ray* : *te* : *wuy* : *way*

One may invent many more exercises to suit one's own taste.

6. *Diatonic melodic subscales. Harmonics from six to ten.*

The chord of the harmonics 3 : 4 : 5 is the same as that of the harmonics

$$6 : 8 : 10 = g : c' : e'$$

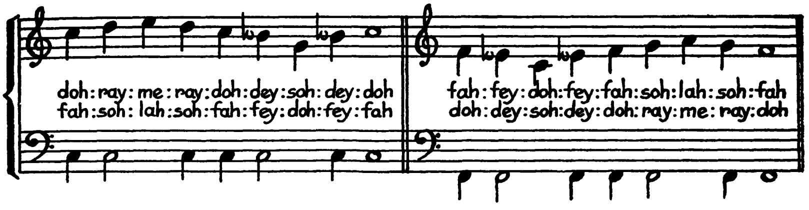
= *soh* : *doh* : *me*

The eighth harmonic is the root and also the centre of this chord. The intervals can be filled in by the seventh and the ninth harmonics. This will result in a group of harmonics, which will serve as a ***diatonic melodic subscale***:

$$6 : 7 : 8 : 9 : 10 = g : b\triangle\flat : c' : d' : e'$$

= *soh* : *dey* : *doh* : *ray* : *me*; or
= *doh* : *fey* : *fah* : *soh* : *lah.*

These melodic subscales still have the eighth harmonic for a centre, which is thus a melodic centre of harmonics. In the examples the centres are *c''* and *f'*, *doh* and *fah* respectively. These centres have a fundamental character.



In the same way, using a high-pitched guiding note one obtains diatonic melodic subscales of subharmonics.

$$\frac{1}{6}:\frac{1}{7}:\frac{1}{8}:\frac{1}{9}:\frac{1}{10} = e : c\natural\sharp : b : a : g$$

= *me* : *tuy* : *te* : *lah* : *soh,* or

= *te* : *wuy* : *way* : *me* : *ray*

These groups of subharmonics have melodic centres of guiding note character. In the examples the centres are *b* and *f* ♯, *te* and *way* respectively.

While singing these diatonic melodic groups one will notice that there are three kinds of whole tones in them.

First one has *superseconds*, having the harmonic ratio 7 : 8. These are found between

7 : 8 = *b* ∠♭ : *c′* = *dey* : *doh* = *fey* : *fah*,
= *b* : *c* ⦣♯ = *te* .: *tuy* = *way* : *wuy*

Next come *pythagorean seconds*, or *major whole tones*, between the eighth and ninth harmonics :

8 : 9 = *c* : *d* = *doh* : *ray* = *fah* : *soh*,
= *a* : *b* = *lah* : *te* = *me* : *way*,

Finally there are *minor whole tones* between the ninth and tenth harmonics :

9 : 10 = *d* : *e* = *ray* : *me* = *soh* : *lah*,
= *g* : *a* = *soh* : *lah* = *ray* : *me*

In the ordinary notation no distinction is made between *a*, belonging to a melodic group having *f* for its fundamental centre, and *a* belonging to melodic groups having *g* or *d* as a fundamental centre. The two notes *a* however, ought to be different. The former is one comma flatter than the latter. This use of the same sign for notes, very slightly different, yet nevertheless distinct, and having different functions, has been called 'double emploi' by JEAN PHILIPPE RAMEAU.

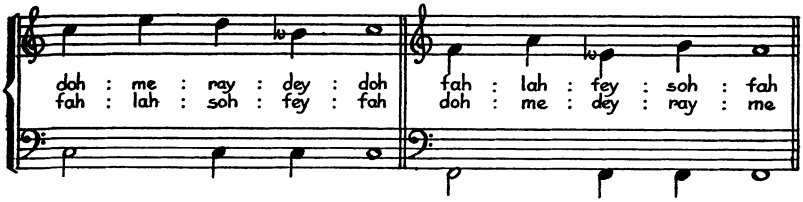
A vocable in the tonic sol-fah system must have one single meaning. Therefore the notes *soh* : *lah* : *te* can only have *te* as a centre (and this is a guiding note centre) but never *soh*. The group *fah* : *soh* : *lah* can only have *fah* for centre (a fundamental centre) but never *lah*. The same argument applies to *doh* : *ray* : *me* which cannot centre in *me*, and to *ray* : *me* : *way* which cannot centre in *ray*.

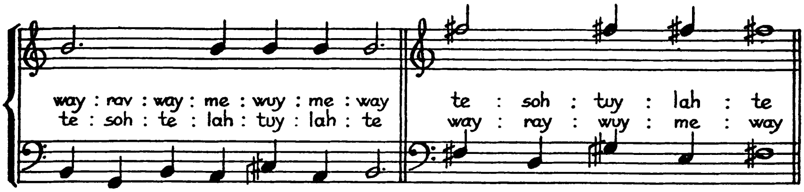
Again, in these melodic subscales there are two kinds of major thirds. One finds *superthirds* between the seventh and ninth harmonics, as

7 : 9 = *b* ∠♭ : *d′* = *dey* : *ray* = *fey* : *soh*,
= *a* : *c* ⦣♯ = *lah* : *tuy* = *me* : *wuy*.

There are *perfect major thirds* between the eighth and tenth harmonics

8 : 10 = *c* : *e* = *doh* : *me* = *fah* : *lah*,
= *g* : *b* = *soh* : *te* = *ray* : *way*.

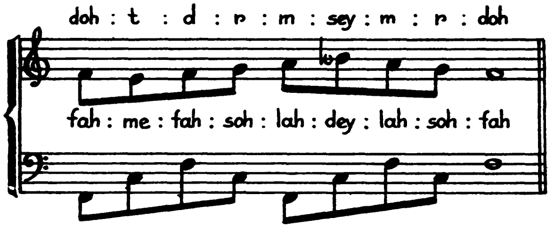




Singing these thirds in immediate succession one will learn to recognize and distinguish them. The difference amounts to one quarter of a whole tone (35 : 36).

7. *Subscales with leading notes.*

To enrich the melody one may add to a diatonic subscale alien notes, belonging to other centres, and not fitting properly in the concordance of the subscale. For example, consider the harmonic subscale having *doh* (*C*) for its fundamental,and next the primary four-note chord having *soh* (*g*) for centre. Here one finds *ray*, *te* and *sey* (*d*, *b* and *f* ∠). The first note, *ray* (*d*), is a regular member of the subscale round *doh*. The second one, *te* (*b*), does not belong to it. Therefore, hearing *te* (*b*) in that connection one instinctively wants to replace it by another note which belongs to the subscale, and preferably by one of the notes nearest at hand. These are either *doh* or *dey* (*c* or *b* ∠♭). Thus *te* (*b*) is providing us



with a so-called *leading note* to the centre and to the seventh. In much the same way the remaining note in the chord of *soh* (*g*), which is *sey* (*f* ∠), is a leading note to *me*, the third from the centre.

The fact that *te* and *sey* with *ray* (*b*, *f* ∠ and *d*) find a common centre in *soh* (*g*) may be shown by the accompaniment. This is already an anticipation of the combination of subscales having separate centres (Cf. Ch. II).

What has been said thus far can be applied to subharmonic subscales as well.

In connexion with the subharmonic scale with the guiding note centre *te* (*b*) one has the leading notes *doh* and *mui* (*c* and *f* ⟍#) taken from *me* (*e*) as the centre of subharmonics. *Doh* (*c*) is leading either to the centre *te* (*b*) or to the subharmonic seventh *tuy* (*c* ⟍#), while *f* ⟍# *muy* is leading to the subthird *soh* (*g*). Again, the origin of the leading notes in the subharmonic subscale may appear in the accompaniment. This of course has to be a high-pitched accompaniment by guiding notes in the treble, not by fundamentals in the bass. It will be superfluous to work out and indicate more melodic lines. The musical imagination has already got much to play with.

8. *Special chords.*

The special thirds (superthirds 7 : 9) which occur in diatonic melodic subscales have been mentioned already. Along with them two special chords must be pointed out. In the harmonic subscales one of these consists of the harmonics

$$\begin{array}{rlll} 6:7:9 = & g & : b\triangle\flat & : d', \\ = & \textit{soh} & : \textit{dey} & : \textit{ray}, \\ = & \textit{doh} & : \textit{fey} & : \textit{soh}. \end{array}$$

This is not an ordinary minor common chord, like

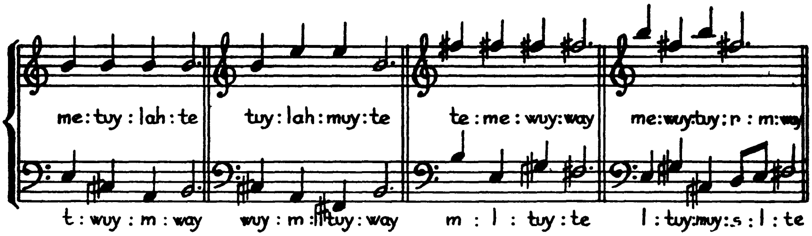
$$g : b\flat : d' = \frac{1}{6} : \frac{1}{5} : \frac{1}{4} = 10 : 12 : 15.$$

It is a different one, which is more quiet in sound. The melodic centre, the supporting fundamental, is always the *C*.

The superthirds 7 : 9 = 14 : 18 can be completed at the other side by a perfect fifth, yielding another special chord,

$$\begin{aligned}14:18:21 &= b\,\angle\flat : d' : f\,\angle' \\ &= \mathit{dey} : \mathit{ray} : \mathit{sey}, \\ &= \mathit{fey} : \mathit{soh} : \mathit{dey}.\end{aligned}$$

This chord, again, is not a major common chord. It lacks the supreme serenity of the major common chord. This new chord also finds its supporting centre and solution in the fundamental (*C*).

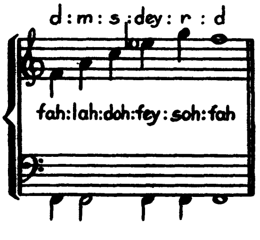


The same chords will be found again in the subharmonic subscales. Here one has :

$$\begin{aligned}\frac{1}{9}:\frac{1}{7}:\frac{1}{6} = 14:18:21 &= a : c'\,\overline{\diagdown}\sharp : e' \\ &= \mathit{lah} : \mathit{tuy} : \mathit{me} \\ &= \mathit{me} : \mathit{wuy} : \mathit{te}. \\ \frac{1}{21}:\frac{1}{18}:\frac{1}{14} = 6:7:9 &= f\,\overline{\diagdown}\sharp : a : c'\,\overline{\diagdown}\sharp \\ &= \mathit{muy} : \mathit{lah} : \mathit{tuy}, \\ &= \mathit{tuy} : \mathit{me} : \mathit{wuy}.\end{aligned}$$

Both chords resolve into their guiding note centre (*b*).

The five notes of the diatonic melodic subscales are easily seen to be the members of primary five-note chords with perfect



sevenths and perfect ninths. There are two kinds of these; the harmonic and the subharmonic chords. It is a crucial case for just intonation to make a clear distinction. The harmonic primary five-note chord has a perfect major common chord at the bottom and on top of it a non-classical minor chord with flattened third. The subharmonic primary five-note chord has a perfect common minor chord at the top and a non-classical major chord with sharpened third at the bottom.

CHAPTER II

COMBINATION OF TWO MELODIC CENTRES

CONTENTS

1. *Leading notes and liaison notes, and connecting leading notes.*

If two centres are being combined the melody is shifting intermittently from notes of the subscale belonging to one centre to notes belonging to the other centre. It is like steering a ship taking heed of one beacon after the other. The two melodic centres can be connected by a shift through some interval, which puts one centre in the other's place, or by an inversion, which transforms a fundamental centre into a guiding note centre and vice versa.

When the melodic line leaves one centre to join the other one, the transition will be most smooth between notes very near one to another. Such notes will be called *leading notes*, because they are leading the melody from one subscale to the other. The classical leading note of the diatonic scale is an example, as will be seen shortly.

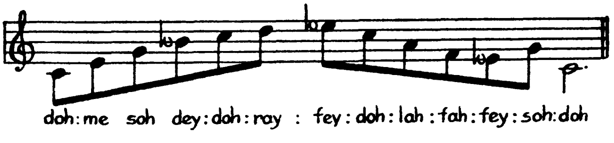
When the centres are inversions of each other the two melodic subscales can have two notes in common. These will constitute *liaison notes.*

Sometimes one meets a case, where the transition occurs with the aid of *connecting leading notes,* which themselves do not belong to the subscales (Cf. Ch. IV, § 2, examples *W* and *X*).

2. *Shift of a centre through a perfect fifth or fourth.*

Taking the centres *f* and *c* (*fah* : *doh* = 2 : 3), one gets the notes

f : *g* : *a* : *c* : *e* ∖♭ or *fah* : *soh* : *la* : *doh* : *fey*
g : *b* ∠♭ : *c* : *d* : *e* *soh* : *dey*: *doh* : *ray* : *me.*



As we saw before, one meets the leading note *e* to *f,* and vice versa (*me* : *fah* = 15 : 16). This is the classical leading note, Rameau's "note sensible". Non-classical is the leading note *b* ∠♭ toward *a,* and vice versa (*lah* : *dey* = 20 : 21), with a smaller interval. Smaller still is the interval between the leading note *e*△♭ to *d*, and vice versa (*fey* : *ray* = 28 : 27). This interval is one-third of a tone. Again there is a pair of leading notes *e*△♭ and *e* (*fey* : *me* = 14 : 15).

Taking the subharmonic subscales, with centres in *b* and *f* ♯ (*te* and *way*) there are the notes :

g ⧹♯ : *b* : *d* : *e* : *f* ♯ *wuy*: *te* : *re* : *me* : *way*
g : *a* : *b* : *c* ⧹♯ : *e* or *soh* : *lah* : *te* : *tuy* : *me*

The classical leading notes are *f* ♯ and *g* (*way* : *soh* = 15 : 16). Smaller intervals are shown by *c* ⧹♯ : *d* (*tuy* : *ray* = 20 : 21) and *g* ⧹♯ : *a* (*wuy* : *lah* = 27 : 28). Somewhat wider is the interval *g* : *g* ⧹♯ (*soh* : *wuy* = 14 : 15).

3. *Shift of a centre through a major third or minor sixth.*

The centres are *f* and *a* in the first example, *f* ♯ and *d* in the second :

f : *g* : *a* : *c* : *e*∠♭ *fah* : *soh* : *lah* : *doh*: *fey*
a : *b* : *c*♯ : *e* : *g*∠ *lah* :*te* : *düh* : *me* : *ley*
or
g⧹♯ : *b* : *d* : *e* : *f*♯ *wuy* : *te* :*ray* : *me* : *way*
e⧹ : *g* : *b*♭ : *c* : *d* *ruy* : *soh* : *too* : *doh* : *ray*.

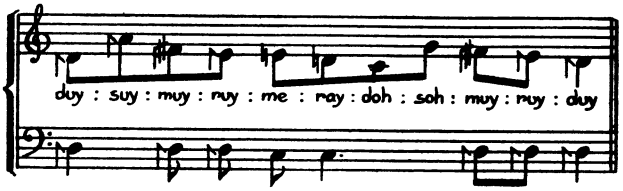
In the examples without harmonic sevenths one meets two kinds of pairs of leading notes. There are classical pairs, *e* and *f* (*me* : *fah* = 15 : 16) in the example with fundamental centres, and *f* ♯ and *g* (*way* : *soh* = 15 : 16) in the example with guiding

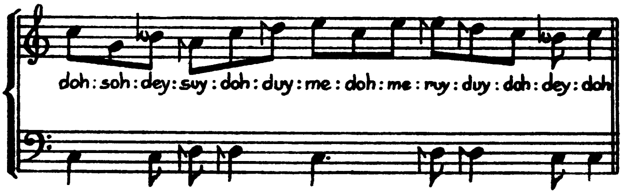
note centres. There are chromatic pairs too. In the first example it is the pair *c* : *c* ♯ (*doh* : *düh* = 24 : 25), in the other it is the pair *b* ♭ : *b* (*too* : *te* = 24 : 25).

In the examples with harmonic sevenths one meets a new phenomenon. There are enharmonic pairs of leading notes. Such a pair is *d* ∠ and *d* (*mey* : *ray* = 35 : 36) and again *a* and *a* ∖̄ (*lah* : *suy* = 35 : 36). The interval between these leading notes is one quarter of a tone.

4. *Shift of a centre through a harmonic seventh.*

The centres chosen are *d* ∖̄ and *c* (*duy* : *doh* = 8 : 7) for the fundamental centres, *a* ∠ and *b* (*tey* : *te* = 7 : 8) for the guiding





note centres. One recognizes the pairs of non-classical semitone leading notes *f* ∖̄# and *g* (*muy* : *soh* = 20 : 21), and in the other example *e* and *f* ∠ (*me* : *sey* = 20 : 21). Again there are pairs of quarter-tone leading notes : *e* and *e* ⦢ (*me* : *ruy* = 35 : 36), and in the other example *g* ∠ and *g* (*ley* : *soh* = 35 : 36).

Beside these one meets pairs of leading notes showing intervals of one-fifth of a tone. They are *a* ⦢ and *b* ∠♭ (*suy* : *dey* = 48 : 49) in the example with harmonics, and *c* ⦢♯ and *d* ∠ (*tuy* : *mey* = 35 : 36) in the example with subharmonics.

Smaller still are the intervals of the pairs of leading notes which have an interval of one-seventh of a tone. These are *d* and *d* ⦢ (*ray* : *duy* = 63 : 64) and *a* ∠ and *a* (*tey* : *lah* = 63 : 64).

These exercises, extremely enharmonic as they are, require the aid of an instrument provided with a special tuning. This will prove practically indispensable.

5. *Review of inversions.*

It has already been stated that in the inversion of a melodic subscale two notes may remain unchanged, thus providing two liaison notes. In our subscale with the harmonics 6 : 7 : 8 : 9 : 10 one has five notes. Thus there are *ten* possibilities from which to choose a pair of liaison notes. Ten inversions will have to be considered. The order might best be chosen according to the degree of difficulty. Here the possibilities will be reviewed in systematic order.

The liaison notes 6 and 7 (*b* and *d* ∠, or *te* and *mey*) define the inversion described in § 15.

The pairs 6 and 8, and 6 and 9 (as *c* and *g*, or *doh* and *soh*) are considered in § 9.

In § 10 the inversion with centres at a distance of a perfect fifth, but without liaison notes, is investigated.

The pair 6 and 10 (*g* and *e*, or *soh* and *me*) define the inversion discussed in § 6.

The pair 7 and 8 (*e* ∠♭ and *f*, or *fey* and *fah*) define the inversion discussed in § 13.

The pair 7 and 9 (*d* ∠ and *f* ♯, or *mey* and *way*) define the inversion discussed in § 16.

The pair 7 and 10 (*f* ∠ and *b*, *c* and *f* ⦢♯, or *sey* and *te*, *doh* and *muy*) define the inversion discussed in § 14.

The pair 8 and 9 (*f* and *g*, *c* and *d*, or *fah* and *soh*, *doh* and *ray*) define the inversion discussed in § 7. In § 8 the notes *c* and *d* are no liaison notes, but the fifth and the subfifth of a double-sided centre in *g*.

The pair 8 and 10 (*c* and *e*, *g* and *b*, or *doh* and *me*, *soh* and *te*) define the inversion discussed in § 12.

The pair 9 and 10 (*g* and *a*, *d* and *e*, or *soh* and *lah*, *ray* and *me*) define the inversion discussed in § 11.

In the examples the notes of a harmonic subscale will be written with tails pointing downward; the notes of a subharmonic subscale will have tails pointing upwards. This indication will save us additional indications on a separate stave.

6. *Inversion, liaison by g and e, soh and me* (3 *and* 5).

The examples do not require much explanation. The notes involved are :

g : *a* : *b* : *c*⧹♯ : *e*   *soh* : *lah* : *te* : *tuy* : *me*
g : *b*△♭: *c* : *d* : *e*   *soh* : *dey* : *doh* : *re* : *me*

There is a series of leading notes *a* : *b*△♭ : *b*♮ : *c* : *c*⧹♯ : *d*, or *lah* : *dey* : *te* : *do* : *tuy* : *ray*, with minor and major semitones (20 : 21, 14 : 15, 15 : 16).

7. *Inversion, liaison by f, c and g, or by c, g and d* (*fah* : *doh* : : *soh* = *doh* : *soh* : *ray* = 4 : 6 : 9).

It is evident that an inversion with liaison notes *c* and *d* (*doh* : *ray* = 8 : 9) will entail a third liasion note, *g* being the fifth of *c* and the subfifth of *d*. The inversion produces the notes

c : *e* : *g* : *b*⊿♭ : *d* or *f* : *a* : *c* : *e*⊿♭ : *g*
c : *e*⊽ : *g* : *b*♭ : *d* or *f* : *a*⊽ : *c* : *e*♭ : *g*
doh : *me* : *soh* : *dey* : *ray* or *fah* : *lah* : *doh* : *fey* : *soh*
doh : *ruy* : *soh* : *too* : *ray* or *fah* : *suy* : *doh* : *moo*: *soh*

There are two enharmonic pairs of leading notes with an interval of a quarter of a tone: *e* and *e*⊽, and *b* ⊿♭ and *b* ♭ (*me* : *ruy* = *lah* : *suy* = 35 : 36, and *dey* : *too* = *fey* : *moo* = 35 : 36).

8. *Inversion by a two-sided centre.*

One can combine the harmonic diatonic group of *g*, where *g* (*soh*) is a fundamental centre, with the subharmonic group, where *g* (*soh*) is the guiding note centre. In the harmonic group we need a major second *a*, which cannot be *lah* for it must be one comma sharper.

We have met the problem before. Let us choose the vocable *lahy* for this note (as in the English *lie* and in the Dutch *laaiend*).

g : *a* : *b* : *d* : *f*∠ : *g* or *soh* : *lahy* : *te* : *ray* : *sey* : *soh*
g : *a*⊽: *c* : *e*♭: *f*♮ : *g* or *soh* : *suy* : *doh*: *moo*: *fah*: *soh*

Here one sees the ordinary minor seconds between the leading notes *b* and *c*, *d* and *e* ♭ (*te* : *doh* = *ray* : *moo* = 15 : 16) and also two very sharp pairs, with an interval of one-seventh of a whole tone, viz. *a* and *a* ∖, *f* ∠ and *f* ♮ (*lahy* : *suy* = *sey* : *fah* = 63 : 64).

9. *Inversion, liaison by c and g* (*doh* : *soh* = 2 : 3 = 6 : 9). In the first place, with centres in *c* and *g* (*doh* : *soh* = 2 : 3), this inversion provides the simple transition from the major common chord to the minor common chord with the same root :

g : *b*∠♭ : *c* : *d* : *e* : *g* *soh* : *dey*: *doh* : *ray* : *me* : *soh*
or
g : *a*∖ : *c* : *e*♭ : *f* : *g* *soh* : *suy* : *doh* : *moo*: *fah*: *soh*

In between the two centres one has the pair *e* ♭ : *e* (*moo* : *me* = 24 : 25) which can serve as leading notes, with a chromatic semitone between.

Again one meets an enharmonic pair of leading notes with an interval of one-fifth of a tone, viz. *a* ∖ and *b* ∠♭ (*suy* : *dey* = 48 : 49).

In the second place there can be a fundamental centre in *f* (*fah*) and a guiding note centre in *d* (*ray*) :

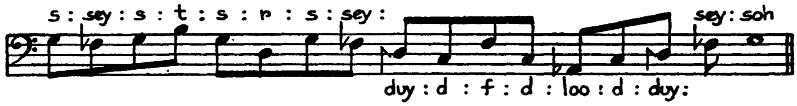
f : *g* : *a* : *c* : *e*∠♭ *fah*: *soh* : *lah* : *doh* : *fey*
or
e∖ : *g* : *b*♭: *c* : *d* *ruy* : *soh* : *too*: *doh* : *ray*

In this case the pair of leading notes *a* and *b* ♭ (*lah* : *too* = 25 : 27) does not show the interval of a minor second. The interval is one comma sharper, a major limma (25 : 27). The pairs of leading notes with an interval of one-third of a tone have been met earlier, in § 2. They are *e*⟍ and *f*, *d* and *e*∠♭ (*ruy* : *fah* = *ray* : *fey* = 27 : 28).

10. *Inversion, centres in c and g without liaison notes* (*doh and soh*).

As before, in § 8, one needs an *a* as major second to the fundamental centre *g*. This must be *lahy*, not *lah*. Moreover *c* is to be a guiding note centre and therefore needs a *b* ♭ as a major subsecond, which cannot be *too*. Let the vocable be *toow*.

b : *d* : *f*∠ : *g* : *a* or *te* : *ray* : *sey* : *soh* : *lahy*
b ♭ : *c* : *d*⟍ : *f* : *a* ♭ *toow* :*doh* :*dui* :*fah* :*loo*

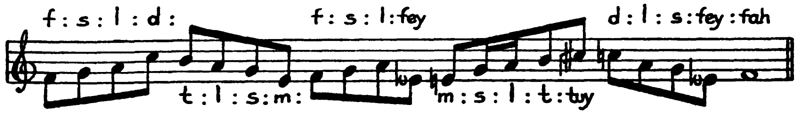


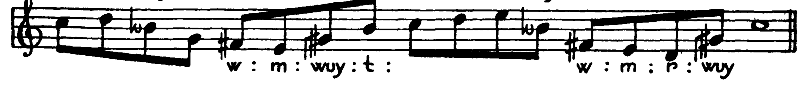
The outstanding feature now is the division of the perfect fifth in three very nearly equal parts by
c : *d*⟍ : *f*∠ : *g* = *doh* : *duy* : *sey* : *soh*
= 16 × 7 : 16 × 8 :21 × 7 : 21 × 8.

The interval in the middle is a trifle sharper (1029 : 1024) than the supersecond, at the very limit of perceptibility.

The pairs of leading notes *b* ♭ and *b*, *a* ♭ and *a* have an interval of a minor limma (*toow* : *te* = *loo* : *lahy* = 128 : 135). The pairs *d* and *d*⟍, *f*∠ and *f* have an interval of one-seventh of a tone (*ray* : *duy* = *sey* : *fah* = 63 : 64).

11. *Inversion, liaison by g and a, or by d and e* (*soh* : *lah* = *ray* : *me* = 9 : 10).





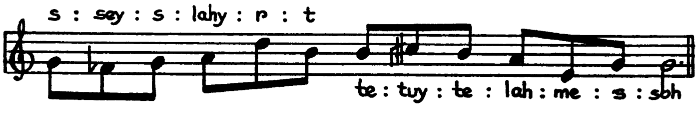
e∠♭ : *f* : *g* : *a* : *c* *fey* : *fah* : *soh* : *lah* : *doh*
e : *g* : *a* : *b* : *c*⟍♯ *me* : *soh* : *lah* : *te* : *tuy*
or
b∠♭ : *c* : *d* : *e* : *g* *dey* : *doh* : *ray* : *me* : *soh*
b : *d* : *e* : *f*♯ : *g*⟍♯ *te*: *ray* : *me* : *way* : *wuy*

One meets again the classical leading notes *b* and *c*, *f* ♯ and *g* (*te* : *doh* = *way* : *soh* = 15 : 16) and the non-classical pairs *b* ∠♭ and *b* ♮, *g* and *g* ⟍♯ (*dey* : *te* = *soh* : *wuy* = 14 : 15). The intervals *f* ♯ : *b* ∠♭ and *g* ⟍♯ : *c* can hardly be distinguished from perfect major thirds (*way* : *dey* = *wuy* : *doh* = 45 : 56 = 180 : 224 instead of 180 : 225 = 4 : 5).

12. *Inversion, centres in c and e, or g and b*(*doh* : *me* = *soh* : *te* = 4 : 5).

The *d*, major second of the centre *c*, is not the same as *d*, major subsecond of *e*. Here again one meets the situation, the "double emploi" of *d* pointed out by Rameau.

In the vocables we shall for clarity make a distinction between *ray* and *rayu*. The latter will be one comma flatter. The distinction between *lah* and *lahy* has already been made, *lahy* being one comma sharp.



b ∠♭ : *c* : *d* : *e* : *g* *dey* : *doh* : *ray* : *me* : *soh*
a : *c* : *d* : *e* : *f* ⦢♯ *lah* : *doh* : *rayu* : *me* : *muy*

or

f ∠ : *g* : *a* : *b* : *d* *sey* : *soh* : *lahy* : *te* : *ray*
e : *g* : *a* : *b* : *c* ⦢♯ *me* : *soh* : *lah* : *te* : *tuy*

After a certain amount of practice the difference of the two *d*'s, of *ray* and *rayu*, will be clearly heard, or at least felt as a distinction of mood. The pairs of leading notes *a* and *b* ∠♭, *f* ⦢♯ and *g* (*lah* : *dey* = *muy* : *soh* = 20 : 21, or *me* : *sey* = *tuy* : *ray* = 20 : 21) have been met before.

13. *Inversion, centres in f and e* ∠♭, *or f and d* ♯ (*fah* : *fey* = 4 : 7).

The inversion in question produces the following notes :

f : *g* : *a* : *c* : *e* ∠♭
f : *a* ∠♭ : *c* ∠♭ : *d* ∠♭ : *e* ∠♭

Because it would be a little troublesome to invent vocables for *d* ∠♭, *c* ∠♭, *a* ∠♭, it is permissible to shift part of the second group through the imperceptible interval of 224 : 225, so as to get *g* ♯ : *b* : *c* ♯ : *d* ♯ and vocalise with

fa : *soh* : *lah* : *doh* : *fey*
fa : *süh* : *te* : *düh* : *rüh*

It is important not to get too high a pitch with *e* ∠♭ (*fey* or *rüh*). Leaving out *c* and *g* ♯ (*doh* and *süh*) one sings the scale of six whole tones.

As to the pairs of leading notes, there are *g* and *a* ∠♭, *c* and *d* ∠♭ (*g* : *a* ∠♭ = *c* : *d* ∠♭ = 27 : 28, but *soh* : *süh* = *doh* : *düh* = 24 : 25). Again, there are *a* ∠♭ : *a* ♮ = *c* ∠♭ : *c* ♮ = 14 : 15 (but *süh* : *lah* = *te* : *doh* = 15 : 16).

14. *Inversion, liaison by b and f* ∠, *or f* ⦢♯ *and c* (*te* : *sey* = *muy* : *doh* = 5 : 7).

g : *a* : *b* : *d* : *f*∠ : *g* *soh* : *lahy* : *te* : *ray* : *sey* : *soh*
a∠ : *b* : *d*∠ : *f*∠ : *g*∠ : *a*∠ *tey* : *te* : *mey* : *sey* : *ley* : *tey*
or
d⦢ : *e*⦢ : *f*⦢♯ : *a*⦢ : *c* : *d*⦢ *duy* : *ruy* : *muy* : *suy* : *doh* : *duy*
e : *f*⦢♯ : *a* : *c* : *d* : *e* *me* : *muy* : *lah* : *doh* : *rayu* : *me*

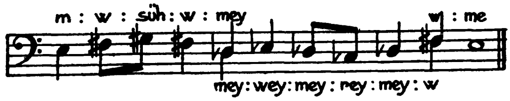
As before, the vocable *lahy* in the one case, and the vocable *rayu* in the other case, is required in order to vocalize this inversion. There are three pairs of leading notes, *a* ∠ and *a*, *d* ∠ and *d*, *g* ∠ and *g*, all of them enharmonic, with an interval of one-quarter of a tone (*tey* : *lahy* = *mey* : *ray* = *ley* : *soh* = 35 : 36). In the chord *b* : *d* : *f* ∠ one has part of a diminished seventh chord. By the inversion the inverted chord *b* : *d* ∠ : *f* ∠ is again part of a diminished seventh chord. This illustrates the modulation so often achieved by means of an enharmonic change in the diminished seventh chord.

15. *Inversion, liaison by b and d* ∠ (*te* : *mey* = 6 : 7).
The centres here are *e* and *a* ∠ (*me* : *tey* = 16 : 21).

d ∠ : *e* : *f* ♯ : *g* ♯ : *b* or *mey* : *me* : *way* : *süh* : *te*
d ∠ : *f* ∠ : *g* ∠ : *a* ∠ : *b* *mey* : *sey* : *ley* : *tey* : *te*

Pairs of leading notes are *e* and *f* ∠, *g* ♯ and *a* ∠ (*me* : *sey* = *süh* : *tey* = 20 : 21). Again *f* ∠ and *f* ♯, *g* ∠ and *g* ♯ (*sey* : *way* = *ley* : *süh* = 14 : 15).

16. *Inversion, liaison by d* ∠ *and f* ♯ (*mey* : *way* = 7 : 9).
Here the two centres are in extreme proximity : *e* and *e* ∠ (*me* : *wey* = 64 : 63).



b : *d* ∠ : *e* : *f* ♯ : *g* ♯ or *te* : *mey* : *me* : *way* : *süh*
c∠ : *d* ∠ : *e* ∠ : *f* ♯ : *a* ∠ *rey* : *mey* : *wey* : *way* : *tey*

Pairs of leading notes are *b* and *c* ∠, *g* ♯ and *a* ∠ (*te* : *rey* = *süh* : *tey* = 20 : 21).

This concludes the survey of the possible combinations of two diatonic melodic subscales.

17. *Conclusion.*

By combining two diatonic melodic subscales, belonging to two different centres, we have assembled groups of eight or nine and occasionally of seven notes, which might be written in the order of rising pitch and might be called scales. I doubt whether this would prove worth while. There would be as many of them as there are combinations of centres. It would be a big job to try to remember them all and this would be an unnecessary labour, because the consciousness of the centres provides all that one needs.

As a result of the combination notes come very near one to another. One meets pairs of *leading notes*. Very frequent is the leading note which leads to the centre, the classical leading note or Rameau's "note sensible", exemplified by *b* and *c* (*te* : *doh* = 15 : 16). Very frequent also is the leading note leading to the third (or subthird) of the centre, such as *f* ∠ leading to *e* (*sey* : *me* = 21 : 20). Again there is the leading note leading to the second (or subsecond) of the centre, such as *e* ∠♭ leading to *d* (*fey* : *ray* = 28 : 27). There is a leading note to the fifth (or subfifth) of the centre, e.g. *g* ♯ leading to *g* (*süh* : *soh* = 25 : 24).

All these are leading notes pointing towards the centre. Others point away from the centre. There is the leading note *a* towards the seventh *b* ∠♭ (*lah* : *dey* = 20 : 21), and *e* ∠♭ towards the third *e* ♮ (*fey* : *me* = 14 : 15). Finally there is the enharmonic leading note towards the second, *d* ∠ to *d* (*mey* : *ray* = 35 : 36).

Another enharmonic pair of leading notes is found with an interval of one-fifth of a tone, viz, *a* ⟍ and *b* ∠♭ (*suy* : *dey* = 48 : 49). Occasionally we meet the minor limma between *a* ♭ and *a* (*loo* : *lahy* = 128 : 135, § 10), and the major limma between *a* and *b* ♭ (*lah* : *too* = 25 : 27, § 9). Intervals of one-seventh of a tone are frequently met, e.g. *f* ∠ and *f* ♮ (*sey* : *fah* = 63 : 64). These pairs of notes are on the border line between leading notes and commatic doublets. These doublets, as *a* and *a*, *d* and *d* (*lah* : *lahy* = *rayu* : *ray* = 80 : 81) are still distinct, and in these cases the symbols *a* and *d* are used with a double meaning. The difference is imperceptible between *e* ∠♭ and *d* ♯, or *e* ⟍♯ and *f* (*fey* : *rüh* = 224 : 225) and can be disregarded.

Thus various kinds of leading notes come into existence by the combination of different melodic centres.

Another result is the discovery of the division of the interval of the perfect fifth into three very nearly equal intervals in *c* : *d* ⟍ : *f* ∠ : *g* (*doh* : *duy* : *sey* : *soh*). This is a feature of existing musical culture in the far East, of the *gamelan slendro* and *gamelan angklun*, in the isles of Java and Bali. I see no reason why it should not become an element of musical culture in Europe and the western hemisphere.

From the artistic point of view it is important to know what this combination of melodic groups means for musical practice. In the fourth chapter it will be shown how one can recognize various combinations of melodic groups in the ancient melodies of the Gregorian liturgy. Again, in a number of four part examples it will be shown how these melodic subscales afford material for modern music. To that end Mr. JAN VAN DIJK has kindly composed a number of exercises which refer to the combinations considered in this chapter.

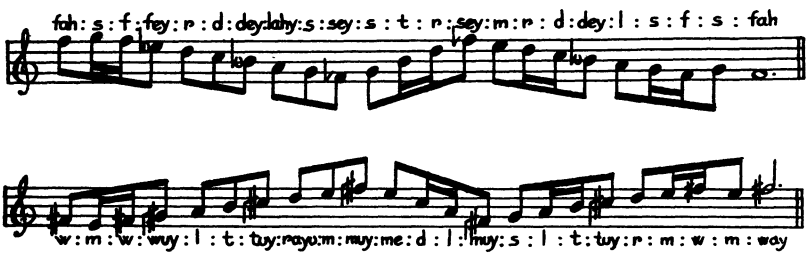
CHAPTER III

COMBINATIONS OF THREE OR MORE MELODIC CENTRES

CONTENTS

1. *Three centres shifted through perfect fifths.*

If the three fundamental centres are chosen in *f*, *c* and *g* (*fah, doh, soh*), one must be aware that *a*, the major second to *g*, is sung one comma sharper than *a*, the perfect major third to *f*. It has already been pointed out that the ordinary symbol *a*



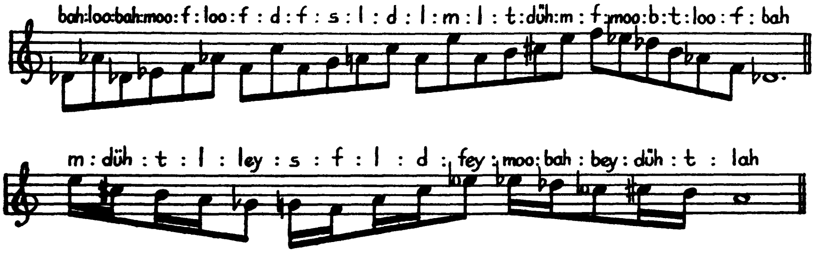
has a double meaning (Rameau's 'double emploi') as *lah* and as *lahy*.

Similarly, putting guiding note centres in *e*, *b* and *f* ♯ (*me*, *te*, *way*) the major subsecond of *e* and the major subthird of *f* ♯ are both represented by the symbol *d*, which stands for the two functions *rayu* and *ray*.

The addition of the harmonic sevenths to the classical substratum gives rise to many leading notes, which have been considered in the preceding chapter.

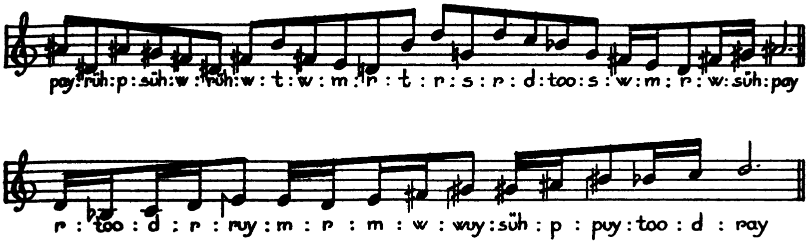
2. *Three centres shifted through perfect major thirds.*

Let us choose our fundamental centres in *d* ♭, *f* and *a* (*bah*, *fah*, *lah*; for *bah* see Ch. I, § 4). Here one ought to distinguish



between *d* ♭ and *c* ♯ (*bah* : *düh* = 128 : 125) which in equal semitone temperament are identified. On the other hand the difference in pitch of *c*∠♭ and *b* (*bey* : *te* = 224 : 225) escapes perception.

For guiding note centres one may choose *a* ♯, *f* ♯ and *d* (*pay*, *way*, *ray*; for *pay* see Ch. I, § 4). Now there must be a perceptible difference between *a* ♯ and *b* ♭ (*pay* : *too* = 125 : 128), obliterated by the current equal temperament.



Undistinguishable are *b* ⧹♯ and *c* (*puy* : *doh* = 225 : 224), and of course this also means a 'double emploi' of one note, in two functions, on a more refined scale.

3. *Three centres shifted through superseconds.*

In choosing for fundamental centres *d* ⧹, *c* and *b* ∠♭ (*duy, doh, dey*) one meets a difficulty. It is the lack of a vocable for the seventh harmonic of *b* ∠♭ (*dey*). The seventh harmonic of *b*-flat being *a*-flat-minus the seventh harmonic of *b*-flat-minus ought to be *a*-flat-double-minus. Twice minus makes a flat, and so we arrive at *a* ♭♭. As for a vocable, one may sing *deh* (*eh* as in *deck*).

Similarly, choosing for guiding note centres *a* ∠, *b, c* ⧹♯ (*tey, te, tuy*), the seventh subharmonic of *c* ⧹♯ must be *d*-sharp-double-plus, or *d*×. As for a vocable, let the subseventh of *tuy* be *tuh* (*uh* as in *but*).

Evidently there will be an abundance of enharmonic pairs of notes. Some show one-seventh of a tone (63 : 64 = *c* ∠ : *c* ♮ = *d* : *d* ⧹ = *b* : *b* ⧹ = *a* ∠ : *a*). Others show a quarter-tone (35 : 36 = *d* ∠ : *d* ♮ = *e* : *e* ⧹ = *a* : *a* ⧹ = *g* ∠ : *g* ♮). These enharmonic subtleties needs must render such exercises extremely difficult, even for the very best singers.

Between *a* ♭♭ and *d* ⟍ (*deh* and *duy*) and again between *a* ∠ and *d* × (*tey* and *tuh*) one has a distance of three superseconds.

a ♭♭ : *b* △♭ : *c* : *d* ⟍ = *deh* : *dey* : *doh* : *duy*.
a ∠ : *b* : *c* ⟍♯ : *d* × = *tey* : *te* : *tuy* : *tuh*.

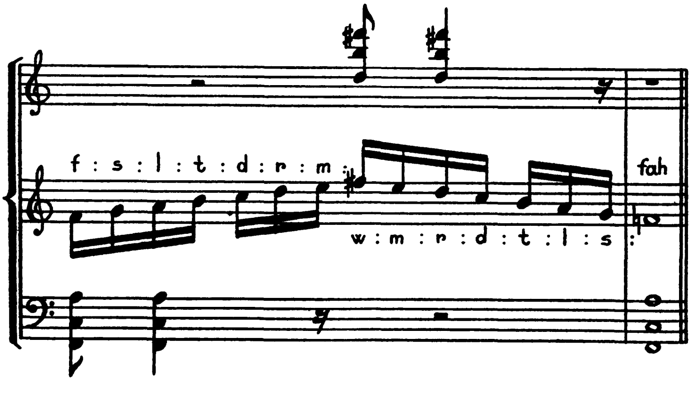
This distance (343 : 512) most probably can only be distinguished from a perfect fifth (342 : 513), by very acute and well-trained hearing.

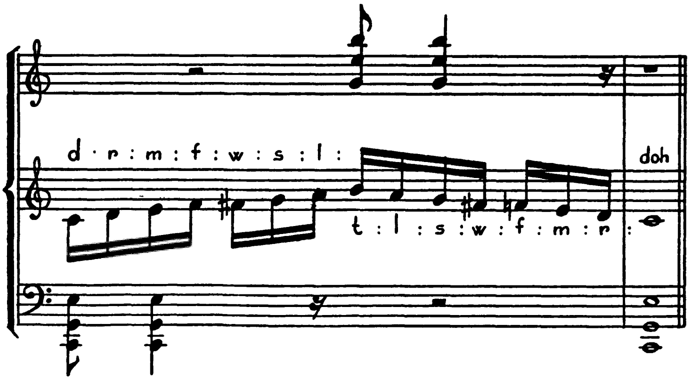
One remembers that in Ch. II, § 10 one met a perfect fifth with a division in three very nearly equal parts (*c* : *d* ⟍ : *f* ∠ : *g*).

4. *Two centres in f and c and their inversions in f ♯ and b* (*fah, doh, and way, te*). *Genus diatonicum.*

Between the centres *f* and *b* there are liaison notes *g* and *a*, between the centres *c* and *f* ♯ there are liaison notes *d* and *e*. These liaison notes are the seconds and thirds of their respective centres. This combination, leaving out harmonic sevenths, produces the classical *genus diatonicum* (Cf. Ch. I, § 3) with the eight basic notes.

These may be sung or played together as a scale in one of two forms. The scale can have its extremities in the centres *f* and *f* ♯ (*fah* and *way*) respectively, and these centres may be emphasized by suitable chords in the bass and in the treble. Again, one can





choose for the extremities of the scale the centres *c* and *b* (*doh* and *te*), both emphasized by their proper chords.

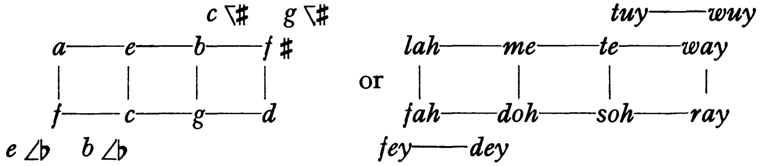
The semitones *e* : *f* : *f* ♯ : *g* (*me* : *fah* : *way* : *soh*) show a succession of two minor seconds or diatonic semitones (*e* : *f* = *f* ♯ : *g* = 15 : 16) and one minor limma (*fah* : *way* = 128 : 135) which is one comma sharp of the chromatic semitone, as explained in Ch. I, § 3.

In both forms the range of the scale can be enlarged by adding the sevenths below the lowest fundamental centre and above the highest guiding note centre.

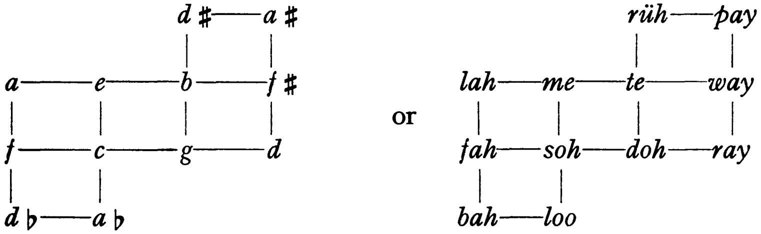
One can also add all the four sevenths among the notes of the scale in the second form. This makes a special non-classical chromatic scale, which can be described as a scale of leading notes with a variety of intervals of major and minor semitones. These twelve notes do not constitute a classical genus diatoni-

co-chromaticum. This is most clearly demonstrated in a diagram showing the harmonic note lattice. This is a diagram in which, starting from the harmonic frame of notes described in Ch. I, § 4, more notes are placed in parallel planes either in front of or behind the plane containing the basic notes, thus affording places for the harmonic and subharmonic sevenths of the basic notes.

Using the harmonic note lattice one gets the representation



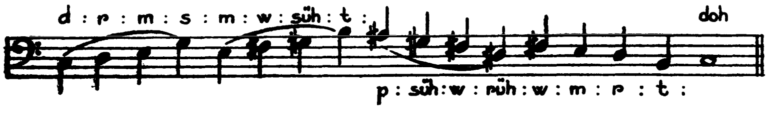
and one sees at a glance that this is not a rectangle 3 × 4 as would be the representation of a genus diatonico-chromaticum. Now it has been pointed out repeatedly, that we are not able to distinguish *e*∠♭ and *d* ♯, *b*∠♭ and *a* ♯, *d* ♭ and *c* ⧹♯ or *a* ♭ and *g* ⧹♯ (*fey* : *rüh* = *dey* : *pay* = *bah* : *tuy* = *loo* : *wuy* = 224 : 225). Thus, by a kind of 'double emploi' one could represent the notes required by the following



as part of the genus diatonico-hyperchromaticum.

5. *Two centres in c and e, and their inversions in a ♯ and f ♯ (doh, me, and pay, way). Genus chromaticum.*

The centres *c* and *f* ♯ will have liaison notes in their seconds and thirds, *d* and *e*, as before, and so will the centres *e* and *a* ♯ with the notes *f* ♯ and *g* ♯.



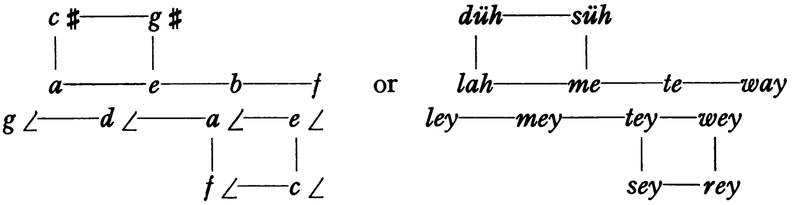
Without the sevenths, one is left with the *genus chromaticum*. From this one can select the seven notes of Tartini's minor scale, the so-called gipsies' scale, or, as another possibility, the six notes of the whole-tone scale. With the sevenths present there is double employment for one note as *c* and as *b* ⧵♯, and for another as *b* ∠♭ and *a* ♯ (*doh* : *puy* = *dey* : *pay* = 224 : 225). The sevenths of the two remaining centres give rise to quarter-tone intervals (*d* ∠ : *d* = *g* ♯ : *g* ⧵♯ = 35 : 36).

Here follows the diagram of the harmonic note lattice:

b ⧵♯ *puy*
g♯—*d*♯—*a*♯ | *süh*—*düh*—*pay* |
| | | *g* ⧵♯ | | | *wuy*
e—*b*—*f*♯ or *me*—*te*—*way*
d∠ | | | *mey* | | |
| *c*—*g*—*d* | *doh*—*soh*—*ray*
b ∠♭ *dey*

6. *Two centres in a and e, and their inversions in e* ∠ *and a* ∠ (*lah, me, and wey, tey*). *Genus enharmonicum simplex.*

In the preceding sections two centres had liaison notes in their seconds and thirds. Now pairs of centres will be chosen, having their seconds and sevenths for liaison notes : thus *a* and *a* ∠ have *b* and *g* ∠ in common. If this pair of centres is shifted through a perfect fifth they show their subscales in the following diagram of the harmonic note lattice



Leaving out the major thirds, one is left with the *genus enharmonicum simplex,* containing eight notes.

There is a series of perfect fourths

f ♯ : *b* : *e* : *a,* or *way* : *te* : *me* : *lah,*

with in between the notes of another similar series of perfect fourths :

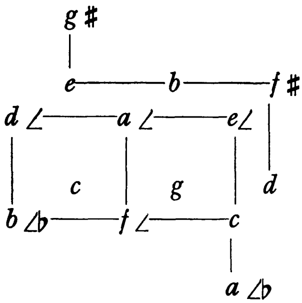
e ∠ : *a* ∠ : *d* ∠ : *g* ∠, or *wey* : *tey* : *mey* : *ley,*

'halving' the intervals in a harmonic way. This reminds one of the series of perfect fifths in the genus diatonicum (*f* : *c* : *g* : *d*) combined with a similar series (*a* : *e* : *b* : *f* ♯), each of the two series alternately 'halving' the perfect fifths of the other.

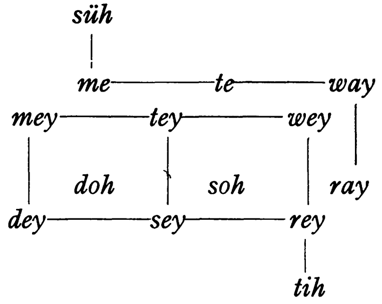
I need not dwell long on the leading notes. They are of the types met before. In the example we recognise *d* ∠ : *c* ♯ (*mey* : *düh* = 21 : 20) and *g* ∠ : *f* ♯ (*ley* : *way* = 28 : 27).

7. *Two centres in c and e, and their inversions in c* ∠ *and e* ∠ (*doh, me, and rey, wey*). *Genus enharmonicum vocale.*

The same pair of centres, *e* and *e* ∠, of the preceding section, sharing their seconds and sevenths as liaison tones, are now being shifted through a major third instead of through a perfect fifth.



or



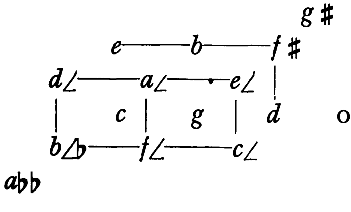
Leaving out the major thirds above *e* and below *c* ∠ one is left with a genus of twelve notes. This is the smallest genus which contains a harmonic melodic subscale. Therefore it might be called *genus enharmonicum vocale* (Cf. Ch. IV, § 2, examples *W* and *X*).

The major third below *c* ∠ leads to *a* ∠♭, the seventh of *b* ♭, or *too*, and a new vocable is required, which must not be *tey*. Let it be *tih* (*ih* as in *tip*).

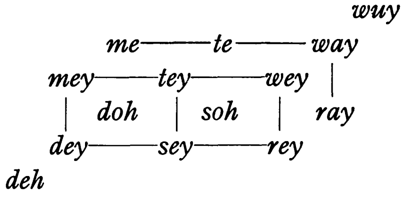
¶ Here again one meets a 'double emploi', for *g* ♯ and *a* ∠♭ (*süh* : *tih* = 125 : 126) have a difference of only two thirds of a comma.

8. *Two centres in c and b ∠♭, and their inversions in e ∠ and f ♯ (doh, dey, and way, wey). Genus enharmonicum vocale.*

Taking again two centres, *c* and *f* ♯, sharing seconds and thirds as liaison notes, one may shift these through a harmonic seventh. The result is shown in the note lattice as follows :



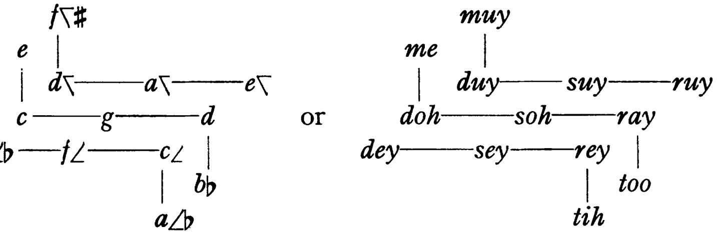
or



As in § 3, one meets the harmonic seventh of the harmonic seventh of *c* (*doh*) in *a* 𝄫 (*deh*). Leaving out *a* 𝄫 and *g* ╲♯, one is left with the same *genus enharmonicum vocale* as before. This time however it is taken as grouped round other centres, here *b* ╲♯ and *f* ♯, instead of *e* and *c* ∠.

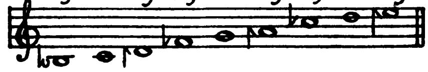
9. *Two centres in d* ╲ *and c, and their inversions in c* ∠ *and d* ♮ (*duy, doh, and rey, ray*). *Genus hyperenharmonicum.*

Finally we shall shift a pair of centres, having their seconds and sevenths as liaison notes (centres *c* and *c* ∠) through a harmonic seventh or supersecond. The result is shown in the diagram:



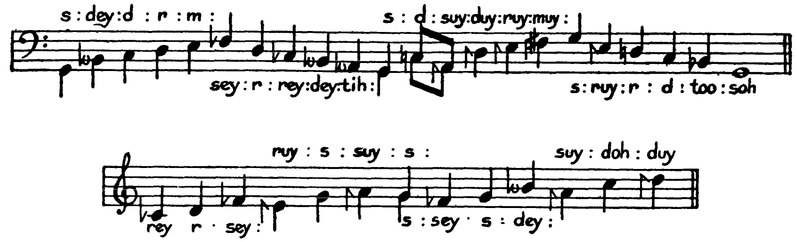
Leaving out the major thirds altogether, nine notes are left in a square in the plane frame of notes with perfect fifths and perfect sevenths. This is a *genus hyperenharmonicum.*

These notes may be arranged in a series proceeding by almost equal steps



b∠♭ : *c* : *d*⟍ : *f*∠ : *g* : *a*⟍ : *c*∠ : *d* : *e*⟍
dey : *doh* : *duy* : *sey* : *soh* : *suy* : *rey* : *ray* : *ruy*
7 : 8 7 : 8 7 : 8
128 : 147 128 : 147
7 : 8 7 : 8 7 : 8

The intervals 128 : 147 are just a trifle sharper than the intervals 7 : 8 (= 126 : 144).

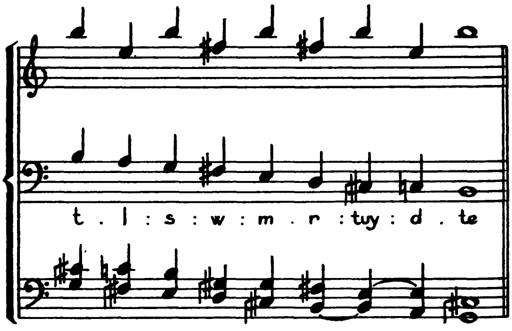


Typical of the genus hyperenharmonicum is the occurrence of intervals of one-fifth of a tone (*a*⟍ : *b*∠♭ = *e*⟍ : *f*∠ = *suy* : : *dey* = *ruy* : *sey* = 48 : 49).

10. *Mixed scales with three shifted centres.*

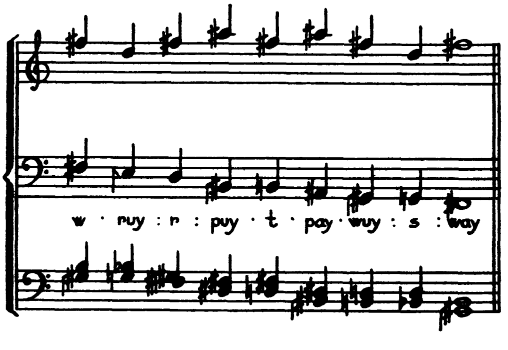
It is possible to arrange series of notes in such a manner that every successive note belongs to another centre. The result is a scale, which will be called a *mixed* scale because members of the various melodic subscales have been mixed up. The first example shown is the major scale of TARTINI, containing eight notes and supported by a perfectly symmetrical bass which shows three centres at distances of a perfect fifth.

The example shows, on a separate stave, other scales which are suitable for use in accompanying parts.



Using three guiding note centres, *f* ♯, *b* and *e* (*way, te, me*), one finds an inverted scale.

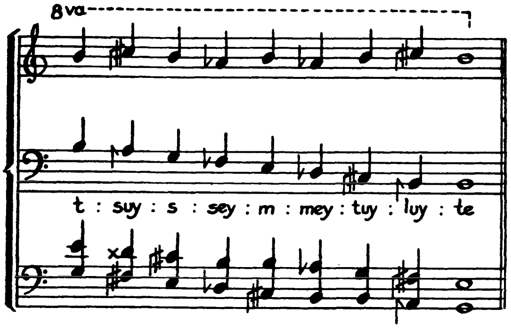
In a similar way three centres at the distance of a major third may serve to compound a scale. In the examples *d* ♭, *f*, *a* (*bah, fah, lah*) have been chosen as fundamental centres, and *a* ♯, *f* ♯, *d* (*pay, way, ray*) for guiding note centres.



In the first case the centre *f* (*fah*) has the function of a tonic. The other centres can in their function be compared to dominant and subdominant.

Companion scales starting on the fifth and on the seventh have been added in the examples. Ninth harmonics and subharmonics have not been used, though there is no reason to avoid them.

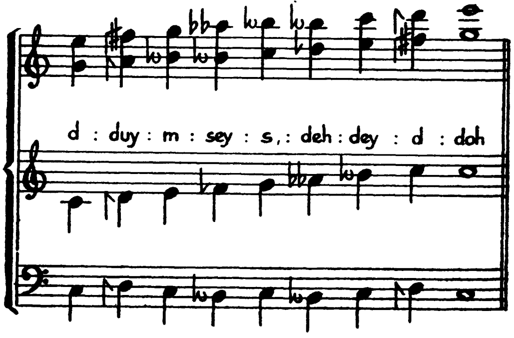
Finally the centres can be taken at distances a supersecond apart. It will be impossible to avoid the scales sometimes proceeding parallel to the bass (or the treble) with equal steps. To compensate for these big strides the steps are in other places

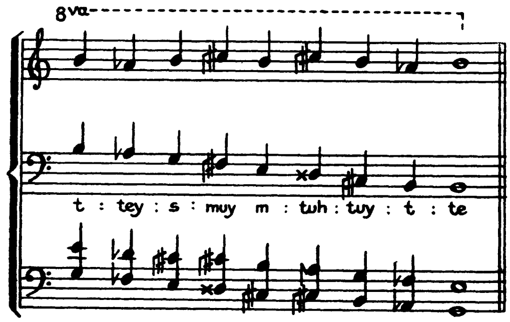


very small. One hears fifths of a tone (*a* ∖ : *b* ∠♭ = *suy* : *dey* = *c* ∖♯ : *d* ∠ = *tuy* : *mey* = 48 : 49), and sevenths of a tone (*c* ∠: *c* ♮ = *ruy* : *doh* = *b* : *b* ∖ = *te* : *luy* = 63 : 64).

Sometimes in the examples the ninth harmonic had to be written in the scales (*c* ∠ on the fundamental *b* ∠♭, and *b* ∖ under the guiding note *c* ∖♯). Companion scales have been added.

By changing the order of succession of the centres it will be found that one of the companion scales will replace the one which started from the tonic, and vice versa. The places of the big strides and of the small steps in the scales are changed.





Similar permutations in the order of the centres can of course be applied to the other scales mentioned earlier. It will not be necessary to work this out explicitly. Neither will it be necessary to attempt a complete enumeration of the scales which might be obtained by taking centres in the bass or in the treble which together form a common chord, ro a four-note chord. The number of possibilities is very great.

CHAPTER IV

EXAMPLES

CONTENTS

1. Examples in one part,
   - *a*) the seventh ecclesiastical mode;
   - *b*) the fifth mode;
   - *c*) the fourth mode;
   - *d*) the sixth mode;
   - *e*) the second mode;
   - *f*) the first mode.
2. Examples in four parts, composed by Mr Jan van Dijk.

1. *Examples in one part.*

a. The *first example* is taken from the music of the Gregorian liturgy for the Mass on the eighth Sunday after Whitsuntide. It is the versus alleluiaticus „Magnus Dominus et laudabilis" in the seventh mode. The fundamental centres are *g* and *d* and in the harmonic note lattice the occurring notes appear like this:

b
g——*d*——*a*——*e*
f ∠——*c* ∠

and they represent the functions of

lah
fah——*doh*——*soh*——*ray*
fey——*dey*

The syllables "*Magnus*" show *d* as the centre. The syllable "*Do-*" shows the centre *g* with *e* as a leading note to *f* ∠. The syllables "*-minus*" descend to the centre *d* with *f* ∠ leading note to *e*. The syllables "*et lau-*" come from the centre *d* down to the centre *g* of the syllables "*-bilis*" after which the melody rises again to *d* centre of "*valde*". The line "*in civitate*" shows *b* leading note to *c* ∠ and *f* ∠ leading note to *e* round the centre *d*; it finally comes down to the centre *g*.

I do not claim that *c* ∠ and *f* ∠ *should* be sung decidedly flat, but one *might* sing them with considerable gain in beauty and clearness of expression in the interpretation put down here, and frequently one actually hears a singer doing it.

In connexion with this example I wish to quote the last line from the second versus alleluiaticus on the first Sunday after Easter.

The syllables *"et dixit"* have *g* for a centre. The word *"Pax"* has the initial notes *f* ∠ and *a* from the centre *g*, but the final pair *c* ∠ and *a* already belong to the centre *d*, which reigns awhile in *"vobis"* until a *c* ∠ proves leading note to *b* and the melody comes down to the centre *g*.

The triad in „*Pax*", *f* ∠ : *a* : *c* ∠, is the special chord 14 : 18 : 21 = *dey* : *ray* : *sey*, refered to in Ch. I, § 8.

b. The second example is the introitus of the Mass on the ninth Sunday after Whitsuntide, in the fifth mode. Again the centres are a fifth apart, on *f* and *c* (*fah* and *doh*), and the centre *c* is accompanied by the leading note *b*. Its seventh *b* △♭ appears in this piece as a leading note to *a*. This is the diagram of the notes and their functions :

a——*e*——*b*
f——*c*——*g*——*d*
b △♭

or

lah——*me* — *te*
fah——*doh*——*soh*——*ray*
dey

The words *"inimicis meis"* temporarily converge on a guiding note centre in *b*. The word *"illos"* is back on the centre *f*.

c. The *third example* is an alleluia in the fourth mode, from the Mass on the third Sunday after Easter.

It has two guiding note centres in *e* and *a*.

The six notes

g———*d*———*a*———*e*
f———*c*

have the functions of

lah———*me*———*te*———*way*
soh———*ray*

It should be clear that *e* : *d* : *c* here is not *me* : *ray* : *doh*. It could be described by *me* : *rayu* : *doh*. Neither can we say *lah* : *soh* : *fah* for *a* : *g* : *f* here. There should be another name here for the major subsecond of *lah*, not *soh*, perhaps *sohu*. It is simpler to describe the functions by means of basic notes, and this has been done here.

d. The *fourth example* is the communio, in the sixth mode, of the first Sunday after Easter. There is a fundamental centre in *f* and a guiding note centre in *a*, both with leading notes *e* and *b* ♭ respectively.

It follows that *g* must have a double function. These are the notes :

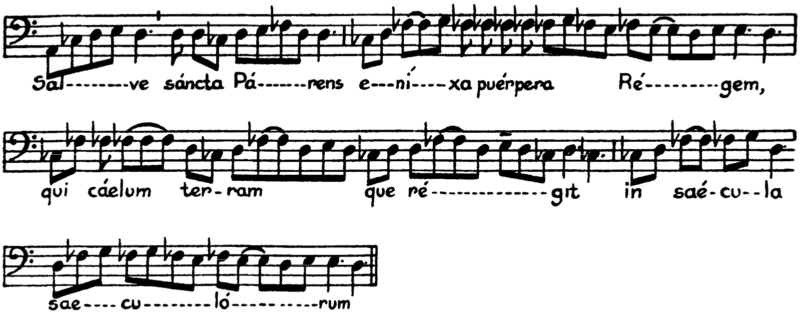
g———*d*———*a*———*e*
b♭———*f*———*c*———*g*

and their functions:

lah———*me*———*te*———*way*
doh———*soh*———*ray*———*lahy*

The first words "*mitte manum tuam*" centre round *f*. The word "*et*" contains a *d*. This belongs, along with *f*, to the guiding note centre *a*. It therefore induces the phrase to rise to the centre *a* in the first "*alleluia*". The phrase comes down to *c* in "*fidelis*", as a temporary resting point. In the last "*alleluia*" the two different pitches of *g* may be heard.

e. The *fifth example* is the introitus of the Mass in the *Commune festorum B. Mariae Virginis.* The introitus is in the second mode. The word *"salve"* displays four notes of the melodic subscale with *d* for centre. This same subscale occurred in the



example *"Magnus Dominus"* quoted before, with leading note *f* ∠ to the second of the subscale, *e.* In this introitus *d* is the dominating centre. In the four notes *c* ∠ : *d* : *f* ∠ : *g,* displayed in *"enixa"* the *d* is counterbalanced by *f* ∠. The diagram of the note lattice is :

g———*d*———*a*———*e*
f ∠———*c* ∠

having the functions of :

fah———*doh*———*soh*———*ray*
fey———*dey*

The *a* appears only once, and the function of *e* is to emphasise *d* as the principal centre.

f. The *sixth example* given here is a *Kyrie eleison,* known under the title *"cum jubilo".* It belongs to the first mode. There is a guiding note centre in *a* with leading note *b* ♭ and a fundamental centre in *d.* The guiding note centre *a* requires an *f.* The fundamental centre in *d* gives rise to *f* ∠ as leading note to *e.* The surroundings of this centre *d* are the same as in the preceding example, belonging to the second mode.

The diagram of the note lattice shows :

g———*d*———*a*———*e*
f ∠———*c* ∠ | |
b♭———*f*

in the functions of :

lah———*me*———*te*———*way*
ley———*mey* | |
doh———*soh*

It is important, in this mode, to realize that by the clearly different intonation of *f* and *f* ∠ the domination of either the centre *a* or the centre *d* mates itself distinctly felt.

2. *Examples in four parts.*

Mr JAN VAN DIJK, teacher at the *Toonkunst Conservatorium* at Rotterdam, kindly composed a number of exercises in four parts exemplifying the various combinations of two melodic centres and their subscales reviewed in Chapter II. My thanks are due to him for the very able manner in which he has broken new ground.

The subscales used have been indicated at the top of each of the exercises. One may see at a glance that the notes of a subscale, placed in a vertical line, show the harmony of the chords of the perfect sevenths and ninths (Ch. I, § 8).

The four parts together are only singing one subscale at a time.

Some examples will prove to require great skill. Example A will not present any difficulty. Example C presents a new feature. After these, the examples B and D may follow, but it is better to leave E and F aside for a while and proceed to G. Example L may follow, then H and K. After the examples M and N one may be prepared for the example Q and the remaining ones.

Example A illustrates Ch. II, § 2 with two centres of fundamental character a perfect fifth apart. This is an almost classical combination.

Example B shows two centres a perfect fifth apart, with a guiding note character and subscales of subharmonics.

In the examples C and D (Ch. II, § 3) the centres are a perfect third apart. In C the enharmonic leading note *g* ∠ to *g* ♮ (*ley* to *soh*) appears, making an interval of a quarter of a tone. In the third bar the bass must make sure not to imitate the soprano's first bar!

Examples E and F (Ch. II, § 4) show centres at a distance of a supersecond. Again enharmonic leading notes appear, *a* ⦣ and *b* ∠♭ (*suy* and *dey*) and *d* ∠ and *c* ⦣♯ (*mey* and *tuy*). These intervals are a fifth of a tone.

Example G is the first to have two inverted subscales with two liaison notes (Ch. II, § 6). It contains part of the diatonic genus (Ch. III, § 4).

Examples H and K (Ch. II, § 7) show quarter-tone leading notes : *e* ∠♭ to *e* ♭ (*fey* to *moo*) in H and *e* ⦣ to *e* ♮ (*ruy* to *me*) in K.

Example L (Ch. II, § 9) again shows leading notes with a fifth of a tone, *a* ⦣ and *b* ∠♭ (*suy* and *dey*).

Examples M and N (Ch. II. § 11) show classical as well as non-classical leading notes with a major semitone: *c* to *c*⦣♯, *e* ∠♭ to *e* ♮ (*do* to *tuy*, *fey* to *me*) and *b* ∠♭ to *b* ♮ (*dey* to *te*). Some intervals are almost perfect major thirds: in M one hears *b* ∠♭ and *f* ♯ (*dey* and *way*). Other intervals are slightly less than a second: *e* ∠♭ and *c* ⦣♯ (*fey* and *tuy*) and *b* ∠♭ and *g* ⦣♯ (*dey* and *wuy*). In the sixth and seventh bars of M the notes *d* : *dey* : *r* are not a mere transposition of *m* : *r* : *w* in the fifth and seventh bars. In N the soprano's symmetrical garland of whole tones in the first and following bars is quite distinct from the garland of supersecond and seconds in the eighth bar!

In example O (Ch. II, § 12) a slight difference of intonation should make itself felt when *d* appears in its two functions of ***ray*** and ***rayu***. ***Lah*** in one of the parts demands ***rayu*** in the other, and ***ray*** is required where ***soh*** appears in another part. Example P, though illustrating a transposition of the same subscales, does not show the ambiguous ***a*** at all. In the interval *c*⦢♯ : *f* ∠ = ***tuy*** : ***sey*** the soprano and the alto produce a third halfway between the minor and the major thirds.

Example Q (Ch. II, § 13) makes a compromise in the notation in so far as *e* ∠♭ is identified with *d* ♯ (***fey*** and ***düh***). The gain is that one thereby avoids the introduction of new vocables. As a consequence ***fah*** has sometimes been written on the stave as *e* ⦢♯.

Example R (Ch. II, § 14) shows the alternating of *d* and *d* ∠ (***ray*** and ***mey***) which is characteristic of the enharmonic transformation of the chord of the diminished seventh in classical music (e.g. *d* : *f* : *a* ♭ : *b* into *d* : *f* : *g* ♯ : *b*). A quarter-tone is involved in this shift. In example S one meets the same situation in the alternating of *a* and *a* ⦢ (***lah*** and ***suy***).

Example T (Ch. II, § 16) is conspicuous by the minor semitones in consecutive chords.

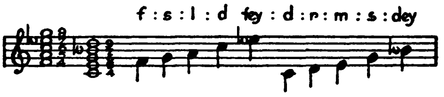
Example U (Ch. II, § 16) looks like a pentatonic piece in *b* : *d* ∠ : *e* : *f* ♯ : *a* ∠ where the scale has been differentiated, duplicating the central note, thus: *b* : *d* ∠ : *e* ∠ : *f* ♯ : *a* ∠, and where leading notes have been added (*g* ♯ to *a* ∠, ***süh*** to ***tey***, and *c* ∠ to *b*, ***rey*** to ***te***).

Examples V and W belong to the genus enharmonicum vocale (Ch. III, §§ 7 & 8). There are two subscales with centres *f* and *a* ∠ (***fah*** and ***tey***) and two connecting leading notes *e* and *b* ∠♭ (***me*** and ***dey***) which have their centres in *c* and in *d* ∠ (***doh*** and ***mey***). Example W will be best suited for soprano and strings.

ACKNOWLEDGMENTS

In conclusion I wish to express my indebtedness to several friends, to the late composer WILLEM PIJPER, to his pupil JAN VAN DIJK, to Professor DOUGLAS R. HARTREE and to Mr URWIN THORNBURN, M.A., for encouragement and much useful help.

EXAMPLE A


f : s : l : d fey : d : r : m : s : dey


Giocoso
S
A
T
B
f
ff
d : d : r : d : dey : l : l : f : f : m : f : f : s : f : fey :
m : s : f : f : l : l : s : l : d :
s : d : d : d : d : d : d : f :
d : m : f : f : f : d : f : l :
r : r : s : s : f : f : s : f : fey : m : r : d : dey : l : s : f : m
dey : dey : dey : dey : l : d : d : dey : s : f : l : s
m : d : r : d : f : s : d : d : d : fey : dey
d : m : m : f : l d : m : m : f : f : d

EXAMPLE B

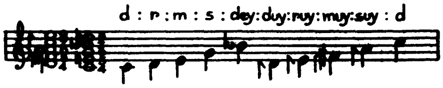

w : m : r : t : wuy : t : l : s : m : tuy
Allegretto
w : w : s : s . t : t : w : r : m : t :
r : r : m : m : m : m : r : w : m :
t : t : t : t : s : s : wuy : t : tuy :
t : t : m : m : tuy : tuy : t : wuy : s :
m : w : r : t : t : w : t : t : w : w :
wuy : w : m : tuy : t : t : t : t : t :
r : t : t : m : r : m : m : w : w :
w : wuy : s : s : t : s : s : r : r :
r : r : t : m : w : r : t : tuy : l : t
w : w : m : wuy : w : m : tuy : r : m : r
wuy : wuy : s : t : wuy : t : m : w : s : w
t : t : m : r : t : s : s : wuy : tuy : wuy


f . s : l : d : fey : l : t : düh : m : ley
Allegretto
p
f : f : fey : d : t : l :
l : ley : l : m : f : ley :
t : l : t : d : düh
l : s : l : m :
espressivo
l t : l : m : t : l
(p)
(mp)
s : f : s : m : s : f : fey : m
fey : düh : d : düh
ley : fey : f : s : l

EXAMPLE D


w : m : r : t : wuy : r : d : too : s : ruy
Allegretto
p
r : t : ruy : r : ruy : r : s : w : m : w : ruy : r :
w : w : s · w : too : t : s : w :
t : r : too : t : ruy : r : ruy : r : r : t :
t : t : s : wuy : s : wuy : too : t :
r : too : s : w : s : w : ruy : r : ruy : r : s : w :
s : s : too : t s : w : too : t :
r : r : r : r : too : t : r : r :
too : s : ruy : wuy : s ; wuy : ruy ; t ·


pp
p : t : m : p : m : p : s : w : s : w : puy : p :
w : w : w : s : too : t : too : too :
t : p : t : too : p : p : p : puy :
t : t : wuy : s : puy : wuy : s : s ;
piu f
allargando
ff
p : s : m : p : m : p : puy : p : puy : p : s : s : w
t : p : t : t : s : w : p : too : t
w : too : w : w · p : p : puy : puy : p : puy : p
w : puy : w : wuy : too : t : too : s : t


d : r : m : s : dey: duy: ruy: muy: suy : d


Tranquillo
d : suy : r : d : suy : m : duy : s : s : m : s : m :
s : muy : s : duy : s : muy : s : s :
m : duy : s : suy : dey : d : r : d : r : d :
d : duy : m : muy : s : suy : dey : d : dey : d :
duy : d : dey : d : suy : dey : s : m : r : duy : d : suy
suy : r : m : duy : r : dey : s : muy : s : muy
(ten.div.)
duy
suy : s : s : muy : s : r : d : dey : suy : s : suy
muy : s : d : duy : s : d : duy : m : duy

EXAMPLE F


tey. ley: sey· mey: t   t : l : s : m : tuy
Allegretto
p
tey:tey· mey:mey:sey : mey : mey : tuy   mey:mey. sey:sey· tey : mey : mey : tuy :
tey . tey · s   tey : s :
sey ; sey : m   sey m :
t : t : t   : t : t .
cresc
piu cresc.
ff
s : s:m.m : tey:tey:sey:sey : s s.m.m : m : mey .
m·m:t.t : sey:sey:tey:tey : m : m : t : t : t · tey .
s : s m : m. tuy. tuy · tey:tey:sey:sey:mey : mey : s : s :m:m. tuy : ·tuy : tuy : mey :
s s tuy:tuy:tuy : s . tey:tey:mey:mey:mey : t : s.s.tuy:tuy. t : s : sey :
(poco f)
mey : mey : tuy : mey:mey:sey:sey . s : m . mey
tey · tey : t : tey:tey:tey:tey : s : s . sey
tey : tey : mey mey : sey : sey : sey : m · mey:mey: t : t : t · t : tey
t : t ; s : sey:sey:mey:mey. m : tuy : mey

EXAMPLE G

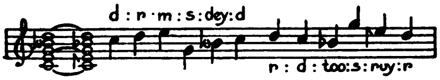

d : r : m : s : dey : d
t : l . s : m : tuy : t
Moderato assai
p-pp
s : s : s : m : r : d : m : m : m : s : l : t : m : s :
p-pp
m : m : m : s : s : s : s : s : m : m : r : d :
p-pp
.d : t : dey : s : t : d : t t : dey : dey :
p-pp
d : m : d : m · m d : m : tuy : s : m : r :
(ad lib.) mp
s : m : r : d : s : m : r : d : r : dey : s : s : s :
(ad lib.) mp
m : s : s : dey : s : s : s : s : r : tuy : r :
(ad lib.) mp
t : s : d : dey : d : dey : r : d : dey : l : dey :
(ad lib.) mp
tuy : m : m : s : m : s : s : s : m : m :


cresc.
f (poco slentando)
s : t : l : s : s : m : r : d : s : m : m : r : d : m : s : m :
m : m : m : m : s : s : dey : s : s : t : s :
t : s : l : t : d : s : m : r : dey : d : tuy : l : t : tuy :
tuy : tuy : d : m : d : s : m : l : l :
piu f
dim
rall............
s : m : s : t : tuy : m : s : m : r : d : r : dey : s : m : m
t : s : t : m : s : t : dey : s : s : s : m : r : d
tuy : t tuy : l : t : tuy : r dey : d : dey : r : r : dey : s
l : l : l : m : s : d

EXAMPLE H


f : s : l : d : fey : f :
s : f : moo : d : suy : s


Andante moderato
p
l : d : l : d : f : moo : d : l : moo : d :
f : s : f : f : l : s : s : f : s : f :
d : s : d : l : d : d : d : d : l :
f : moo : f : f : d : moo : f : d : fey :
f
poco allarg.
ff
moo : d : s : suy : d : fey : moo : d
s : s : s : l : s : f
d : d : moo : d : d : l
d : moo : d ; f : s : f

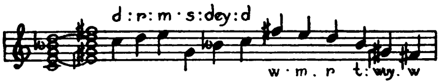

d : r · m : s : dey : d
r : d : too : s : ruy : r


Allegretto
d : d : r : m : ruy : r : s : m : d : d : r :
poco rall.
m : s : dey : too : d : dey : s : s : dey . too :
poco rall.
s : dey : s : r : ruy : ruy : s : s : m : d : r : ruy :
poco rall.
d : s : s : m : s : dey : s :
poco rall.
a tempo
r : d : r : m : ruy : r : s : ruy : m : (pp)
rall.
a tempo
s : too : s : s : d : too : too : m : s : dey
rall.
pp
a tempo
ruy : r : d : r : d : r : ruy : r : r : dey : r : d
rall.
pp
a tempo
d : dey : d : s : ruy : s : d : too : s : d
rall.
pp

EXAMPLE L

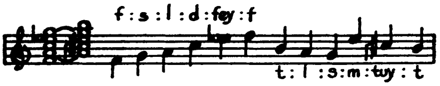

d : r : m : s : dey : d
s : f : moo : d : suy : s
Con moto
mp
s : moo d : s : r : s : d : moo :
s : suy : dey : s :
d : moo : m : moo : s :
.d : m : s : d : d :
s : moo : r : s : d : m : s :
s : s : s : s :
moo : r : d : d : dey : m : s : d : r : m :
d : s : moo : d : s : m : s : d :
dim.
rall.
s : moo : r : moo : r : d : s : d : dey : d
dey : d : s : s : dey : suy : s m
d : r : d : s : r : d : r : moo : r : moo : r : d : r : d
m : s : d : suy : dey : suy : dey : d : m : s : s : m : s : d

EXAMPLE M


d : r : m · s : dey : d
w · m . r t : wuy . w


p
w : wuy : r : r : r : t . r . m · r · m : d · dey . r : d :
m . w : s : s : s :
dey . t : dey : d : dey .
s : wuy : s : m : s :
poco f
cresc
m : r : m : r : w · m : r d s : t . d : s : t :
cresc.
w : d dey r : m : w : s : m : w :
poco f
t
m . r : w : m : r . r :
wuy
d · dey : r : dey : wuy :
f
fz
p
(f)
d : r : m : m : r : m : w : wuy : r
d : r m :
ffz
dey : w : w : m : s : m : s
m r · d : t : dey : d : dey : r : dey
s : wuy : s : m : s : t : d

EXAMPLE N

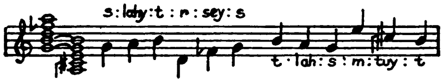

f : s : l : d : fey : f
t : l : s : m : tuy : t


Andante
p
l : t : l : s : f : s : l : t : l : s : f : s : l : t : l : s : f : s :
p
l : d : tuy : d :
l : t : l : s : f : s : l : t : l : s : f : s : l : t : l : d : f : f : fey : t :
cresc.
poco f
p
m : fey : m : fey : m : s : fey : fey : l : s :
cresc.
poco f
tuy : d : tuy : d : tuy : fey : d : d . f : m :
cresc.
poco f
p
l : f : s : l : s : l : d : tuy :
cresc.
poco f


p
dim.
rit. tempo
t:tuy : t : l : s : l : t:tuy : t : l : s : l : l : t : l : s : f : s : l : f :
s : tuy : m : fey : s : fey : m : fey :
m : t : tuy : d : tuy : d : tuy : d :
l : l : s : f : m : f : s : f :
espressivo
allarg. mf
f : s · f : s : l : s : f : s · f . s : l : l : s : f . s : l
fey . tuy : s : fey
d : t : fey . d
l : d : l : tuy : d : l : m : f


d : r : m : s : dey : d
m : rayu : d : l : muy : m
Con moto
p
cresc.
d:ray:m : m:rayu·d : d:ray:m : m:ray:d : s:m:ray:m:ray:d : s:m:r:m:r:d :
m : m : m : s : dey:s : s:s : s : s:s :
s : l : s : d : d:dey : d:d : dey : d:d :
d : l : d : m : m:s : d:m : s : d:m :
f
marcato
ray : d:rayu:m : m:rayu:d : m:rayu : s : m : m : d : d:l : m:d :
s : m : l : l:l : s:dey : l : l : m:muy : m:l :
dey : l : d : d:d : dey:d : m:rayu:d : d:rayu:m : l : d:rayu:d:rayu:m :
s : l : l : s:muy : s : d : d : l : muy:l : l:l :
allargando-maestoso
cresc.
poco f
cresc. ff
l : m:d : l : dey:dey : l:r:d:m : m:d:l : d
muy : m:l : muy : s:s : muy:dey : l:muy : s:m:rayu : m
l : d:rayu:d:rayu:m : rayu : ray:r : d:m:muy:d : dey : l : muy : s
d:l : l:l : rayu : s:m : m:d:m:l : s:l:rayu : d

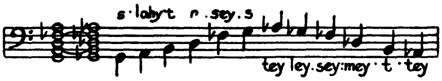
EXAMPLE P


s : lahy : t : r : sey : s
t · lah : s : m : tuy : t


Allegretto
p
mp (poco agitato)
mp
poco f
tranquillo
s : t : t : s : s : sey : sey : s : t : t : s : sey : sey : m : r :
p
mp (poco agitato)
mp
poco f
tranquillo
r : m : m : r : m : r : r : m : tuy : tuy : r : s : s : t : t :
p
mp (poco agitato)
mp
poco f
tranquillo
t : tuy : tuy : t : t : s : s : t : r : r : m : s : sey :
p
mp (poco agitato)
mp
poco f
tranquillo
s : m : s : s : m : m : m : sey : t : t : s :
espr. molto
p
piu p
p
rallent.
tuy : r : tuy : sey : m : sey : m : r : tuy : r : tuy : sey : m : m : m : r
p
piu p
p
rallent.
t : t : t : t : t : t : t : t : t
p
piu p
p
rallent.
m : sey : m : r : tuy : r : tuy : sey : m : sey : m : s : sey : s : sey
p
piu p
p
rallent.
s : s : s : s : s

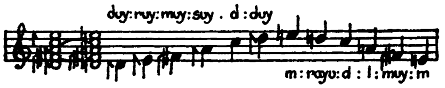
EXAMPLE Q


f : s : l : d : fey : f
rüh : düh : t : süh : fah : rüh
Adagio
p
f : s : l : d : fey : rüh : düh : t : süh : t : düh : rüh : süh : fah :
d : f : l : süh : süh : süh : rüh : süh : süh : t :
l : d : l : d : t : t : düh : düh : t : rüh : fah : fah : rüh : rüh :
f : f : f : fah : f : f : süh : süh : t : süh :
cresc.
f : fey : d : l : t : düh : rüh : s : f : f : fey : d : l
d : l : s : f : s : f : rüh : fah : süh : l : d : r : d : süh : l : s : f
d : d : l : d : süh : t : fey : fey : t : fey : t : d
flattened fey
l : s : f : d : fey : fey : f : fah : süh : fah : d : l : f : f : f : f

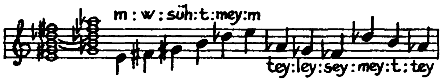

s · lahy · t r . sey . s
tey ley . sey : mey · t · tey


Moderato
mf
s : t : tey : t : mey : t : sey : sey : r : t :
r : sey : mey : sey : tey : sey · tey · t : t : s : sey
t : r · t : tey : sey : mey : mey : r : s : sey : r :
s · sey : r : sey : ley : sey : mey : t · mey : t · s : s : s .:
cresc
ff
dim. .... (p)
tey : t : mey : tey : ley : lahy : t : r : t : sey : r : r : t
mey : sey : mey : sey : sey · (tey) (lahy) : s : s : sey : r
t : tey : t · tey : t . s : lahy : s : (mey) r : t lahy : s
sey : mey : sey : r : t : r : t : sey : r : s

EXAMPLE S


duy : ruy : muy : suy . d : duy
m : rayu : d : l : muy : m


Larghetto
pp
m : m : l : d : mey : duy : suy : m : m : l : d muy : m : l : duy :
pp
d : d : muy : suy : duy : d : muy : suy :
pp
div.
l : l : m : m : d :
duy : duy : suy : muy : d : l : m : muy : d : muy : muy : d : d : suy :
pp
muy : d : duy : suy : muy :
p
poco f
rall.
div.
rayu
l : l : m : m : m : m : l d : d : muy : m : l : l
p
poco f
rall.
duy : duy : suy : muy : d : muy : muy : d : d : d : d : muy : suy : suy duy : d muy : m
p
(unis.)
poco f
rall.
m : m : l : d : muy : l : muy : d : l : l : m : m : d
p
poco f
rall.
duy : duy : suy : muy : d : muy : l : rayu : muy : d : duy : suy : muy muy d : d : l


m : w : süh:t:mey:m
tey:ley:sey:mey:t:tey


Poco maestoso
p
süh : süh : tey : süh : t : mey : t :
p
m : m : sey : m · m : sey : m :
p
t : t : t : t : süh : tey : süh :
p
m : mey : mey : m : m : mey : m ;
poco f
mf
mey : t : t : tey : mey : t : mey
poco f
mf
sey : mey:mey : mey : sey : m : sey
poco f
mf
tey : tey:süh : tey : tey : süh : tey
poco f
mf
mey : sey : m : sey : mey : m : mey

Erratum. In the fifth bar the alto should sing *rey* (c-minus), *not ray.*


f s l d:tey·f d dey
m
deyteydey seymeyttey·mey·|m


Allegretto tranquillo
(p)
f : fey : s . sey : m : mey · mey.m : sey : fey : s :
(p)
l : d : tey : t : tey : t : dey : l :
(p)
d : f : mey . m · sey : sey .m : mey · f : fey :
(p)
f : l t tey : t · tey : d : f :
sey : mey : sey : d : mey . fey : mey : m : sey · m : mey : d . l
t : tey : s : dey : l : t : tey : t : s : f
mey : sey : fey : s : sey : s : mey : m : sey : dey : f
suy : t : d : d : tey : t : tey : m : f

EXAMPLE W


f : s : l : d:fey : f : d:dey
dey:tey:ley:rey:mey:t:tey:mey:m
Misterioso
p
m : f : m : fey : m : sey :
piu p
mey : dey : l : l : dey :
mp
non troppo p
mf
dim.
pp
tey : dey : t : d : l
poco f
(Longa)