

## CHAPTER 10

## "ARIEL"

In 1925, a new publishing house appeared in Leipzig called the Neunzehn Stufen Verlag. Citing as its purpose nothing less than the reform of music, the house issued one book and the unfulfilled promise of two additional works and then faded into immediate and lasting oblivion. The one published volume is Das Relativitätsprinzip der Musikalischen Harmonie by an anonymous author writing under the pseudonym "Ariel." A detailed description of a 19-tone instrument with three rows of keys was to follow, along with a proposed system of notation, in a second volume. Volume III was to be a harmony text for 19-tone music based on the so-called principle of relativity. It is to be presumed that financial difficulties postponed the publication of these successor volumes and that political upheavals destroyed all traces of the company and its benefactors.<sup>1</sup>

## THE 59 BASIC INTERVALS

Das Relativitätsprinzip is, from the start, a violently partisan work. Ariel insists that all genuine

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<sup>1</sup>The real name of Ariel appears in none of the catalogues where the book is listed. The copyright was issued to the Neunzehn Stufen Verlag, with a Mme. Boas de Jouvencel signing the necessary papers. Whether she was the author or simply an agent is unknown, but the latter would appear the more likely assumption.

music, if its tone relationships are to be logical, must arise out of certain pure intervals. Ariel quite arbitrarily selects 59 such "pure" intervals, and they range from such fundamental intervals as 3:2 to such complex intervals as 20736:15625. Ariel then shows by an appended chart how much more logically the 59 intervals group together in 19 separate units than in 12. One can question several aspects of Ariel's reasoning. Why should 20736:15625 be considered as important as 3:2 in establishing the validity of a temperament? Why, indeed, should 20736:15625 be considered at all?

Reduced to simple intervals, 20736:15625 is the interval created when two major thirds are deducted from four minor thirds. A process as complicated as this would hardly be warranted in the quest for "basic" intervals. What is basic about four superimposed minor thirds? The minor third, of course, possesses exceptional accuracy in 19-tone temperament. In the absence of a meaningful explanation to the contrary, Ariel would seem to be guilty of circular reasoning. He justifies 19-tone temperament on the basis of a group of intervals which seem themselves to have been chosen because they are themselves based on combinations of those intervals which are known to be effectively duplicated by 19-tone temperament. Because Ariel's 59 intervals appear doubtful and arbitrary, they can probably be dismissed from consideration as a serious basis for 19-tone temperament.

## Example 44: Ariel's 19 Intervals

1:1	6:5	64:45	27:16
128:125	3125:2592	36:25	128:75
643:625	768:625	625:432	216:125
25:24	100:81	4608:3125	125:72
16:15	5:4	40:27	27648:15625
27:25	32:25	3:2	16:9
625:576	162:125	15625:10368	9:5
3456:3125	125:96	192:125	3125:1728
10:9	20736:15625	125:81	1152:625
9:8	4:3	25:16	50:27
15625:13824	27:20	8:5	15:8
144:125	3125:2304	81:50	48:25
125:108	864:625	625:384	625:324
75:64	25:18	5184:3125	125:64
32:27	45:32	5:3	2:1

## THE FOUR REQUIREMENTS OF A TONAL SYSTEM

Ariel moves to a more tenable position when he considers the basic requirements of a system of temperament. He maintains that one must make four demands on any tempered system:<sup>2</sup>

1: That the smallest intervals where there is the possibility for direct progressions (direkte Tonbewegungen) be available.

2: That the consonances should be as pure as possible.

3: That the proportions between the sizes of the basic scalar intervals should resemble those of the natural intervals from which they are taken.

<sup>2</sup>Das Relativitätsprinzip, page 134. "Man kann für die Anforderungen an ein temperiertes System vier Thesen aufstellen: I. Die kleinsten Intervalle, in denen direkte Tonbewegungen noch möglich sind, müssen vorhanden sein. II. Die Konsonanzen müssen möglichst rein sein. III. Die Tonfolgen müssen in den Verhältnissen ihrer natürlichen Abstufungen zum Ausdruck kommen. IV. Konsonanzen und Dissonanzen müssen als solche gegensätzlich wirken. Natürliche Dissonanzen dürfen nicht zu Scheinkonsonanzen wirken."

4. That consonances and dissonances should function as such against one another. Intervals which are by nature dissonant should not be tempered so as to appear consonant.

On all four counts he finds 19-tone temperament to be far superior to 12.

Smallest Interval: That 19-tone temperament should be preferable to 12- on the first count, that of supplying the smallest possible interval, is self-evident. But Ariel makes a rather important point about the special significance of that particular smallest interval. The natural chromatic semitone, he asserts, is 25:24. This interval he calls the "Terzendifferenz," as it represents the difference between the major third, 5:4, and the minor third 6:5. It is Ariel's contention that this Terzendifferenz is of great importance in music; this appears quite sound in view of the role of major and minor in our musical heritage. But the smallest interval in 12-tone temperament is 100 cents, while the natural Terzendifferenz is 70 cents. For this reason, Ariel strongly rejects 12-tone temperament, even for the performance of the current repertory. He turns, instead, to 19-tone temperament, whose smallest interval, 63 cents, is a much closer approximation of the Terzendifferenz.

The Purity of the Consonances: According to Ariel, the following are the consonant intervals of music: 10:9, 6:5, 5:4, 4:3, 25:18, 36:25, 3:2, 8:5, 5:3, and 9:5. Of these ten consonances within the octave, six are traditionally

regarded as consonances (the minor and major thirds and sixths and the perfect fourth and fifth). In accepting the major second and minor seventh as consonances, Ariel would appear to be justified by much 19th as well as 20th century practice, wherein the minor seventh and/or the major ninth is often added to triadic sonorities with little sense of dissonance. What may appear subject to question about Ariel's use of the major second as a consonance is his choice of 10:9 over the more traditional 9:8. In making this choice he takes the same position as Kornerup, but for his own very different reasons.

He is quite emphatic in his choice. 9:8, he insists, is a dissonance. It is produced by the artificial process of superimposing two fifths.<sup>3</sup> The inversion, 16:9, involves relatively high numbers and is also a dissonance. On the other hand, 10:9 is the inversion of the ratio 9:5, representing the combination of the perfect fifth and minor third. In performing the consonant dominant seventh chord, asserts Ariel, singers attempt to make both the fifth G-d and the minor third, d-f pure. This produces the consonant minor seventh, which must be 9:5. The consonant dominant seventh chord, as the basic generator of consonant harmony, is a

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<sup>3</sup> Ariel seems in this respect to be implying that the combination of like intervals tends to produce dissonance. This is Zarlino's basic view. In the case of the tritone, however, he appears to contradict this view. Perhaps the one consistent way in which Ariel's view might be interpreted is this: the combining of like consonances in itself does not produce a consonance.

theoretical phenomenon with a rather respectable history, and Ariel is in good company in asserting that this consonant seventh chord is comprised of the relationships 20:25:30:36. It is the first and last members, 20:36 which relate to one another as 5:9, and invert to 10:9. Since Ariel is determined to reject the natural harmonic seventh as the basis for any music,<sup>4</sup> there may indeed be acoustical validity to his choice of 9:5, the simplest remaining ratio, as his consonant seventh.

Ariel's "consonant" tritone 25:18, and its inversion, the so-called "Doppelterz" 36:25, are, like the minor seventh, drawn from the consonant seventh chord 20:25:30:36. The Doppelterz is therefore declared consonant in spite of its being the sum of like intervals. It may be somewhat difficult to accept the tritone as a consonance. It is generally referred to as a dissonance, and Hindemith gives the tritone a special classification as the most dissonant interval.<sup>5</sup> However, it should be remembered that the tritone (like the perfect fourth as well as the diminished fifth) was treated as a semi-consonance, in the Renaissance, usable between the upper voices as a consonance. It is only

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<sup>4</sup> Ariel is most adamant in rejecting the 7th partial outright. He argues against its incorporation into music in several ways, concluding with the statement on page 51 that when two singers sing an interval of 7:6, the result is neither a musical interval, nor the temperament of such an interval. It is, "überhaupt keine Musik."

<sup>5</sup> Hindemith, The Craft of Musical Composition.

since the era of 12-tone equal temperament, with an interval 31 cents removed from Ariel's Doppelterz, that the harmonic use of the tritone has come to imply the ultimate in instability and dissonance.

Having established the above intervals as consonant, Ariel compares their representation in 19- and 12-tone temperaments.

3:2 and 4:3: Ariel admits to the superiority of the 12-tone fifths. He insists, however, that the 7-cent error of the fifth in 19-tone temperament is far from transgressing the limits of agreeable sound,<sup>6</sup> and warns against basing a judgment on the fifths alone.

5:4 and 8:5: Ariel complains of the bad thirds in 12-tone temperament, and points out that the major thirds in 19-tone temperament are almost twice as pure. He makes no distinction between the positive error of the 12-tone major third and the negative error (considered by Barbour and others to be far more objectionable) of the 19-tone third.

6:5 and 5:3: The error of this interval in 19-tone temperament is less than 0.1 cents. To Ariel this is the most important of intervals. He refers to it as the child of an eternal father (the perfect fifth) and an eternal mother (the major third). As the offspring of so propitious a union, as the Urkind, the minor third is the foundation

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<sup>6</sup>Op. cit., page 144 "... Sie bedeutet aber noch lange nicht die Grenze für die Trübung des Wohlklanges."

for a higher order of music.<sup>7</sup> Without the knowledge of this fundamental significance of the minor third, maintains Ariel, harmony, and especially modern harmony, is not to be understood. The error in 12-tone temperament is over 15-cents.

10:9 and 9:5: If one is able to agree with Ariel that these are the proper ratios for the major second and minor seventh, and that they are consonances, then 19-tone temperament has a clear edge over 12- in their representation. The approximate margin is 7 cents to 18.

25:18 and 36:25: As in the case above, whatever controversy might exist as between the two systems lies in whether or not these intervals are the correct ratios for tritone and diminished fifth. The error in their representation is a minute fraction of a cent in 19-tone temperament, and about 31 cents in 12-tone temperament. No alternative ratios involving smaller numbers are generally offered for 12-tone temperament, since the interval is generally conceded to be a dissonance in that system. Fokker, on behalf of 31-tone temperament, argues quite persuasively that the consonant tritone is 7:5 and that all other approximations for this interval are dissonant.

For the ten intervals which Ariel has selected as

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<sup>7</sup>Ibid., page 74, "Jener Ausgleich, der aus Gegenbewegung von Quinte und grosse Terz hervorgeht ist die kleine Terz. Diese, gleichsam das Kind von Ur-Vater (Quinte) und Ur-Mutter (gr. Terz) ist Grundintervall höherer Ordnung."



his consonances, the average error in 19-tone temperament is 4.2 cents, in 12, 16.2. Granted the premise of his intervals, Ariel's case for the superiority of the consonances of 19-tone temperament is quite solid indeed.

The Mutual Relationships of the Small Intervals:

According to Ariel, there are five essential small intervals in our musical system, the chromatic and diatonic semitones, the whole-tone, the diminished third (to which he is evidently referring by the name of its enharmonic equivalent in 19-tone temperament, the augmented second), and the minor third. In 12-tone temperament the two types of semitone are enharmonic equivalents, as are the augmented second and the minor third. This is, according to Ariel, a serious distortion of the logic of their basic relationship to one another. Ariel considers the mutual relationships singly and in groups. The chromatic and diatonic semitones relate to one another and to the whole-tone<sup>8</sup> approximately as 37:63:100. The twelve tone scale can do no better at reproducing this formula than 50:50:100. Markedly better is the 19-tone approximation, 33:67:100. Ariel shows, similarly, that the 3:4:5 relationship of 19-tone temperament is a rather close approximation for the series 10:9, 144:125, 6:5, while 12-tone temperament is powerless to represent such gradations. In the end, it is Ariel's preference for two

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<sup>8</sup>Ibid., page 147. Compare these figures with the golden ratio of Kornerup which is approximately 38:62:100.

kinds of semi-tone and for a distinct interval larger than the whole-tone but smaller than the minor third which leads him strongly again to endorse 19-tone temperament.

The Polarity of Consonance and Dissonance: Ariel's fourth demand on a tempered system is that it differentiate clearly between its consonances and its dissonances. In this respect, Ariel objects to 12-tone temperament on the grounds that it possesses only one set of dissonances (the minor second and major seventh) which isn't so clearly removed from consonant intervals that it cannot be falsely mistaken for a set of consonances. In contrast, there are four such sets of dissonances in 19-tone temperament. There are, to begin with, two sets (instead of one) of semitones and sevenths. The two sets are  $1/19 - 18/19$ , and  $2/19 - 17/19$ . In addition there are two sets,  $7/19 - 12/19$ , and  $4/19 - 15/19$ , which Ariel considers to be unambiguously dissonant in 19-tone temperament,<sup>9</sup> and which lack any counterpart in 12-tone temperament.

Ariel illustrates the advantage of unambiguous dissonance by showing that the augmented triad and diminished seventh are dissonances in 19-tone temperament, while they are ambiguous through the lack of any completely dissonant intervals in 12-tone temperament. The diminished seventh

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<sup>9</sup>As just representations for  $4/19$  and  $7/19$ , Ariel gives  $144:125$  and  $32:25$  respectively. The deviation of these intervals from their tempered equivalents is somewhat greater than is the deviation of the consonant intervals.

in 19-tone temperament must contain an interval of  $4/19$  or  $15/19$ , while the augmented triad must have either  $7/19$  or  $12/19$ .

There appear to be a number of weak points in Ariel's theory of consonance and dissonance employed as polar opposites. It is difficult to see why  $25:16$  should be regarded as a dissonance while the more complex formulation  $36:25$  is considered consonant. At best it is hard to imagine why two such similar intervals as  $25:16$  and  $25:18$  should have effects which function as polar opposites.

Ariel's theory of consonance and dissonance falls completely if his fundamental rejection of the 7th partial is denied. He has established as consonant a number of intervals containing integers higher than 7. This has not been done by the traditional supporters of the senario (and nothing more). The decision to include as consonances intervals involving integers larger than 6 makes somewhat questionable on logical grounds the rejection of 7 as a basis for consonance.

Despite the theoretical weaknesses of Ariel's hierarchy of consonances and dissonances, I must add that my own subjective appraisal of the characteristics of the intervals in 19-tone temperament concurs with Ariel's theory. This may be less the result of valid acoustical principles than a matter of familiarity. Ariel has named as dissonances  $4/19$  and  $7/19$ , which are the two basic sounds in 19-tone

temperament that possess no equivalent in present-day music. They may simply sound "dissonant" by reason of their lack of familiarity.

#### THE INTERRELATION AND EVOLUTION OF TONAL SYSTEMS

In common with the other major 19-tone advocates, Ariel finds "good" tonal systems to be related. Where Kornerup and Yasser find the ultimate relationship between desirable systems to lie in a single Fibonacci series, Ariel is concerned with other aspects of continuity between the different systems. His approach is close to Opelt's (with whose work he is familiar) and even closer to Würschmidt's. Like Würschmidt, Ariel sees the problem as one of finding three suitable constructing intervals, based on pure fifths and thirds, which will not differ too greatly in size. Ariel is not as systematic as Würschmidt,<sup>10</sup> relying more on instinct and on trial and error. His results are quite similar to those of Würschmidt, and remarkably similar to those obtained by Viggo Brun using a carefully worked out mathematical formula.<sup>11</sup>

Reduced to its fundamentals, Ariel's view of the relatedness of desirable musical systems is as follows. Tempered

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<sup>10</sup>Würschmidt's views are dealt with at considerable length in Chapter 1, and again briefly in chapter 8. Opelt is considered in Chapter 8.

<sup>11</sup>Viggo Brun's mathematical formula, first presented in 1919, is still virtually unknown. It is taken up in detail in Chapter 13.

systems are necessary to music, but they are of value only to the extent that they succeed in the approximation of just intervals. It must therefore be possible to formulate any complete musical (tempered) system in terms of the just intervals on which it is built. The basic just intervals remain the same in every "good" system. The systems with many tones result simply from the subdivision of the intervals of the systems with fewer tones.

The seven-tone system, for example, has three constructing intervals, 9:8, 10:9, and 16:15. These intervals are used three times, twice, and twice, respectively. This is, as all will recognize, the just diatonic scale, which Ariel recognizes as the basis for 7-tone temperament. This same just diatonic scale is present in all "good" systems involving additional tones; the accuracy of its representation increases in the temperaments involving more tones. The 12-tone system is formed by retaining the smallest of the intervals of the 7-tone system, 16:15, and subdividing (factoring the ratios of) the other intervals. As there can only be three constructing intervals (a basic premise of Ariel's which might be subject to some criticism), an effort must be made to use the held-over interval (16:15 in this case) as a factor, and to find common factors for the remaining intervals. In the conversion from the 7- to the 12-tone system, 9:8 is subdivided into 25:24 and 27:25, the former member of which is also used, together with 16:15,

as factors of 10:9. The complete 12-tone system is thereby found to contain five intervals 25:24, four intervals 16:15, and three intervals 27:25. By similar processes, further subdivisions yield 19-, 31-, 34-, 53-, 65-, 118-, 171-, 289-, 323-, 441-, 612-, 730-, and 1053-tone temperaments.<sup>12</sup>

The full details of each of the simpler systems are in Das Relativitätsprinzip.<sup>13</sup> Here it would seem relevant only to note Ariel's just 19-tone scale and how it is derived from the just 12-tone scale. As might be expected, the Terzendifferenz, 25:24, is the most important interval in the system. The five intervals, 25:24, which appear in the 12-tone system, are held over. In addition, 25:24 is used as a factor in the subdivision of each of the other intervals. This brings to twelve the number of intervals of that size employed in the 19-tone scale. The remaining seven intervals represent the left-over factors of 16:15 and of 27:25, and are 128:125 and 648:625 respectively. These two new intervals represent on the one hand the

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<sup>12</sup>It is fascinating to compare Ariel's seemingly empirical deductions with Brun's which develop a priori from a mathematical formula. Not only are the systems similar with respect to the number of tones used (with a few exceptions), but also with respect to the constructing intervals in a number of cases. For example, the syntonic comma first becomes a constructing interval for Ariel in 34-tone temperament. Note, in Chapter 13, that it is the graduation of the syntonic comma to an important position in Brun's equation that makes 34- a member of his series.

<sup>13</sup>Beginning page 160.

difference between the octave and three major thirds, and on the other, the difference between the octave and four minor thirds. With the other constructing interval representing the difference between the two kinds of third, it can well be seen that the thirds play the primary role in the construction of the 19-tone scale.

Ariel's "system of systems" is quite impressive, especially as it contains all of the most sought-after of the equal tempered systems. Ariel's is one of the few systematic combinations which link the 53-tone system with the 19- and 31-tone systems. Ariel's hierarchy of systems possesses the considerable advantage of having one universal just intonation which is equally adapted to all of the member systems.

There are, however, several grounds on which Ariel's sequence of systems is subject to question. It is difficult to see why three should have to be the limit to the number of constructing intervals. It is quite possible that a more stable system might be possible with two or four such intervals. Ariel's total reliance on empirical methods and his refusal to draw precise limits can be questioned. He rejects Opelt's proposal for 22-tone temperament, for example, on the grounds that the three constructing intervals must vary excessively in size,<sup>14</sup> the largest being 25:24,

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<sup>14</sup>Würschmidt disputes Ariel on this point, offering an alternative construction which does not use the syntonic comma as a constructing interval. The constructing intervals are, instead, (25:24)<sup>9</sup>, (250:243)<sup>3</sup> and (648:625)<sup>10</sup>.

and the smallest the syntonic comma, 81:80. But Ariel never states precisely what the proper limits for size variation should be. Finally, in Ariel's series of systems, the absence of the 7th partial as a factor stands out as a possible fault as much as in his considerations of the necessary properties for the individual system.

### CULT OF NUMBERS

In his attempt to leave no stone unturned under which might be found an argument for the 19-tone scale, Ariel indulges freely in a kind of astrology of numbers. He points out that the Arabian shiek, Mudschial Eddin ibn Arabi (1172-1240), considered 19 to be the number which explained the development of the world.<sup>15</sup> Its double, 38, is a much-mentioned mystical number (mehrfach erwähnte mystische Zahl), according to Ariel. There is also some significance to its square, 361, and the proximity of this square to 360, the number of degrees in a circle and the product of  $2^3$ ,  $3^2$  and 5. Ariel reminds us further that the Asian Go-board is 19x19! Although he disclaims any interest in the Occult as a basis for 19-tone temperament, his citation of traditional mystical and occult sources to support his numerical theories is both interesting and somewhat disquieting. It is a measure of his fanaticism.

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<sup>15</sup>Op. cit., page 43, "die Zahl der Weltentwicklung."



## SUMMARY

Behind a mask of fiery polemics and mystical fervor, Ariel presents a number of ideas of considerable importance on the interrelation of musical systems, the universal validity of a single just intonation, the function and adequacy of consonance and dissonance in different tempered systems, and the proper demands a musician can make upon a tempered system. Many of his ideas seem quite arbitrarily established, such as his choice of consonances and dissonances in 19-tone temperament. But, as with his choice of consonances in 19-tone temperament, many of his ideas stand up surprisingly well under subjective evaluation. If Das Relativitätsprinzip has not proved the revolutionary manifesto its author intended it to be, it is still an interesting document.