SUPPLEMENT: THE GENERALIZED KEYBOARD

The literature on multiple division abounds in illustrations and descriptions of instruments producing more than twelve sounds per octave. There are the famous historical examples: Vicentino's 31-tone Archicembalo; the 19-tone "Clavicembalo Universale" described by Praetorius; the special instruments built under the instructions of Zarlino and later of Mersenne. There are the numerous efforts of 19th century theorists to build the perfect instrument for just intonation. The methods of tuning these instruments varied, as did the means of tone production and the design for the keyboards. The 20th century has added its own vast number of instruments with more than 12 tones to the octave. There is the collection of quarter-tone instruments dating from the first three decades of this century. There is Harry Partch's arsenal of stringed and percussion instruments.

Many of these instruments can be found in various museums around the world. Detailed studies of their operating principles and the state of their preservation would be extremely useful. It is beyond the scope of this paper to make such a detailed study of the instruments of multiple division. However, because of its direct bearing on the central problem of reducing the difficulty of performing music involving many tones to the octave, a brief consideration of a small group of instruments based on a single principle of keyboard design is here appended.

The principle of the generalized keyboard is essentially simple. The keys must be arranged in such an order that for any given sonority, regardless of key, a single basic position for the hands is employed. Attempts have been made, most notably by Paul von Jankó around 1900, to incorporate this principle into pianos for everyday use. The widespread technical mastery of the regular piano keyboard has rendered such an instrument unnecessary. However, the complexities of multiple division would probably make the luxury of a keyboard as inefficient as that of the piano impossible.

In order to achieve a generalized keyboard three fundamental principles must be observed, none of which is applied to our present-day instruments. (1) The keys must be arranged in an absolutely regular pattern from bank to bank (they would probably best be of uniform shape as well). (2) The succession of keys along any bank or in any given direction must involve always the same interval. (3) There must be a substantial number of extra keys in order that any pitch in the system can be sounded by any part of a hand configuration.

Three keyboards of actual instruments will be examined. In each, the interval between adjacent members of a bank of keys is a whole-tone of the system in use. Example 61 is a diagram of the keyboard of a 19-tone electric organ built by Tillman Schafer and James Fiehl during the late 1940's. The arrangement of keys is reminiscent of the typewriter keyboard,

a keyboard which has also fascinated Partch.[1] There are five parallel rows of keys, separated from one another by a single unit of the temperament, 1/19. Consecutive keys in a given row are separated by a whole-tone or 3/19. The arrangement of black and white keys is based on the hexads of Yasser's supra-diatonic scale, 12 of the 19 tones in each octave being white. The 7 black keys form a scale equivalent to our present diatonic scale. The basic hand configuration for the diatonic scale is the same, no matter on what tone it is begun. It will be further observed that this basic pattern remains essentially unchanged on the 31- and 53-tone instruments to be examined later.

Assuming the black keys in Example 61 to represent the scale of D major (for purposes of illustration--in actuality they do not), the key at the upper left-hand corner is D. The scale is formed by proceeding horizontally for whole-tones and diagonally for semitones. It should be pointed out that the two different diagonal directions involve different intervals. Consecutive tones along the diagonal from the top left are diatonic semitones (2/19) from one another, while consecutive tones along the diagonal from the bottom left are a full 4/19 from one another.

---

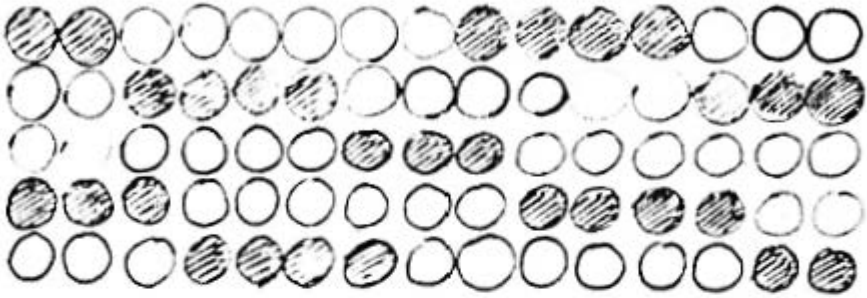
[1] Partch has temporarily abandoned experiments involving the use of an instrument with a keyboard resembling that of a typewriter. In Genesis of a Music, however, he compares favorably the efficiency of the keyboard of the typewriter (the instrument which any stenographer can learn to play in six weeks) with that of the piano.

Example 61 shows the full depth of the keyboard (five rows) but only a cross-section of its full length, which is about four octaves. All of the tones could be represented by three banks (rows) of keys, but the extra two rows are essential to the generalization of the keyboard, as they permit any standard combination of sounds to be produced in any key by a single configuration of the hand.

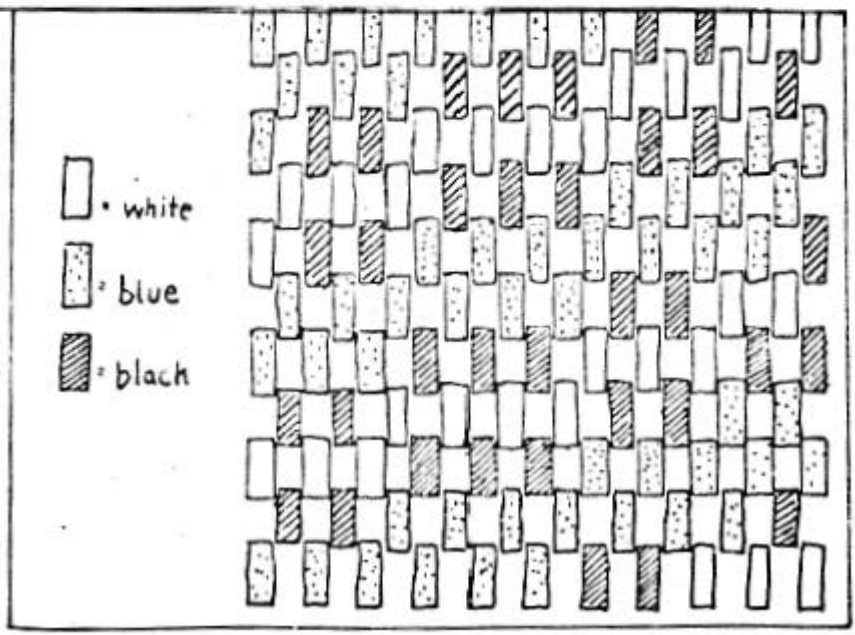
Example 62 shows a similar cross-section of one of the two identical manuals of Fokker's 31-tone organ in Haarlem, built during the middle 1940's. This instrument has received the most widespread use among the modern multiple division instruments. A special console with traditional 12-tone keyboard is also attached to the pipes of the instrument for the rendering in meantone temperament of ancient music. The main console, shown in Example 62, contains 11 banks of keys on each manual. The white keys form the diatonic scale of C major, just as they do on the piano. It will be noted that Fokker's keyboard differs from Schafer's in that the ascending diatonic semitone uses the upward instead of the downward diagonal. The result is a kind of mirror inversion of the pattern Schafer uses.

Unlike Schafer's keyboard, in which the keys are placed on parallel planes perpendicular to the rows of keys, Fokker's keyboard places the keys in alternate rows diagonally in such a way that a vertical line of keys (that is to say a line of keys proceeding perpendicular to the rows or banks)

Example 61: Schafer and Plehl's 19-Tone Keyboard--Cross-Section



Example 62: Fokker's 31-Tone Keyboard--Cross-Section

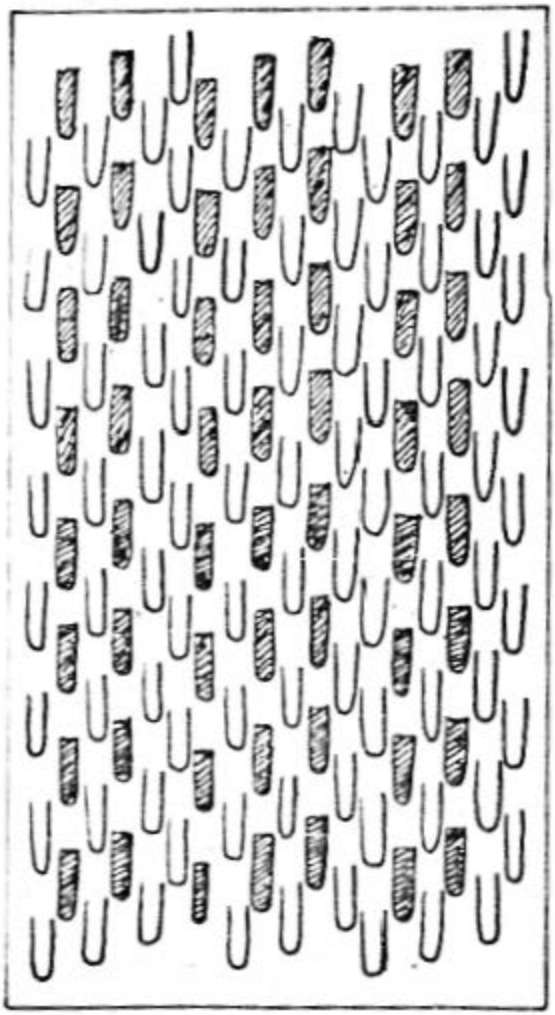


uses alternate rows only. Such a vertical line gives the smallest interval, 1/31. The rising diagonal yields the diatonic semitone, 3/31, while the falling diagonal yields the chromatic semitone, 2/31. The interval between consecutive tones on a given bank is a whole-tone, 5/31.

The black keys represent each sharp and flat currently obtainable on the black keys of the piano keyboard. There are ten black keys in each octave, since there are separate keys for F♯ and Gb, etc. The fourteen keys in each octave which are neither black nor white are blue; they represent the tones which lie between the tones of the present piano. The 11 banks of keys offer ample alternative fingering possibilities for complete generalization. Five banks would be adequate for the single rendering of each tone.

Bosanquet's keyboard, shown in Example 63, some 70 years earlier than the other two, is the most complex of the three, but the principle is much the same. Where the other two contain level banks of keys in which the tones are a whole-step apart, Bosanquet's whole-tones form a slanting row. This is the result of recessing each fifth slightly (the whole-tone has twice the recess of the fifth). After twelve fifths the recess is sufficient that a new bank of keys is begun. As the fifths are almost exactly pure in 53-tone temperament, the next higher bank sounds one comma higher than the first bank or, more precisely, 1/53 higher. The result is rather like the Fokker system slightly tilted

Example 63: Bosanquet's 53-Tone Keyboard--Cross-Section



and inverted (as in Schafer's instrument, the diatonic semitone is achieved by a downward diagonal). The downward diagonal yields the diatonic semitone (4/53 in Bosanquet's Pythagorean system), while the upward diagonal yields the larger chromatic semitone (5/53). The row of whole-tones (9/53) is itself a gently sloping diagonal.

Counting every sloping row of semitones as a separate bank, there are about 16 banks of keys at most parts of the instrument (9 would suffice for every pitch), providing for a high level of generalization. The sloping character of the whole-tone rows is particularly efficient for diatonic scale-production, for in completing such a scale the hand remains at a constant distance from the edge of the keyboard nearest the player.[2]

Example 63 shows a cross-section of Bosanquet's keyboard. The seven-to-five ratio of white keys to black is retained; comma alterations do not change the color of the keys. Bosanquet appears to have seven sets of twelve keys to the octave, 84 in all. By an effective arrangement, however, Bosanquet has an even greater number of keys per octave in his middle registers.

It should be added that on Bosanquet's instrument (and on Fokker's as well) as the keys are recessed from the player

---

[2] In contrast, the ascending diatonic scale on Fokker's instrument brings the hands away from the player (and eventually off the keyboard unless the player leaps to a new position), while a similar scale on Schafer's instrument brings the hands toward the player.

they are elevated. In the case of Bosanquet's instrument this means that no two keys within an octave are at precisely the same elevation.

The three examples shown in this short supplement offer, it seems to me, the best basis for a starting point in the performance of tempered music using more than 12 tones per octave. It would be quite possible to use the more complex instruments, such as Bosanquet's or Fokker's, for the simpler musical systems such as Schafer's, although the reverse, of course, would be impossible. A keyboard of about the complexity of Fokker's could be tuned to yield a more complex temperament, such as 53-tones to the octave, without any essential changes in the hand positions for the approximations of given just sounds. However, the use of a keyboard for a more complex system than that for which it was designed will result in the sacrifice of some or all of the generalization. Certain chords can then no longer be transposed to certain keys without a radical change in the position of the hand.

The School of Music of Indiana University has just purchased the equipment to build an instrument with up to 60 different (variable) pitches to the octave. I strongly recommend that a generalized keyboard be built for this instrument along the lines of Bosanquet's or possibly Fokker's. The successful completion of such an instrument would represent a valuable contribution to the mastery of the complex field of multiple division by offering the opportunity to many

students to experiment with all of the recommended tuning systems. Such an instrument, for maximum effectiveness, should possess at least 90 keys to the octave.

GLOSSARY

Area of complexity.... used in Chapter 13 to group together tempered systems possessing approximately the same number of tones. There are four such areas considered, with 24, 36, and 53 tones representing the maximum number of tones in each of the first three areas.

Autochthonous.... a term used by Fokker and others in referring to already existing Folk music.

Average Deviation.... a statistical measurement of the differences between two musical tuning systems computed by measuring the intervals between each of the tones of each system and a central tone, finding the differences between each interval of one system and the comparable interval of the other system, and taking the average of these intervals. The method is applicable only where the two systems have the same number of tones to the octave, and is meaningful only where one system is intended as a representative of the other, as when a tempered system is proposed as the representative of a just system.

Cents.... in music this term is used in the measurement of intervals. Cents are computed by multiplying the logaritha of the interval-ratio by a constant approximately equal to 3986.3, in order that an octave contain exactly 1200 cents; each semitone in 12-tone equal temperament thus contains 100 cents. Cents are

used throughout this paper as a means of measuring intervals, since they enable the reader to relate most easily the relationship between the interval discussed and the intervals of 12-tone temperament which are the most familiar to him.

Combined error factor.... a concept used in Chapter 13 in which the errors of intervals are measured both in terms of absolute size and in terms of how large they are in relation to the size of each unit of the system. The combined error factor is obtained by multiplying the size of an error (measured in cents) by the fraction whose numerator is this same error and whose denominator is half the size of one unit of the system. Where E represents the error of an interval measured in cents, and n is the number of tones in a tempered system, the combined error factor can be represented by the formula

$$\frac{E^2 n}{600}$$

Constructing interval.... a just interval which is made theoretically equal to one unit of a tempered system. Such an interval, in 12-tone temperament, is 25:24. (from Wörschmidt)

Defining interval.... a just interval of small size which is made to equal zero in the establishment of a tempered system. Wörschmidt uses this term because he defines

each tempered system by the largest just intervals which must be considered equal to zero. In 12-tone temperament it is the various dieses which are the largest defining intervals.

Departure.... Bosanquet's term for the difference between a given just interval and a specified tempered interval used as a point of reference. The departure of the just fifth is about two cents (in the 12-tone tempered system).

Difficulty factor.... in the selection and evaluation of tempered systems, the role played by the difficulty in recognizing and performing all of the tones, hence determined by the number of tones in the system.

Ditonic comma.... The Pythagorean comma, the difference between 12 fifths and 7 octaves; about 23.5 cents, or 2 cents larger than the syntonic comma. The term "comma" when used alone refers not to the ditonic but to the syntonic comma.

Dodecaphony.... the term applied to serial 12-tone composition by the Schönberg school and its successors.

Error.... Bosanquet's term for the difference between a given tempered interval and a specified just interval used as a point of reference. The error of the fifth in 12-tone temperament is about two cents.

Fibonacci series.... a series of numbers so arranged that any term represents the sum of the two preceding terms. An example is 1,1,2,3,5,8,13,21,34....

Golden ratio.... the numerical ratio between two terms such that the same ratio exists between the larger term and the sum of the two terms. $\frac{a}{b} = \frac{b}{a+b}$ By cross-multiplication, $b^2 = a^2 + ab$. Dividing both sides by b^2, $\frac{b^2}{b^2} = \frac{a^2}{b^2} + \frac{ab}{b^2}$. Simplifying, $1 = \frac{a^2}{b^2} + \frac{a}{b}$. Deducting 1 from both sides, $\frac{a^2}{b^2} + \frac{a}{b} - 1 = 0$. Considering (the ratio) to be equal to $\frac{a}{b}$, $w^2 + w - 1 = 0$. $w = .618$.

Golden tone system.... Kornerup's "ideal" musical system in which the intervals of different sizes, diesis, chromatic semitone, diatonic semitone, tone, minor third, and perfect fourth, relate to the next larger interval by the golden ratio .618...

Infra-diatonic.... Yasser's name for what he alleges to be the musical system from which ours is essentially evolved. It consists of a five-tone scale with two additional auxiliary tones.

Mean deviation.... a concept used by Barbour in evaluating various tuning systems according to how close they are to equal temperament. The difference between each constructing interval of a system and a unit of the equivalent tempered system is taken, and the average found.

Meantone temperament.... in its narrowest sense that particular temperament in which the fifth is made smaller by exactly 1/4 of a syntonic comma so that the tone

represents the mean between 9:8 and 10:9. In the general sense that the term is used in this paper, meantone temperament is any regular but non-cyclical system wherein the fifths are made smaller by from 1/3 to 1/6 of a syntonic comma.

Negative systems.... Bosanquet's term for regular systems whose fifths are smaller than 700 cents (the size of the fifth in 12-tone temperament).

Ornamental scales.... scales based on the arithmetical division of a string or pipe. Such a scale might have as consecutive tones, $\frac{1}{12}\ \frac{1}{11}\ \frac{1}{10}\ \frac{1}{9}\ \frac{1}{8}\ \frac{1}{7}\ \frac{1}{6}$.

Positive systems.... Bosanquet's term for regular systems whose fifths are larger than 700 cents (the size of the fifth in 12-tone temperament).

Pythagorean tuning.... any system composed entirely of just perfect fifths.

r.... Bosanquet's symbol for the order of a regular cyclical system, representing the number of units of the system by which 12 perfect fifths (as represented in the system) exceed or fall short of 7 octaves.

Regular cyclical system.... an equal-tempered system. (from Bosanquet)

Regular system.... a system whose perfect fifths are always of the same size. If, as in meantone or Pythagorean tuning, the tones at the two ends of such a chain of equal fifths do not themselves form a fifth of the

same size, the system, although regular, is not cyclical. (from Bosanquet)

senario.... the numbers from 1 through 6. According to theorists representing much of the best writing on music for several centuries, all true consonance in music resides in intervals formed by ratios between numbers in the senario.

senarist.... a person who believes all musical consonance to derive from the relationships of numbers in the senario.

Small-number ratio.... A ratio between two rational numbers. There is no exact limit to what can be considered a "small" number in this sense, but the implication in the use of this term is that the ratio will involve numbers sufficiently small that a perceptible sense of consonance will result from the simultaneous use of two sounds whose vibration frequencies are related by such a ratio.

Superparticular ratio.... a ratio involving consecutive integral numbers such as 16:15, 81:80, 3:2.

Supra-diatonic.... a term used by Yasser to describe what he considers to be the next musical system in an evolving series. The supra-diatonic scale is a 12-tone scale and 7 additional auxiliary tones complete a 19-tone system.

Syntonic comma.... the interval 81:80, or 21.5 cents; the difference between the just third and the third formed by four fifths. Where the term "comma" is used without further explanation, it may be assumed that the reference is to the syntonic comma.

Tricesimoprimal.... relating to the number 31. Fokker uses this term in reference to music using 31 tones to the octave.

12-tone system.... the twelve-tone equal-tempered tuning. In this paper, the expression "12-tone system" does <u>not</u> refer specifically to the procedures of the serial technique of Schönberg's disciples.

Unit.... the smallest interval in any given tempered system.

BIBLIOGRAPHY

Apel, Willi. Harvard Dictionary of Music. Cambridge: Harvard University Press, 1944.

Ariel (pseudonym). Das Relativitätsprinzip Der Musikalischen Harmonie. Leipzig: Neunzehn Stufen Verlag, 1925.

Asuar, José Vicente. "De los Microtonos y su Aplicación como Sistemas temperados," Revista Musical Chilena, XI (October-November, 1957), 59.

Awramoff, A. M. "Jenseits von Temperierung und Tonalität," Melos, I (1920), 131.

Babbitt, Milton. "Twelve-Tone Invariants as Compositional Determinants," Musical Quarterly, XLVI (April, 1960), 246.

Barbour, J. Murray. "Just Intonation Confuted," Music and Letters, XIX (1938), 48.

-- "Music and Ternary Continued Fractions," American Mathematical Monthly, LV (1948), 545.

-- "The Persistence of the Pythagorean Tuning System," Scripta Mathematica, I (1933), 286.

-- Tuning and Temperament. East Lansing: Michigan State College Press, 1951.

Bellermann, Heinrich. Die Tonleitern und Musiknoten der Greichen. Berlin: A. Förstner, 1847.

Bender, G. "Le Futurisme Musical," Le Guide du Concert, III (February, 1912).

Bindel, Ernst. Die Zahlengrundlagen der Musik. Stuttgart: Verlag Freies Geistesleben, 1950.

Blaukopf, Curt. Musiksoziologie. St. Callen: Zollikofer, 1950.

Bosanquet, R. H. M. An Elementary Treatise on Musical Intervals and Temperament. London: Macmillan and Company, 1876.

-- "Temperament; or the Division of the Octave," Proceedings of the Musical Association, (1874-75), 4.

Brandsma, Engbert. "Über die Tonverhältnisse in der Alten und Neuen Musik," Bericht von III Kongress der Internationalen Musikgesellschaft, (1909), 337.

Brun, Vigo. "Algorithmes Euclidiens pour trois et quatre nombres," Sartryck ur Trettonde Skandinavska Matematikerkongressen, XIII (1957), 45.

-- "En generalisation av kjedebroken," Videnskapsselskapets skrifter, I-II (1919-20).

-- "Music and Ternary Continued Fractions," Det Konelige Norske Videnskabers Selskab, Forhandlinger Bd. XXIII, (1950), 38.

See also p 376

Busoni, Ferruccio. Sketch of a New Esthetic of Music. Translated by Th. Baker. New York: G. Schirmer, 1911.

Carrillo, Julián. "El sonido 13," El Sonido 13, a series of editorials appearing in each issue. The quotation on page 154 is from Año I, Número 2, (February, 1924), 6.

-- "Preludio a Cristobal Colón," New Music, XVII:3 (April, 1944).

Carter, Elliott. "Shop Talk by an American Composer," Musical Quarterly, XLVI (April, 1960), 189.

Cavallini, Edoardo. "Breve nota sul pluricromatismo," Rivista Musicale Italiana, (1943), 247.

-- "Il pluricromatismo nell evoluzione musicale," Rivista Musicale Italiana, (1946), 130.

Cornu, A. and E. Mercadier. "Sur les intervalles musicaux," Comptes Rendus de l'Académie des Sciences de Paris, (1869), 301.

Danielou, Alain. Introduction to the Study of Musical Scales. London: The India Society, 1943.

Drobisch, M. W. "Über musikalische Tonbestimmung und Temperatur," Abhandlung der mathematisch-physischen Classe der Königlich sächsischen Gesellschaft der Wissenschaften, IV (1855), 1.

Dupont, Wilhelm. Geschichte der musikalischen Temperatur. Erlangen: 1935.

Eggen, Erik. Skala-Studier. Oslo: Eberh. B. Oppi's Forlag, 1923.

-- "Zur Entstehung und Entwicklung der Skala," Gedenkboek aangeboden aan Dr. D. F. Scheurleer, s Gravenhage, 1925, 103.

Eitz, Carl. "Von den natürlich reinen Stimmungsverhältnissen," Melos, I (1920), 293.

Ellis, Alexander J. "On the Musical Scales of Various Nations," Journal of the Society of Arts, XXXIII (1885), 485.

-- Supplement to Helmholtz' Sensations of Tone. (see under Helmholtz.)

Engel, Gustav. Das mathematische Harmonium; ein Hulfsmittel. Berlin: Verlag von Carl Habel, 1881.

-- "Eine mathematisch-harmonische Analyse des Don Giovanni von Mozart," Vierteljahrsschrift fur Musikwissenschaft, III (1887), 491.

Euler, Leonard. Tentamen novae theoriae musicae. (see listing under Smith, C. S.)

Fickenscher, Arthur. "The 'Polytone' and the Potentialities of a Purer Intonation," Musical Quarterly XXVII (July, 1941), 356.

Fokker, Adriaan D. "Les cinquièmes de ton," Acoustique Musicale, LXXXIV (1958). Extrait, Editions du Centre National de la recherche scientifique, Paris, 1959.

-- "Equal Temperament with 31 Notes," Organ Institute Quarterly, V:4 (Autumn, 1955), 41.

-- Just Intonation and the Combination of Harmonic Diatonic Melodic Groups. The Hague: Martinus Nijhoff, 1949.

-- Les Mathématiques et la Musique. Extrait du Tome X, Archives du Musée Teyler. Le Haye: Martinus Nijhoff, 1947.

-- "Optelakkoorden," Mens in Melodie, (May, 1960), 145.

Fokker, Adriaan D. Recherches musicales théoriques et practiques (includes compositions by Jan van Dijk, an article by B. J. A. Pels, and the following articles by Fokker: "Expériences musicales avec les genres musicaux de Leonhard Euler contenant la septième harmonique," "La gamme, la musique et le tempérament égal," "Un orgue a demitons majeurs et mineurs placé au Musée Teyler à Haarlem," and "Les possibilities d'une notation musicale de plus grande précision.") Extrait du Tome X, Archives du Musée Teyler, Le Haye: Martinus Nijhoff, 1951.

-- Rekenkundige Bespiegeling der Muziek, Gorinchem, J. Noorduijn en Zoon, 1944.

Forte, Allen. Contemporary Tone Structures. New York: Teachers College, Columbia University, 1955.

Fox-Strangways, Arthur H. "Whence? Whither?" London Observer, (February 18, March 4, March 11, 1934).

Frost, Thomas P. "A Matter of Records," Organ Institute Quarterly, V:3 (Summer, 1955).

Frye, R. M. and E. W. Tipple. A Graphic Introduction to the Harmon, private printing. (Library of Congress catalogue number ML 3809 .T 56 G 7). P183, 331

-- Data Supplement to A Graphic Introduction to the Harmon, private printing. (Library of Congress catalogue number ML 3809 .T 557).

Gevaert, François Auguste. Histoire et Théorie de la Musique de l'Antiquité. Gand: C. Annoot -- Braeckman, 1875-81.

Giovanetti, Gustavo. Genesi del Nuovo Sistema Musicale Tricromatico Naturale a Terzi di Tono. Firenze: Leo S. Olschki, 1955.

Gmelch, Joseph. Die Vierteltonstufen im Messtonale von Montpellier. Freiburg: Gregorianischen Akademie zu Freiburg, 1911.

Goldmann, Richard Franko. "Current Chronicle," Musical Quarterly, XLVI (April, 1960), 260.

Combosi, Otto Johannes. Tonarten und Stimmungen der Antiken Musik. Kopenhagen: Einar Munksgaard, 1939.

Groven, Eivind. "My Untempered Organ," Organ Institute Quarterly, V:3 (Summer, 1955) 39.

Haagh, Raymond Herbert. "Hugo Riemann's 'Geschichte der Musiktheorie im IX--XIX Jahrhundert,' Books I and II," Unpublished Doctoral dissertation, Indiana University, Bloomington, Indiana, 1961.

Haba, Alois. "Kwart-tonen-problemen," De Muziek, (Amsterdam: 1927), 109.

-- "Meine Vierteltonoper Die Mutter," Anbruch, Monatsschrift für Moderne Musik, 1931.

-- Musique pour violon seul, Wien: Universal-Edition, 1923.

-- Neue Harmonielehre. Leipzig: F. R. Kistner and C. F. W. Siegel, 1927.

-- Phantasie pour violon seul. Wien: Universal-Edition, 1923.

Handschin, Jacques. "Akustisches aus Russland," Gedenkboek aangeboden aan Dr. D. F. Scheurleer. s Gravenhage, Martinus Nijhoff, 1925.

-- Der Toncharakter, eine Einfuhrung in die Tonpsychologie. Zurich: Atlantis, 1948.

Hanson, Howard, Harmonic Materials of Modern Music; Resources of the Tempered Scale. New York: Appleton-Century-Crofts, 1960.

Hauptmann, M. "Temperatur," Jahrbücher für Musikalische Wissenschaft, I (1863), 25.

Helmholtz, Hermann L. F. On the Sensations of Tone as a Psychological Basis for the Theory of Music. Translated by Alexander J. Ellis, with supplement by Mr. Ellis. 6th Edition, New York: Peter Smith, 1948.

Hindemith, Paul. A Composer's World, Horizons and Limitations. Cambridge: Harvard University Press, 1952.

-- The Craft of Musical Composition. Translated by Arthur Mendel. New York: Associated Music Publishers, 1942.

Ives, Charles. "Some Quarter Tone Impressions," Pro Musica Quarterly, (March, 1925), 24.

Janko, Paul von. "Über mehr als zwölfstufige gleichschwebende Temperaturen," Beiträge zur Akustik und Musikwissenschaft (1901), 6.

Jonquière, Alfred. Grundriss der Musikalischen Akustik. Leipzig: Grieben's Verlag, 1898.

Kallenbach-Greller, Lotte. "Die historischen Grundlagen der Vierteltöne," Archiv für Musikwissenschaft, VIII (September, 1927), 473.

Kaufmann, Henry. "Vicentino's Arciorgano, an Annotated Translation," Journal of Music Theory, (April, 1961).

Kennan, Kent. The Technique of Orchestration. New York: Prentice-Hall, 1952.

Kinkeldey, Otto. "The Harmonic Sense: Its Evolution and its Destiny," Papers and Proceedings of the Music Teachers' National Association, XVIII (1923), 9-26.

Klein, Sigmund. "Quarter-Tone Data," Pro Musica Quarterly, (March, 1925), 21.

Kolinsky, Mieczyslow, "A New Equi-Distant 12-Tone Temperament," Journal of the American Musicological Society XII (1959), 210.

Kornerup, Thorvald. Acoustic Methods of Work. Translated by Marie Baruel. Copenhagen: Aschehoug Forlag, 1934.

-- Acoustic Valuation of Intervals. Translated by Jean Ferguson. Copenhagen: Aschehoug Forlag, 1938.

-- Akustische Gesetze für die Akkordbildung. Copenhagen, 1930.

-- Das Goldene Tonsystem. Copenhagen: Aschehoug Forlag, 1935. 1272

-- Musical Acoustics Based on a Pure Third System. Translated by Augusta Petersen. Copenhagen: Wilhelm Hansen Musik-Forlag, 1922.

-- Das Tonsystem des Italieners Zarlino. Copenhagen: J. Jorgensen, 1930.

-- Die Vorläufer der gleichschwebenden Temperaturen mit 19 oder 31 Tönen in der Oktave. Translated from Danish to German by P. Friedrich Paulsen. Copenhagen: J. Jorgensen, 1930.

Krenek, Ernst. "Extents and Limits of Serial Techniques," Musical Quarterly, XLVI (April, 1960), 210.

Kuttner, Fritz A., and J. Murray Barbour. Meantone Temperament in Theory and Practice. Introductory Notes. New York: Musurgia Records A-3, 1958.

-- The Theory and Practice of Just Intonation. Introductory Notes. New York: Musurgia Records A-2, 1958.

-- The Theory of Classical Greek Music. Introductory Notes. New York: Musurgia Records A-1, 1958.

Lenormand, René. A Study of Twentieth Century Harmony. Translated by Herbert Antcliffe. London: J. Williams. Boston: B. T. Wood, 1940.

Lursen, Martinus J. Modi Antichi Musiche Nuove. Haarlem: Erven F. Bohn, 1947.

Malherbe, Edmond. "Systeme musical et clavier a tiers de tons," Le Ménestrel, XXIX (July 19, 1929), 329.

McClure, A. R. "An Extended Mean-tone Organ," The Organ, (January, 1951).

-- "Studies in Keyboard Temperament," Galpin Society Journal, I.

Meyer, Leonard B. Emotion and Meaning in Music. Chicago: University of Chicago Press, 1956.

Mollendorff, Willi. Musik mit Vierteltönen. Leipzig: Verlag von F. E. C. Leuckart, 1917.

Norden, N. Lindsey. "A New Study of Intervals," Organ Institute Quarterly, V:1 (Winter, 1955), 31.

Novarro, Augusto. Sistema Natural de la Música. Mexico D. F., 1951.

Opelt, Friedrich Wilhelm. Allgemeine Theorie der Musik. Leipzig, 1952.

Partch, Harry. Genesis of a Music. Madison: University of Wisconsin Press, 1949.

Parrett, Wilfrid. Some Questions of Musical Theory. Cambridge: W. Heffer and Sons, 1926.

Pfrogner, Hermann. Die Zwölfordnung der Töne. Zürich: Amalthea-Verlag, 1953.

Poole, Henry Ward. "Essay on Perfect Intonation and the Euharmonic Organ," American Journal of Science and the Arts, 1850, 77.

-- "Perfect Harmony in Music," American Journal of Science and the Arts, XLIV (July, 1867).

Reese, Gustav. Fourscore Classics of Music Literature. New York: Liberal Arts Press, 1957.

Riemann, Hugo. Geschichte der Musiktheorie. See under Haagh.

Sabaneev, Leonid. "The Possibility of Quarter-tone and other New Scales," translated by S. W. Pring, The Musical Times, (June 1, 1929).

Sachs, Melchior E. "Das temperierte 19-Tonsystem und eine dafuer passende Schrift," Report of the Fourth Congress of the International Musical Society--London, (May, 1911), 279-291.

Salinas, Francisco de. De Música, 1577. Faksimile-Nachdruck, hrsg. von Macario S. Kastner. Kassel: Bärenreiter Verlag, 1958.

Sauveur, Joseph. "Méthode Générale pour Former les Systêmes tempérez de Musique et du choix de celui qu'on doit Suivre," Histoire de l'Académie Royale des Sciences, avec les Mémoires de Mathematique et de Physique, (1707), 117, 203.

-- "Systême Général des Intervalles des Sons et son Application à tous les Systêmes et à tous les Instruments de Musique," Histoire de l'Académie Royale des Sciences, 1701, avec les Mémoires de Mathematique et de Physique, (1704), 297.

-- "Table Générale des Systêmes Tempérez de Musique," Histoire de l'Académie Royale des Sciences, avec les Mémoires de Mathematique et de Physique, (1711), 309.

Schafer, Tillman H. The Music of Tomorrow: The Supra-Diatonic Scale, A New Concept of Timbre, The Electronic Musical Instrument. Unpublished honors thesis, Mills College, Oakland, 1941.

Schafer, Tillman and W. A. Piehl. "Musical Instruments in Nineteen-Tone Equal Temperament," Acoustical Society Journal, (1947), 730.

Scherchen, Herman. The Nature of Music. Translated by William Mann. Chicago: Henry Regnery, 1950.

Schlesinger, Kathleen. The Greek Aulos. London: Methuen and Co., 1939.

Schoenberg, Arnold. "Problems of Harmony," Modern Music. XI (November, 1924), 167.

Sessions, Roger. "Problems and Issues Facing the Composer Today," Musical Quarterly, XLVI (April, 1960), 159.

Slonimsky, N. Thesaurus of Scales and Melodic Patterns. New York: Coleman-Ross Co., 1947.

Smith, Charles Samuel. "A Translation of Tentamen Novae Theoriae Musicae by Leonhard Euler." Unpublished doctoral dissertation, Indiana University, Bloomington, June, 1960.

Smith, Robert. Harmonics or the Philosophy of Musical Sounds. London: T. and J. Merrill Booksellers in Cambridge, 1759.

Steiner, Joachim. "Einfluss der reinen Stimmung auf die Entwicklung der Musik," Bericht von III Kongress der Internationalen Musikgesellschaft. Wien: Artaria and Co., (1909), 387.

Stravinsky, Igor and Robert Craft. Conversations with Igor Stravinsky. Garden City: Doubleday, 1959.

Strunk, Oliver, Source Readings in Music History. New York: W. W. Norton, 1950.

Stumpff, Karl. Beitraege zur Akustik und Musikwissenschaft Hefte I-IV. Leipzig: Verlag von Johann Ambrosius Barth, 1898.

Tovey, Donald F. "Harmony," Encyclopaedia Brittanica, XI, 203. Chicago: University of Chicago Press, 1950.

Waller, St. Die Grundtheorie des Vierteltonsystems. Budapest: Edition Richtner, 1936.

Tonaka, Shohe, 1890? [cited p 269 this volume]

Wallot, J. "Bemerkungen zu der Arbeit von J. Wurschmidt über Logarithmische und graphische Darstellung der musikalischen Intervalle," Zeitschrift fur Physik, IV (1921), 157.

Woolhouse, W. S. B. Essay on Musical Intervals, Harmonics, and the Temperament of the Musical Scale. London: J. Souter, 1835.

Wurschmidt, José. "Buchstabentonschrift und Von Oettingenshes Tongewebe," Zeitschrift für Physik, V, 111.

-- "Logarithmische und graphische Darstellung der musikalischen Intervalle," Zeitschrift für Physik, III (1920), 89.

-- "Die 19-stufige Skala," Neue Musik-Zeitung, XLII (1921), 215.

-- "Die rationellen Tonsysteme im Quinten-Terzengewebe," Zeitschrift für Physik, XLVI (January, 1928), 527.

-- "Vierteltel- und Sechsteltonmusik, eine kritische Studie," Neue Musik-Zeitung XLII (1921), 183.

Wyschnegradsky, Ivan. Manuel d'Harmonie à Quarts de Ton. Paris: La Sirène Musicale, 1933.

-- "Problèmes d'Ultrachromatisme," Polyphonie, Cahier 9, 129.

-- "Quartertonal Music, Its Possibilities and Organic Sources," Pro Musica, (October, 1927), 19.

Yasser, Joseph. "A Letter from Arnold Schoenberg," Journal of the American Musicological Society, (1953), 60.

-- "A Revised Conception of Tonality," Music Teachers National Association Journal, (1935), 100.

-- A Theory of Evolving Tonality. New York: American Library of Musicology, 1932.

-- "The Highways and the Byways of Tonal Evolution," Journal of the American Musicological Society, XI (September, 1948), 100-121.

-- "Tonality and Atonality as Synthesized by Supra-Tonality," Modern Music, (November, 1930), 347.

Young, Robert. "Some Problems for Postwar Musical Acoustics," The Journal of the Acoustical Society of America, XVI (October, 1944), 103-107.

-- "Tuning: Pianoforte," Grove's Dictionary of Music and Musicians, Fifth edition, VIII, 595. London: Macmillan Co., 1954.

ADDENDUM

Jones, Clark, "Remarks on Just Intonation and Musical Scales," Acoustical Society Journal, 1947, p. 727.

Kent, Charles S, An Introduction to Tuning and Temperament, unpublished, Bloomington, Ind., 1961.

Vogel, Martin, Die Zahl Steben in der spekulativen Musiktheorie, Bonn, 1955.

Vita

EDUCATION:

Horace Mann-Lincoln High School, New York, N.Y.

A.B. magna cum laude, Harvard University, 1953.

M.F.A. with high honor, Brandeis, University, 1957.

Fulbright grant to Berlin Germany, 1957-8.

Indiana University.

EXPERIENCE:

Instructor in Music Theory, U.S. Naval School of Music, 1955.

Teaching Assistant in Theory and Music History, Brandeis and Indiana Universities.

HONOR SOCIETIES:

Phi Beta Kappa

Pi Kappa Lambda

COMPOSITIONS:

Two one-act operas

String Quartet

Woodwind Quintet

Piano Suite

Piano Concerto

Orchestral work

Choral works

Sonata for flute and piano

Song cycle for soprano