CHAPTER 6

UNUSUAL SUGGESTIONS FOR EQUAL TEMPERAMENTS

In this short chapter appear briefly temperaments with many tones in the octave which have been regarded as ideal but impractical, and temperaments with relatively few tones belonging to unusual orders of r and containing serious defects in their thirds or fifths.

"Ideal" Systems

Example 36 shows the basic acoustical features of seven of the leading systems of multiple division involving more tones to the octave than even their firmest supporters have deemed practical.

It is obvious from a quick examination of this chart that no truly ideal temperament in which the partials through 13 are accurately represented has been found. It is patently absurd to use the same kind of standards as are used in 12-tone temperament when measuring the discrepancies of systems having over a hundred tones per octave. With 100 tones to the octave, the individual pitches are only 12 cents apart, bringing any imaginable pitch within 6 cents of a member of the system. One would expect "ideal" temperaments such as these to have a particularly high degree of correspondence between tempered and just ratios to compensate, at least in some measure, for their

EXAMPLE 36: "IDEAL" SYSTEMS

Number of Tones:	74	87	113	118	171	347	612
Cents per unit:	16.22	13.79	10.62	10.17	7.02	3.46	1.96
Size of the fifth	697.30	703.45	700.88	701.69	701.75	702.01	701.96
Error of the fifth	4.64	1.50	1.07	0.26	0.20	0.06	0.005
% of possible error	57	22	20	05	06	04	00
Size of the third	389.2	386.2	382.3	386.4	386.0	387.3	386.3
Error of the third	3.1	0.1	4.0	0.1	0.3	1.0	0.0
% of possible error	38	01	75	02	08	59	00
Size of the seventh	973.0	965.5	966.4	966.1	968.4	968.3	968.6
Error of the seventh	4.2	3.3	2.4	2.7	0.4	0.5	0.2
% of possible error	52	48	45	53	11	29	20
Size of 11:8	551.35	551.7	552.2	549.2	554.4	549.8	551.0
Error of 11:8	00.0	0.4	0.9	2.1	3.1	1.3	0.3
% of possible error	00	06	17	41	88	75	31
Size of 13:8	843.2	841.4	838.9	844.1	842.1	840.3	841.2
Error of 13:8	2.7	0.9	1.6	3.6	1.6	0.2	0.7
% of possible error	33	13	30	70	46	12	71

impracticability. Otherwise, why propose them in the first place?

74-tone temperament can be disposed of quickly, since it was proposed not as an ideal temperament but rather as part of a mathematical formula for obtaining satisfactory temperaments.[1] It represents the approximate fulfillment of a system of 2/9-comma temperament, a temperament which did figure in some late Renaissance theory. It combines the assets and liabilities of 31- and 43-tone temperaments, which are too much alike to be worth combining, since the resulting temperament is too similar to both to be worth the many additional tones involved. It can well be dismissed from further consideration, with only the regret over losing so fine a representation of the 11th partial.

87-tone temperament is the only temperament between 28 (!) and 559 obtained by Barbour using ordinary ternary continued fractions.[2] Since ternary continued fractions relate only to the octave, fifth, and major third, it is not surprising to find the error in the 7th partial rather large. The 5th and 11th partials are excellent, and the 13th partial is quite adequately approximated. However, in addition to the 7th partial the 3rd partial is not as well represented as one would wish in as complex a system as

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[1] By Drobisch as reported in Tuning and Temperament, p. 129.

[2] Ibid., p. 130.

this one.

113-tone temperament is suggested by Partch as the simplest temperament in which all of the partials through 11 are adequately represented.[3] There appears to have been an error in his calculations, because there are thirds that he would never tolerate in a musical system, even one involving far fewer tones than 113-tone temperament. It is true that many scale units of Partch's system (see chapter 7, below) are quite close to intervals representing multiples of 10.62 cents (the unit in 113-tone temperament), but many others are not.

118-tone temperament has enjoyed the most widespread approval of all the "ideal" temperaments among theorists. Bosanquet, Würschmidt, Ariel and Barbour are among the writers who have reported favorably on this temperament. Both the fifth and the third are obtained within small fractions of a cent, rendering the discrepancies for these two intervals small, even for a system involving such minute units. Far less fortunate in 118-tone temperament are the upper prime partials, each missing its mark by more than the comparable interval in Partch's 113-tone system. The ideal temperament is indeed an elusive matter.

171-tone temperament is Perrett's choice among the

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[3] Genesis of a Music, p. 307.

equal temperaments.[4] He is concerned with the partials through 7, and, indeed, for the 3rd, 5th, and 7th partials, this temperament is the first to approach "ideal" standards. 0.4 cents is the maximum deviation for any of the partials below 11, an impressive attainment even for a temperament with as many tones as this. With the 11th partial the system becomes as useless as 12-tone temperament for the same partial; the desired tone lies almost midway between two members of the tempered system.

347-tone temperament bears the seal of approval of Von Janko and von Hornbostel. With this many tones in the system, a single cent's discrepancy must be considered serious, and the 3rd partial is out of tune by that much. It is hard to understand why these men selected 347 in preference to, say, 171. It is likely that the discovery of 347 was by chance and that before Perrett nobody had explored the advantages of 171-tone temperament.

The last of the charted temperaments, 612, is endorsed by such diverse writers as Barbour, Bosanquet, and von Janko. The fifths and thirds are obtained with a

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[4] Perrett, op. cit., p. 149. Perrett admits the weakness of the 11th and 13th partials in 171-tone temperament, but regards them as unimportant, using instead the 17th partial. "If the 11th and 13th harmonics can be dispensed with, 171 may claim to be the perfect scale," he concludes.

[5] Op. cit., from p. 6. von Janko considers 41- and 53-tone temperaments to be the best until 347. He then also mentions 400, 453, 506, 559, and 612-tone temperaments, the numbers each removed from one another by 53 integers.

precision that would be remarkable in any equal temperament. Even the seventh partial is closer in the more meaningful category (% of possible error) than in any of the "ideal" systems except for 171-tone temperament. It may be of some consolation in an imperfect world to consider that if some kind of living creature in a distant solar system should develop ears and a mind of incomprehensible powers, nature has, in 612-tone temperament, an almost perfect system for him. Almost perfect, that is, if he is content to forego the 11th, and especially the 13th, partials.

SYSTEMS WITHOUT FIFTHS OR THIRDS OR BOTH

From time to time, systems lying outside the field of consonant thirds and fifths have been proposed, usually for an isolated, specific use. With the availability of electronically produced music, not dependent on a fixed tuning system, the practice of adopting unique tuning systems for individual pieces may well increase. In his recent work, Spiritus intelligentiae sanctus, Ernst Krenek makes use of 13-tone equal temperament in an electronic interlude.[6] In this interlude he makes use of conjunct heptachords in a serial manner. It would be no less logical to construct an electronic interlude out of 11-tone temperament, 23-tone temperament or any other system. As long as

[6]Musical Quarterly, April 1960, p. 248.

consonance is not an objective, such irregular systems commend themselves as very possibly superior to systems with good fifths or thirds.

Augusto Novarro has fretted guitars in 15-tone temperament, thereby obtaining considerable improvement in the minor thirds and natural sevenths as against those of 12-tone temperament. He uses these guitars in conjunction with other instruments, traditionally tuned, in order that he may also have access to well-tuned fifths. To close the combined system would require 60-tone temperament, but Novarro envisages no such closing of the system; rather the simple coexistence of 12- and 15-tone temperaments. One would imagine that the three pitches common to both systems would have special importance to Novarro, and that the 400-cent interval between them would be of particular significance in music so conceived.

Busoni's supposed 18-tone system has been discussed above. It is in reality a 36-tone system. It can, however, be thought of as combining with 12-tone temperament in an incomplete 36-tone system much like Novarro's incomplete 60-tone system above. Busoni might have tried to maintain the 18-tone scale as a distinct entity, in which case he would have had a fifth-less scale.

The list of proposed equal temperaments without third or fifth is, indeed, quite small. It would seem that once the basic consonances of the harmonic series are eliminated, one system would do quite as well as another.