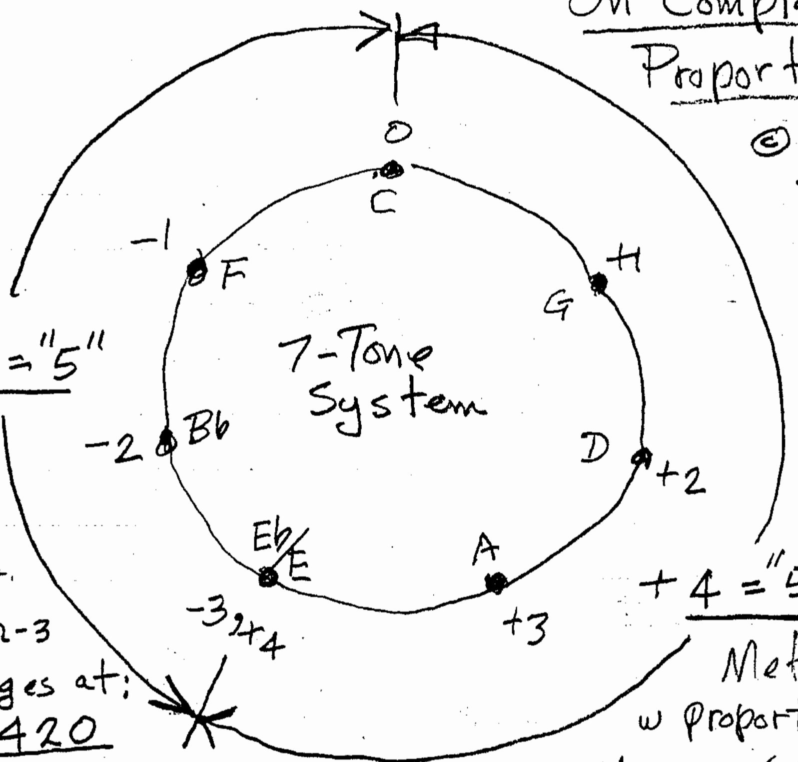
On Complementary Proportional Triads



Meta-Maliva
w. proportional Triad.

$P_n=(2P_{n-4})+P_{n-3}$

P_n/P_{n-1} converges at:

1.35320996420

$\log_2$ .436385705396

Example of recurrent sequence:

4.5, 6, 8, 11, 15, 20, 27, 37, 50, 67, 91, 124, 167, 225, 306, 415, 559, 756, 1027, 1389, 1874, 2539, 3443, 4652, 6287, 8521, 11538, 15591, 21095 etc.

MOS at: $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{7}{16}$, $\frac{10}{23}$, $\frac{17}{39}$, $\frac{24}{55}$, $\frac{31}{71}$ etc

Meta-meantone
w proportional triad

$H_n=2(H_{n-4}+H_{n-3})$

H_n/H_{n-1} converges at:

1.49453018048

$\log_2$ .579692031034

Example of Recurrent Sequence:

1, 2.5, 3, 5, 7, 11, 16, 24, 36, 54, 80, 120, 180, 268, 400, 600, 896, 1336, 2000, 2992, 4464, 6672, 9984, 14912, 22272, 33312, 49792, 74368, 111168, 166208, 248320, 371072, 554752, 829056, 1238784 etc

MOS at: $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{7}{12}$, $\frac{11}{19}$, $\frac{18}{31}$, $\frac{29}{50}$, $\frac{40}{69}$, $\frac{51}{88}$ etc

$\boxed{P_n = 2P_{n-4} + P_{n-3}}$ This is the recurrent sequence for 4-"5"-"6" arith. mean (−3 = 5)

©1995 by Erv Wilson, dated June 10, 1995

$(54 \times 2) + 72 = 180$

	27	36	48	135					
40.5	54	72	96	135	180	240	327	450	600
	1	2	3	4	1,	2,	3,	4,	

Converges → 1.353209964 20

$\log_2$ .436385705 396

Reference: Linear Tuning of 4-"5"-"6" arithmetic mean (−3 = 5) by Erv Wilson 1989

$$\boxed{P_n = 2P_{n-4} + P_{n-3}}$$

Ref: Chopi Scale from Mavila

```
——··——
RCL 1        RCL 4
 ×            ×
 2            2
 =            =
 +            +
RCL 2        RCL 1
 =            =
STO 1,       STO 4,
RCL 2         ÷
 ×           RCL 3
 2            =
 =           STO 5
+ RCL 3      ——··——
 =
STO 2,
RCL 3
~~3~~
 ×
 2
 =
 +
RCL 4
 =
STO 3,
```

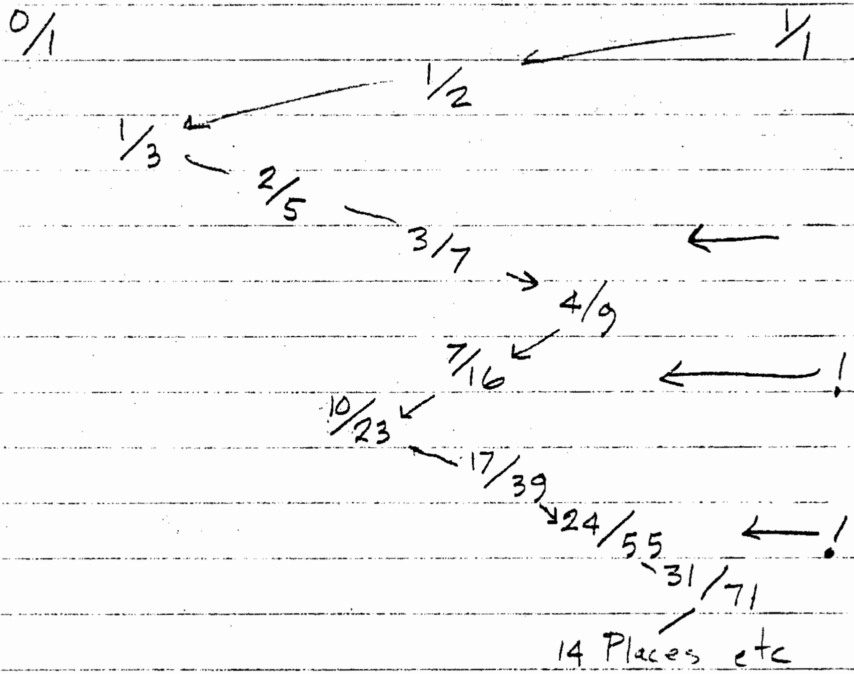
⇨ Also try: $^{3.25}$ $^{4.5}$ 5, 6, 8, 11, 15, 20, 27, 37, 50, 67, 91, 124, 167,
225, 306, 415, 559, 756, 1027, 1389, 1874,
2539, 3443, 4652, 6287, 8521, 11538, 15591, 21095

Meta-Mavila, ($P_n = 2P_{n-4} + P_{n-3}$) 1.35320996420

©1995 by Erv Wilson

$\log_2$ .436385705396

	1/n Pattern	
←	2	.291
→	3	.429
←	2	.325
→	3	.068
	14	.688
	1	.452
	2	.209
	4	.782
	1	.277
	3	.607



Note: see Linear Tuning of 4-'5"-"6" arithmetic mean (-3=5) by Erv Wilson 1989.

Sheet 2

The Recurrent Sequence for 4"-5"-"6" Arithmetic Mean (-3=5)

$P_n = 2P_{n-4} + P_{n-3}$, which converges on 1.35320996420

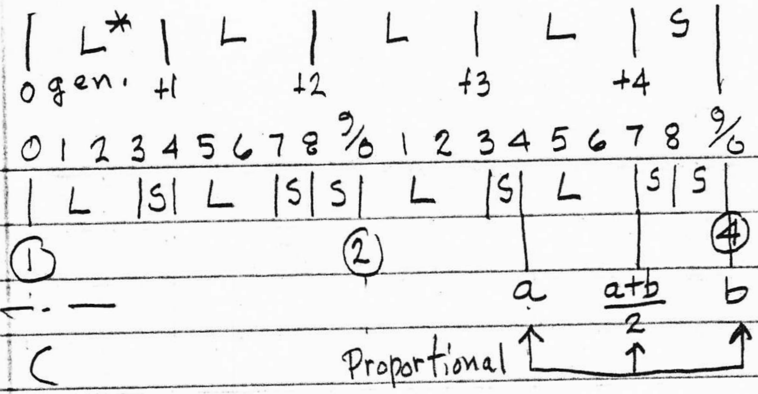
$\log_2 = .436385705396$

0 ⓪ 8, 256 (+23)
1 (+7) 67, 268
2 (+14) 559, 279.5
3 (-2) 4.5, 288 (+21) 4652, 290.75
4 (9) (+5) 37, 296
5 (+12) 306, 306
6 (+19) 2539, 317.375
7 (+3) 20, 320
8 (+10) 167, 334
9 (+17) 1389, 347.25
10 (+1) 11, 352 (+24)
11 (+8) 91, 364
12 (+15) 756, 378
13 (-1) 6, 384 (+22) 6287, 392.9375
14 (+6) 50, 400
15 (+13) 415, 415
16 (-3) 3.25, 416 (+20) 3443, 430.375
17 (13) (+4) 27, 432 ✓
18 (+11) 225, 225
19 (+18) 1874, 234.25
20 (+2) 15, 240
21 (+9) 124, 248
22 (+16) 1027, 256.75
23/0 ⓪ 8, 256 (+23)

Ref: Chopi scale from Mavila

(Continuing) Notes on Meta-Mavila

16 Jul 97 E.W.



LsLss

Please see P.7b

$$\text{gen.} * L = \left(\left(2 \cdot L + 4\right) / 2\right)^{\left(\frac{1}{4}\right)}$$

decimal approx. = 1.353209964 20...

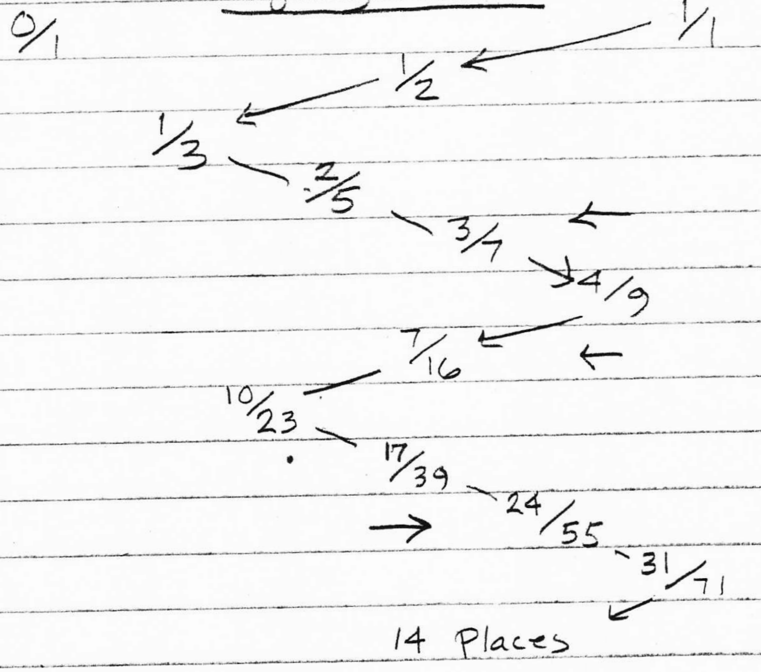
$\log_2$ = .436385705396...

```
— · —
(
(
2
×
RCL 1
+
4
)
÷
2
)
y^x
(
1
÷
4
)
=
STO 1
— · —
```

1/x pattern

	.436...
2	.291
3	.429
2	.325
3	.068
14	.688
1	.452
2	.209
4	.782
1	.277

Zig Zag Pattern

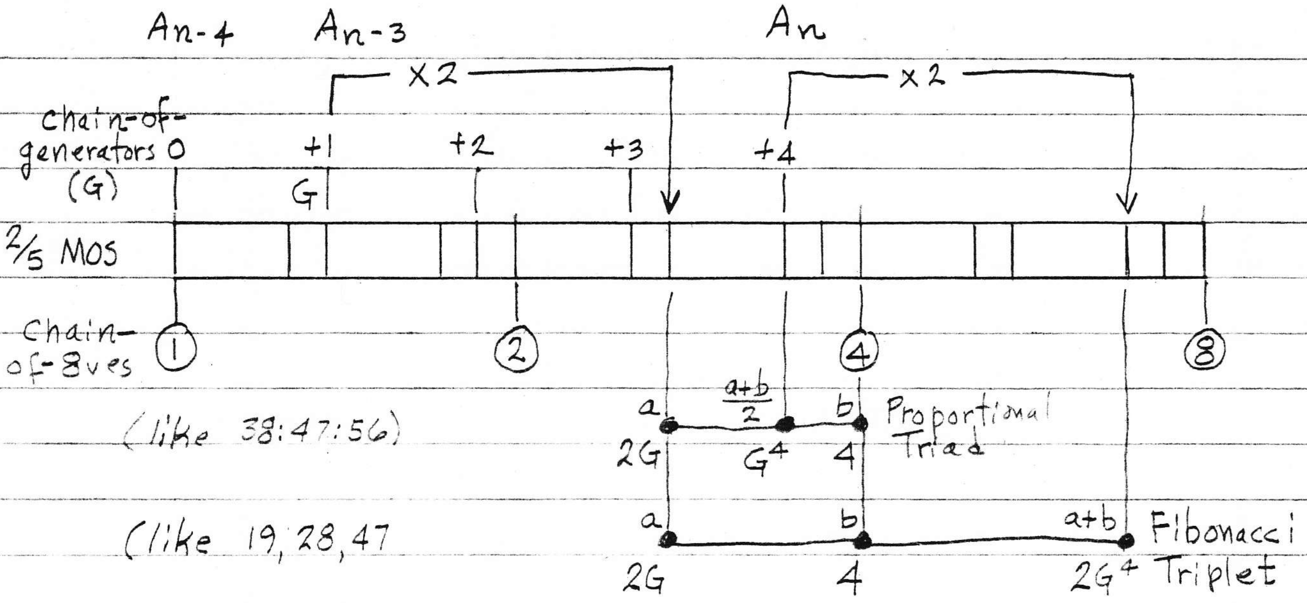


14 Places

Ref; Linear Tuning of 4-"5"-"6" arithmetic mean (-3=5), 1989 Erv Wilson

Notes on Meta-Mavila

18 Aug 97 - E.W.,
P. 7b



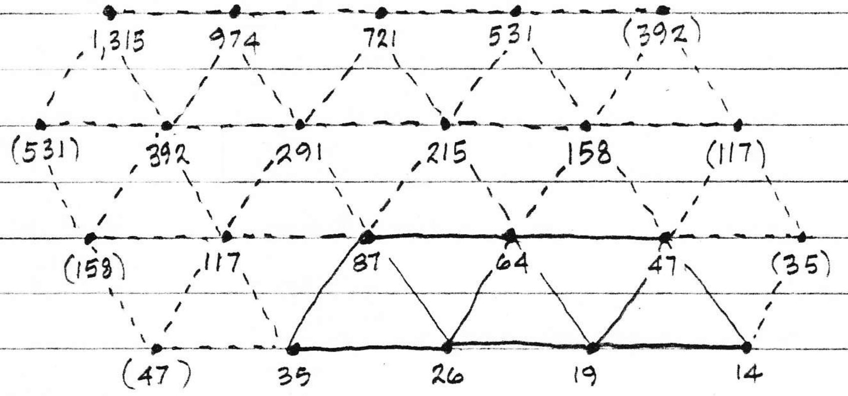
Recurrence;

$$(4A_{n-4} + 2A_{n-3})/2 = A_n$$
$$= 2A_{n-4} + A_{n-3} = A_n$$

$$\Rightarrow \quad G = ((4+2G)/2)^{(\frac{1}{4})}$$
$$= (2+G)^{(\frac{1}{4})}$$
$$= \underline{1.35320996420\cdots}$$
$$\text{Log}_2 = \underline{.436385705396\cdots}$$

Example

Seed; 14 19 26 35 47 64 87 117 158 215 291 392 531 721 974 1,315
1,783 2,416 3,263 4,413 5,982 8,095 10,939 14,808 20,023 27,129



16-Tone Scale where; $2A_{n-4} + A_{n-3} = A_n$ (Meta-Mavila)

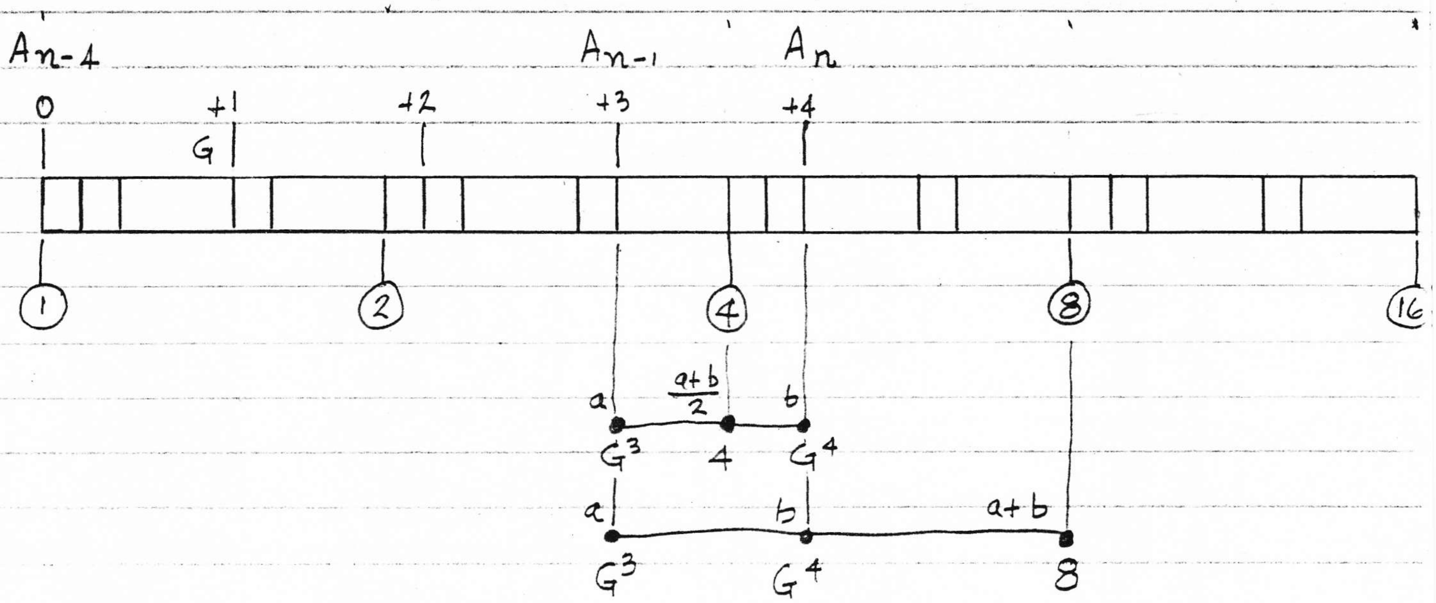
See also; <u>On Complementary Proportional Triads</u>, 1995 by Erv Wilson

Notes on Meta-Mavila

19 Aug 97 - E.W.

P.7c

This is an interesting try, - see P.7b for an elegant approach, and fundamental.



$$8A_{n-4} - A_{n-1} = A_n$$

$\cancel{(8-G^4)^{(\frac{1}{3})}}$ No compute

$\Rightarrow G = (8-G^3)^{(\frac{1}{4})}$ OK

$= 1.47796724301\cdots$

$\text{Log}_2 = .563614294605\cdots$

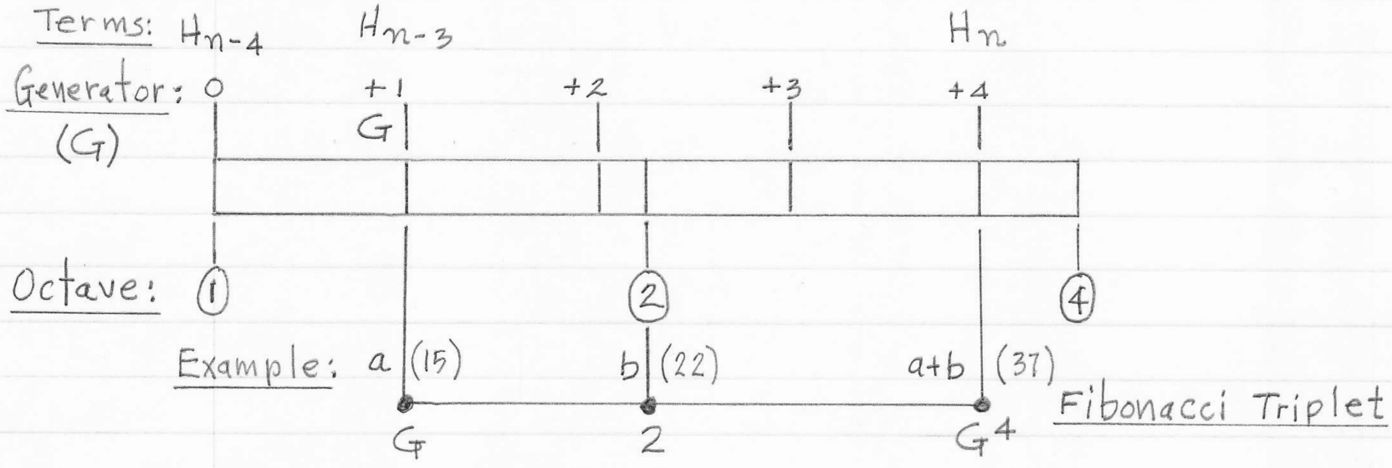
32 47 70 104 152 224 336 496 720 1,072

32 47 70 104 152 224 (31) 31 45 67

$G = (2 + G)^{(1/4)}$, Meta-Mavila

10·Oct·97 - E.W.

(P. 7d)



Recurrence Relation:

$$2H_{n-4} + H_{n-3} = H_n$$

(1, 1, 1, 1, 3, 3, 3, 5, 9, 9, 11, 19, 27, 29, 41)

- NLIS -

G Paraphrase:

$\Rightarrow$ $G = (2 + G)^{(1/4)}$

$= 1.35320996420\cdots$

$\log_2 = .436385705396$

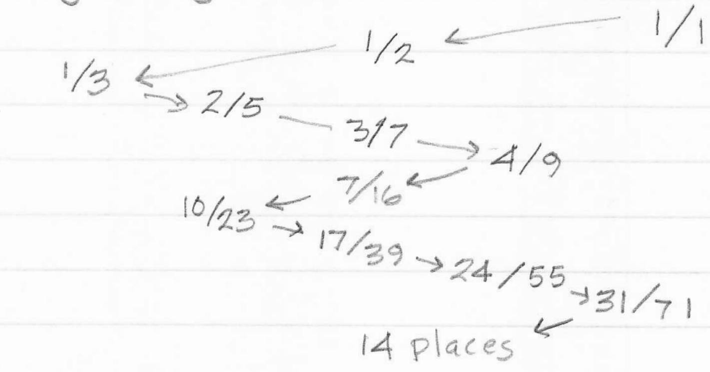
Re-Seed Example:

6, 8, 11, 15, 20, 27, 37, 50, 67, 91, 124, 167, 225, 306, 415, 559, 756, etc.

1/N Pattern

		.43638···	0/1
←	2	.291	
→	3	.429	
←	2	.325	
→	3	.068	
	14	.688	
	1	.452	
	2	.290	
	4	.782	
	1	.277	

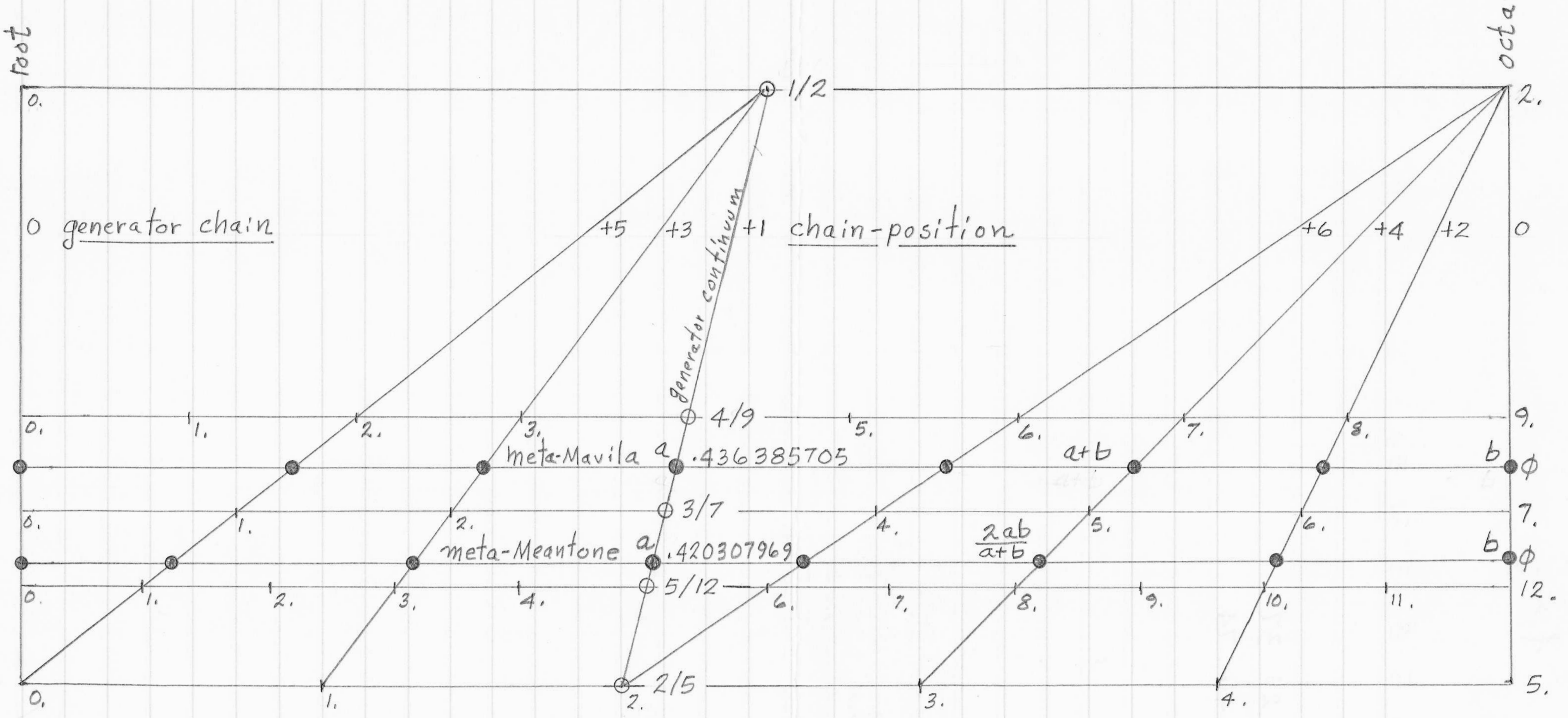
Zig-Zag Pattern



Ref. Linear Tuning of 4-"5"-"6" Arithmetic Mean (-3 = 5), 1989, Erv Wilson

16OCT97-E.W.

Enantiodromia of Meta-Meantone into Meta-Mavila



Proportional "4, 5, 6" Sequence

dated
Oct 16, 1993
Erv Wilson

Meta-Meantone

© 1993 by Erv Wilson
ref John Harrison's scale

0	+1	+2	+3	+4
2	3	4.5	6,75	10,125

$\frac{11}{7}$ $\frac{16}{11}$ $\frac{24}{16}$ $\frac{36}{24}$ $\frac{54}{36}$ $\frac{80}{54}$ $\frac{120}{80}$ $\frac{180}{120}$

$\frac{27}{18}$ $\frac{40}{27}$ $\frac{60}{40}$ $\frac{90}{60}$ $\frac{134}{90}$ $\frac{200}{134}$ $\frac{300}{200}$

$H_n = 2(H_{n-4} + H_{n-3})$, Recurrent Sequence

	H_{n-4}	H_{n-3}			H_n						(67)		÷4			
1.5	1	2.5	3	5	7	11	16	24	36	54	80	120	180	268	400	600
			(1)	(2)			(1,)			Proportional Triad(s)						
				(2)	(3)			(2,)								
					(3)	(4)			(3,)							
						(4)	(1,)			(4,)						

H_n / H_{n-1} Converges on 1,494530180 48

(See also proportional "4,5,6" 1992)

To get a quick convergence

```
(            (            (            (
RCL 1        RCL 3        RCL 1        RCL 3
+            +            +            +
RCL 2        RCL 4        RCL 2        RCL 4
)            )            )            )
×            ×            ×            ×
2            2            2            2
=            =            =            =
STO 1,       STO 3,       STO 1,       STO 3,
(            (            [ ÷          [ ÷
RCL 2        RCL 4          RCL 4        RCL 2,
+            +              =            =
RCL 3        RCL 1,         STO 5 ]      STO 7 ]
)            )            (            (
×            ×            RCL 2        RCL 4
2            2            +            +
=            =            RCL 3        RCL 1,
STO 2,       STO 4,       )            )
             ÷            ×            ×
             RCL 3,       2            2
             =            =            =
                          STO 2,       STO 4,
                  [ ÷ RCL 1, = STO 6 ]  [ ÷ RCL 3, = STO 8 ]
```

This stores the ratios for viewing at 5,6,7,8

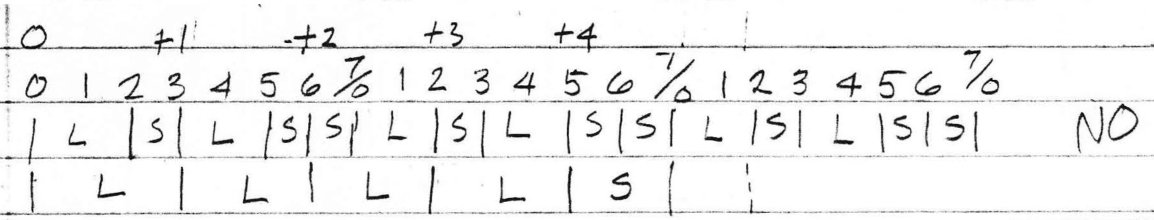
rev Feb 20, 1994 E.W.

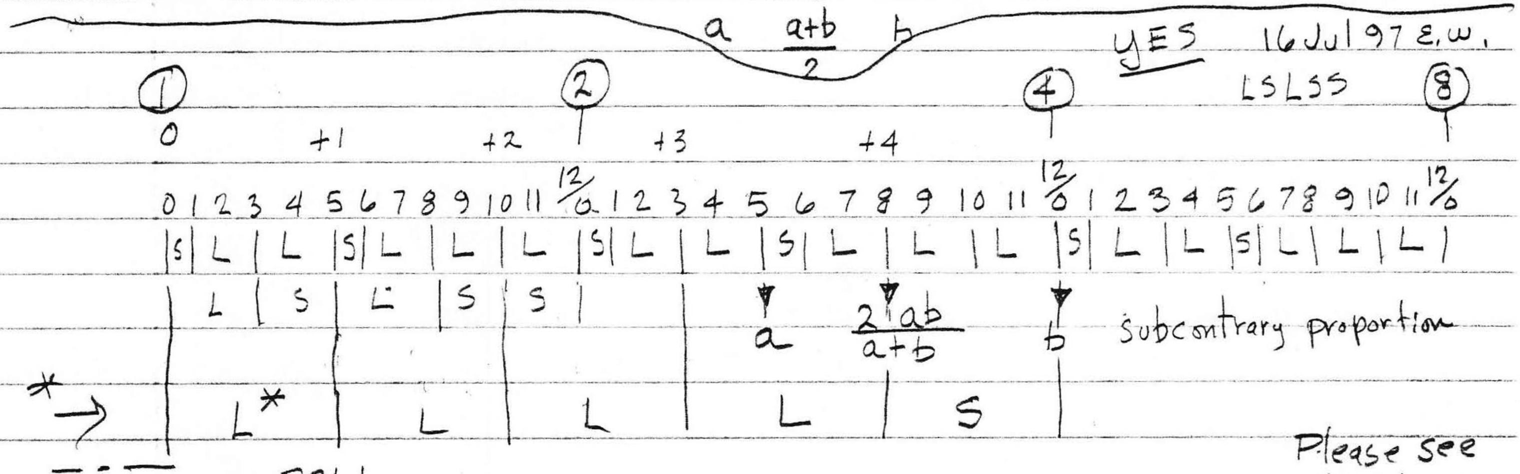
Try this: 2 3 4.5 6.5 10 15 22 33 50 74 110 166 248 368

(Continuing)

Notes on Meta-Meantone

16Jul97 E.W
P.6a





```
              RCL1
 C            +4       1/x pattern
 C             )        .420...
 2x            )      ← 2 .379
 C            y^x     → 2 .637
RCL 1.         (      ← 1 .569
 ×             1      → 1 .755
 2             ÷      ← 1 .323
 ×             4      → 3 .088
 4             )        11.320 %1
 )             =        3.125
 )            STO 1     8.096
 ÷            iterate
 (
 2
 ×
RCL 1.
```

Please see P.6b ← this ⇦

$$*L = \sqrt[4]{\frac{2(L\cdot 2\cdot 4)}{2L+4}}$$

$$((2(L\cdot 2\cdot 4))/(2L+4))^{(\frac{1}{4})}$$

decimal approx = 1.338213318975...

$\log_2$ = .420307968965...

~~ZigZag pattern~~

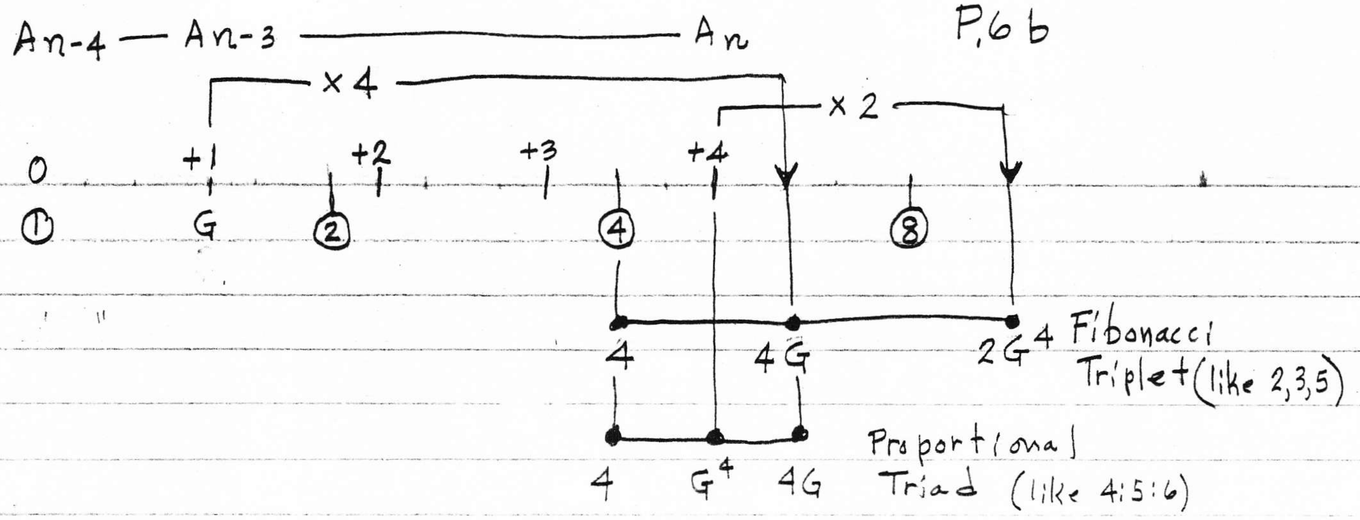
1/1
1/2
1/3
2/5
3/7
5/12
8/19
13/31
21/50
29/69
37/88
11 places

Ref; Linear Tuning of the 4"5""6" Arithmetic Mean (+4=5), 1989, E.Wilson

compare with Harrison's π tuning.

Notes on Meta-Meantone

17 Aug 97 - E.W.
P.6b



Recurrence;

$$(4A_{n-4} + 4A_{n-3})/2 = A_n$$
$$= 2A_{n-4} + 2A_{n-3} = A_n$$

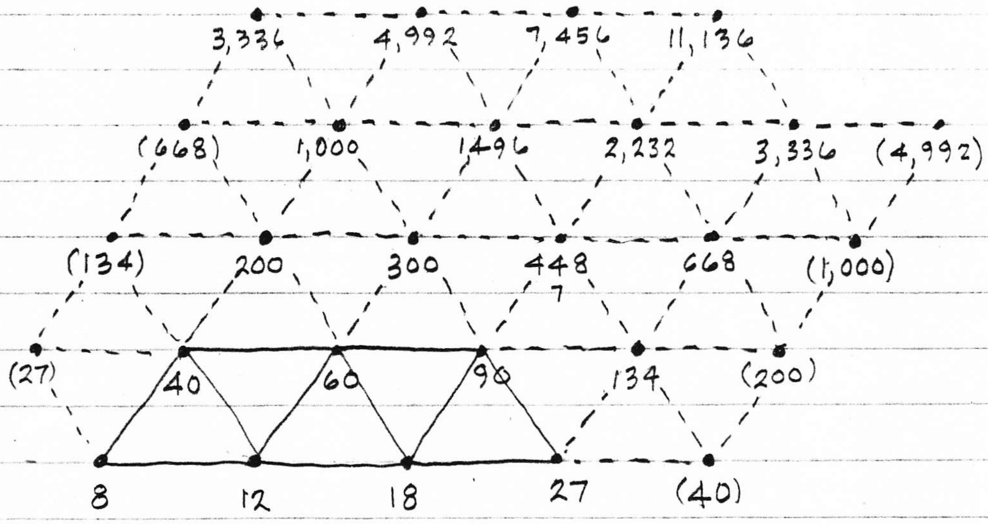
$$G = ((4+4G)/2)^{(\frac{1}{4})}$$
$$\Rightarrow = (2+2G)^{(\frac{1}{4})}$$
$$= 1.49453018048$$
$$\log_2 = .579692031034$$

example

Seed: 8 12 18 27 40 60 90 134 200 300 448 668 1,000 1,496

(7)

2,232 3,336 4,992 7,456 11,136

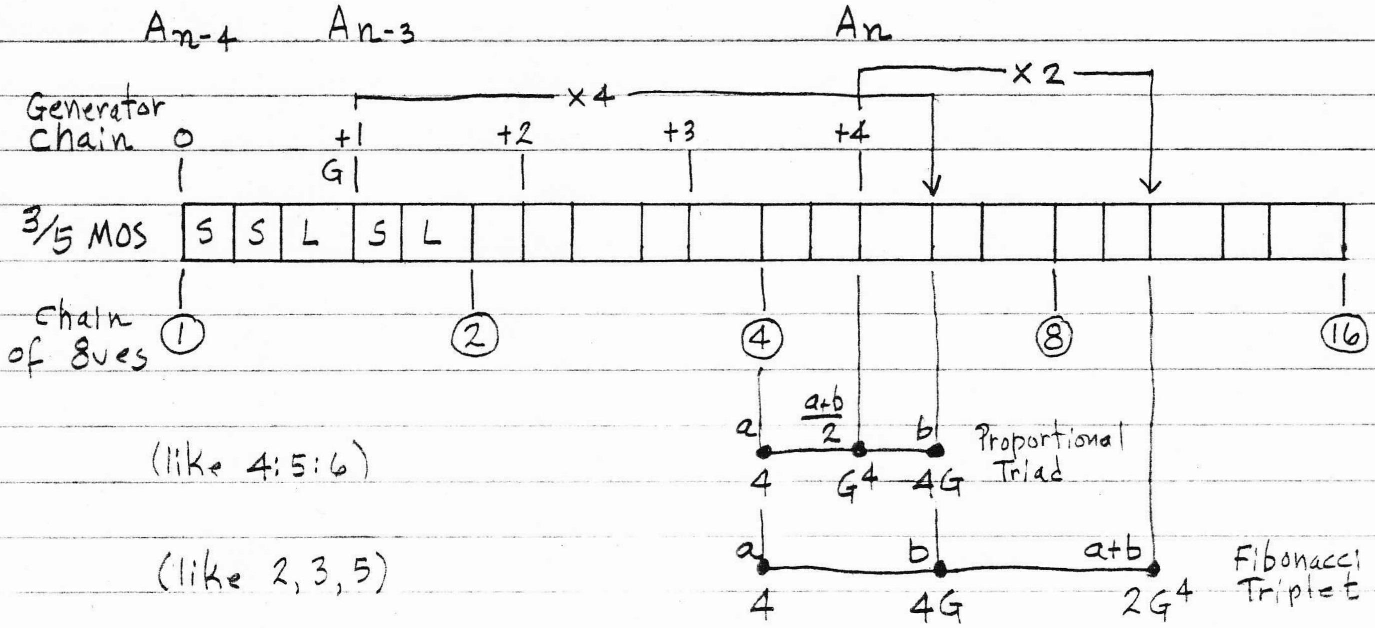


19-tone Scale where; $2A_{n-4} + 2A_{n-3} = A_n$ (Meta-Meantone)

Notes on Meta-Meantone

18 Aug 97 - E.W.
p. 6c

This sheet clarifies p.6b



Recurrence:

$$(4A_{n-4}+4A_{n-3})/2 = A_n$$

$$(=)\ \underline{2A_{n-4}}+\underline{2A_{n-3}}=\underline{A_n}$$

$$\Rightarrow \quad G = ((4+4G)/2)^{\left(\frac{1}{4}\right)}$$

$$= (2+2G)^{\left(\frac{1}{4}\right)}$$

$$= \underline{1.49453018048}$$

$$\log_2 = .579692031034$$

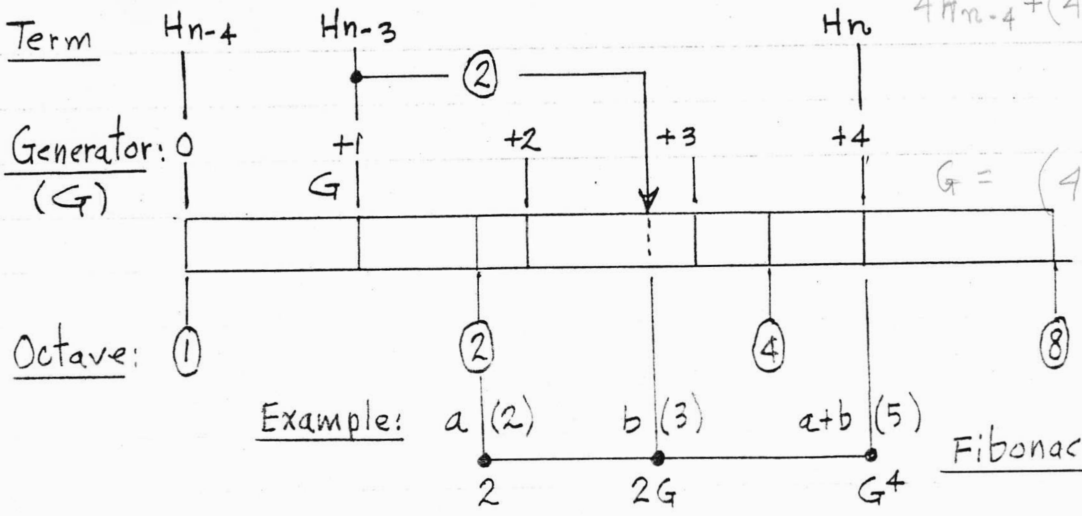
	A_{n-4}	A_{n-3}	A_{n-2}	A_{n-1}	A_n								
Example Seed:	8	12	18	27	40	60	90	134	200	300	448	668	etc

Please see p. 6b

$G = (2 + 2G)^{(\frac{1}{4})}$, Meta-Meantone

10·OCT·97-E.W.

$4H_{n-4} + (4H_{n-4} - (2H_{n-3}) = H_n$
(p 6d)



Fibonacci Triplet

Save

$G = (4 + (4 - (2G)))^{(\frac{1}{4})}$

$= 1.49595350624$

$\log_2 = .581065337419$

1	.720
1	.387
2	.583
1	.712
1	.403
2	.478
2	.089
11	.180

Recurrence Relation:

$2H_{n-4} + 2H_{n-3} = H_n$

G Paraphrase:

$\Rightarrow G = (2 + 2G)^{(1/4)}$

$= \underline{1.49453018048}$

$\log_2 = .57969203034$

Example re-seed:

8 12 18 27 40 60 90 134 200 300 448 668 1000 1496 2232
3336 4992 7456 11136 etc, *

1/N Pattern

		.57969... 0/1
←	1	.725
→	1	.379
←	2	.637
→	1	.569
←	1	.755
→	1	.323
←	3	.088
	11	.320
	3	.123

Zig-Zag Pattern

1/2 ← 1/1

1/2 → 2/3

3/5 ← 2/3

4/7 ← 3/5

4/7 → 7/12

11/19 ← 7/12

11/19 → 18/31

29/50 ← 18/31

40/69 ← 29/50

! → 51/88 ← 40/69

51/88 → 11 places

18/31 — 25/43

25/43 — 43/74 ← !

43/74 — 61/105

61/105 — 11 places

Ref: Linear Tuning of 4-"5"-"6" Arithmetic Mean (+4 = "5"), 1989, Erv Wilson

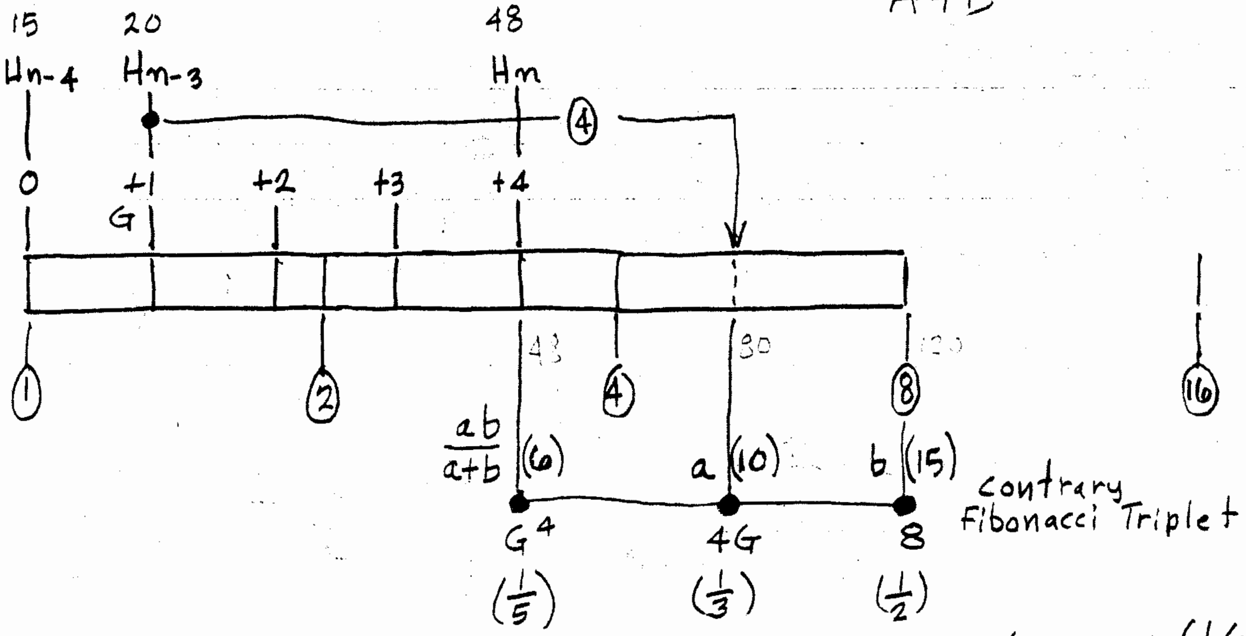
* 1, 1, 1, 1, 4, 4, 4, 10, 16, 16, 28, 52, 64, 88, 160, etc NLIS.

135	180	240	320	432	579.2
27	108	144	192	259.2	
405	540	720	960	1080 1296	
3	4				

Study on Contrary Meta-Meantone

$$\frac{2AB}{A+B}$$

24 Oct 97-E.W.



$$(8H_{n-4} \times 4H_{n-3})/(8H_{n-4}+4H_{n-3}) = H_n$$

is not an integer sequence

$$G = ((8 \cdot 4G)/(8+4G))^{(1/4)}$$

$$\left(\frac{2/G}{1.49..}\right) = 1.33821318975$$

$$\log_2 = .420307968965$$

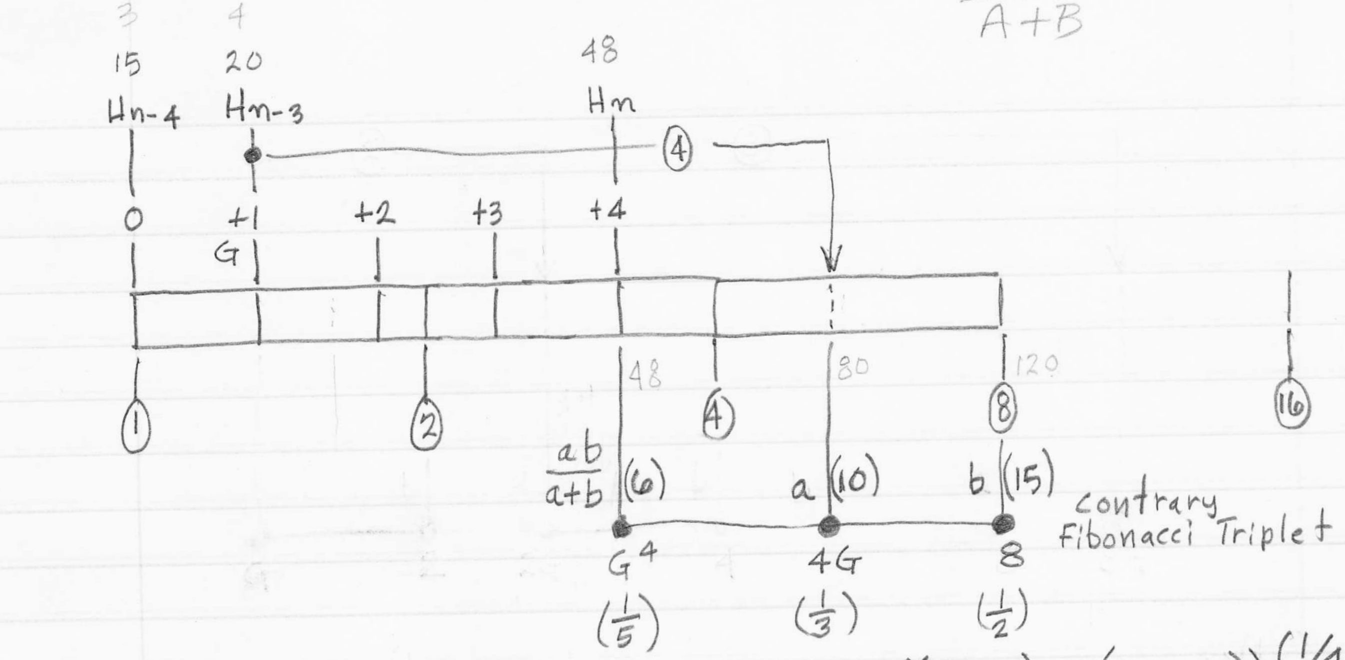
Ref: $G = \underline{(2+2G)^{(1/4)}}$ Meta-Meantone 10 OCT 97-E.W.

135 180 240 320 432 579.2
27 108 144 192 259.2
405 540 720 960 1280
1296

Study on Contrary Meta-Meantone

24 Oct 97 - E.W.

$\frac{2AB}{A+B}$



$(8H_{n-4} \times 4H_{n-3})/(8H_{n-4}+4H_{n-3}) = H_n$
is not an integer sequence

$G = ((8\cdot 4G)/(8+4G))^{(1/4)}$

$= 1.338213189\,75$
$\log_2 = .420307968965$

Ref: $G = \underline{(2+2G)^{(1/4)}}$ Meta-Meantone 10 OCT 97 - E.W.

$\boxed{-3 = 19/16}$

dated March 2, 1995

(5,106/12,276) ref only
actually 5105.94357515!

27	36	48	64	85.5	114	152	203
a	b	c	d	a_1	b_1	c_1	d_1
STO1	2	3	4				

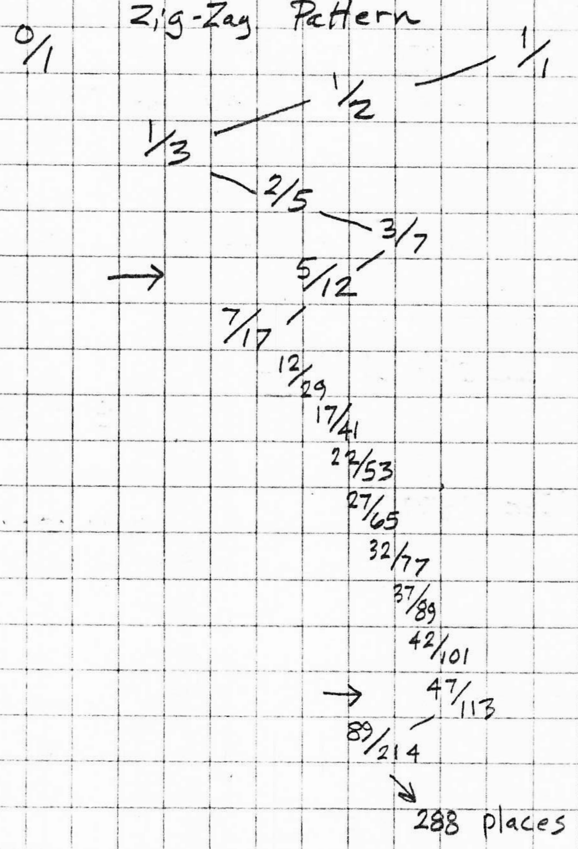
$$T_n = \frac{T_{n-4} + 4(T_{n-3})}{2}$$

```
(
RCL 1
+
(
RCL 2
x
4
)
)
÷
2
=
STO 1,
(
RCL 2
+
(
RCL 3
x
4
)
)
÷
2
=
STO 2,
```

```
(
RCL 3
+
(
RCL 4
x
4
)
)
÷
2
=
STO3,
(
RCL 4
+
(
RCL 1,
x
4
)
)
÷
2
=
STO 4,
÷
RCL3,
=
STO 5
```

Converges on 1.33415744713
Log_2 = .415928932482

1/x	
← 2	.404
→ 2	.473
← 2	.111
→ 8	.996
← 1	.003
! 288	.029



(Ref only
PA = 5095/12276
This = 5106/12276
ET 5/12 = 5115/12276)

This sequence is suggested by Tom Stones (19-16=3) version of the minor triad.

3 - 16 - 19 Fibonacci Triplet

27Jul98-EW

3, 16, 19 a, b, (a+b) Fibonacci Triplet

$6, 19, 32$ $a, (\frac{a+b}{2}), b$ Proportional Triad

$\underbrace{\quad\quad}_{13}\ \underbrace{\quad\quad}_{13}$

The difference tones are identical, at 13.
This gives us 2 new Fibonacci Triplets:

6, 13, 19 → Fibonacci Triplet

12 19 26 → Proportional Triad

$\underbrace{\quad\quad}_{7}\ \underbrace{\quad\quad}_{7}$ → Difference tones at 7

and

19 13 32 → Fibonacci Triplet

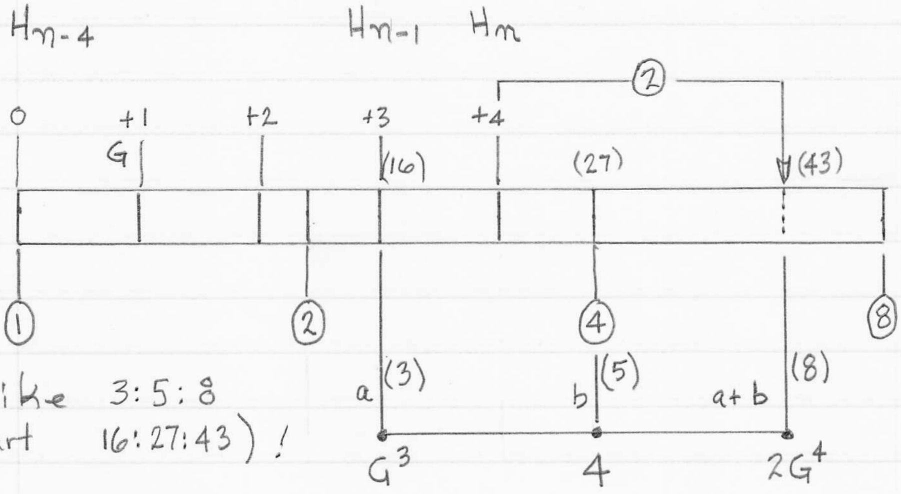
19 16 13 Proportional Triad

$\underbrace{\quad\quad}_{3}\ \underbrace{\quad\quad}_{3}$ Difference Tones at 3

$G=((4+G^3)/2)^{(1/4)}$ meantone

9 Mar 98 · EW

Sheet 1 of 2



Like 3:5:8
(start 16:27:43) !

epimore note:

$\frac{4}{3}\ e\ \frac{43}{32} = \frac{129}{128}$!

Recurrence relation:

$$(4\cdot \underbrace{H_{n-4}} + \underbrace{H_{n-1}})/2 = \underbrace{H_n}$$

example: 27 36 48 64 86 115 153.5

$G=((4+G^3)/2)^{(1/4)}$
$= 1.33693994609$
$\text{Log}_2 : .418934662571$
$2/G = 1.49595350625$

1/x Pattern

		.418934···	0/1
←	2	.387	
→	2	.583	
←	1	.712	
→	1	.403	
←	2	.478	
→	2	.089	
	11	.180	
	5	.544	
	1	.836	→

Zig-Zag Pattern

1/1
1/2
1/3
2/5
3/7
5/12
8/19
13/31
18/43
31/74
44/105
11 places

Ref. Straight-Line Patterns and Proportional Triads 1992, Ervin M. Wilson

Meta-1/5 comma meantone (3)

closer to 31 equal

(1.4957713478 2)

7131.00138573

(7131/12276!)

(A+B)×3=F

A 8 B 96 F

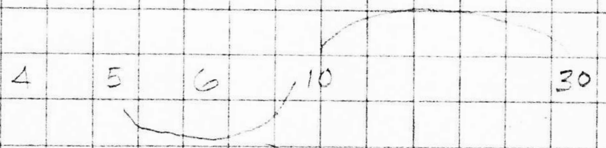
3	5	7	11	16	24	36	54	81	120	180	270	405	603
				(1)	(2)	(3)	(4)	(5)	1,	2,			
									30	45	135		

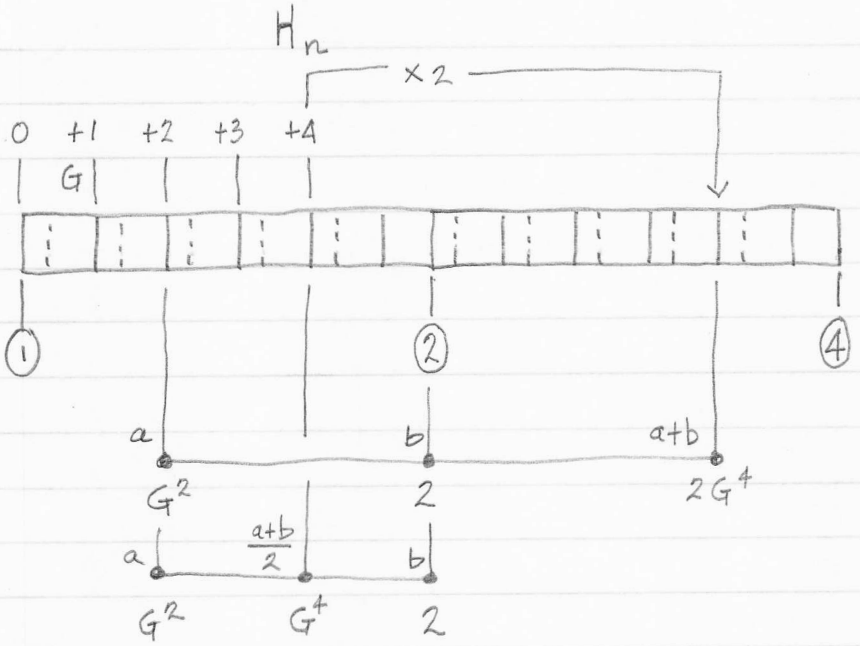
$$J_n = 3(J_{n-4} + J_{n-5})$$

This was done about Oct 16, 1993, Erv Wilson

```
— -- —
(
RCL 1
+
RCL 2
)
×
3
=
STO 1,
(
RCL 2
+
RCL 3
)
×
3
=
STO 2,
(
RCL 3
+
RCL 4
)
×
3
=
STO 3,
```

```
(
RCL 4
+
RCL 5
)
×
3
=
STO 4,
(
RCL5
+
RCL 1,
)
×
3
=
STO 5,
÷
RCL 4,
=
— -- —
```





This cannot be a prime
reccurrence relation

$G = ((2 + G^2)/2)^{(1/4)}$

$= \underline{1.13171392428}$

$\log_2$ .178509318434

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ref $G = ((2 + G)/2)^{(1/2)}$

$= 1.56155281280$

for comparison (G^2) only

$^1/_x \times 2 = 1.28077640641$

See 1992