To Understand this Discussion

Peirce Series, has much in common with the Farey Series, which springs from the Lambdoma.

also related to

Pascal's Triangle, by the sums process

Bruns Algorithm will solve

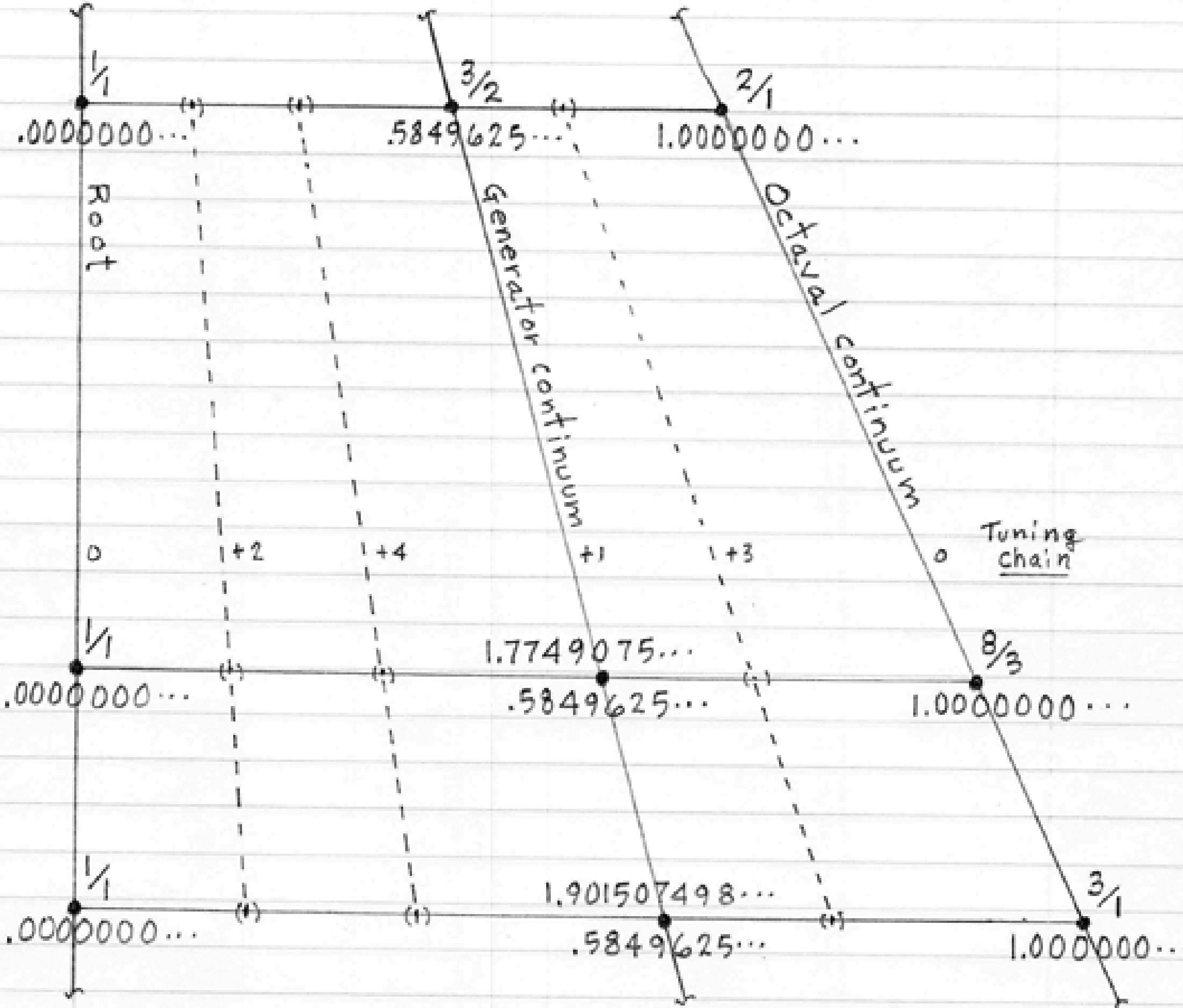
Moments of Symetry (MOS) with great accuracy.

14 Dec 1999

$\log_3$, 8ve utterly mutable, $\log_{1.61803398875}$ out to 12 significant digits, or whatever the computer will handle

8ve; any interval which behaves in the role of the 8ve. May be stated; "$\log_{8ve}$" regardless of octave size. This has the very useful effect of keeping the Scale-Tree relevant. Consider the root at $\frac{1}{1}$ the gen at $\frac{2}{1}$ and the 8ve at $\frac{3}{1}$ for example.

Some Notes on Alternate-Octaves (Octaval Continuum)



8ve $\quad$ n $\quad$ generator

Example:

$$\left(\frac{2}{1}\right)^{.5849625\cdots} = 1.5000000\cdots \quad \left(\frac{3}{1}\right)$$

$$\left(\frac{8}{3}\right)^{.5849625\cdots} = 1.7749075\cdots$$

$$\left(\frac{3}{1}\right)^{.5849625\cdots} = 1.901507498$$

or, $(8ve)^{n}$ = Generator. "n" is continuous, "8ve" is continuous. This ties the tuning of Alternate Octaves (Octaval Continuum) into the Scale-Tree and the associated Gral Keyboard Guide.

844 N. Ave 65
Los Angeles, CA 90042
July 2, 1994

Dear Brian,

Thanks for the delectable morsels in non-octaval tunings. For early use of non-octaval tunings you may wish to review Augusto Novaro, in ~~See~~ Sistema Natural de la Musica about p. 29 some noteworthy examples of what he was doing. The significant details are in arabic notation. If you know Enrique Moreno, he might be willing to translate the Spanish. This the 1951 edition, Library of Congress call # ML3805 N69 1951.

Moreno's Ave Maria (on 12 root of 3) is outstanding. Has anyone considered the 65th root of 3?

$$3^{\left(\frac{5}{65}\right)} = 1.08818224346$$

compares with $3^{\left(\frac{1}{13}\right)} = 1.08818224346$, (which compares with $2^{\left(\frac{5}{41}\right)} = 1.088205647658$). (This is a minutely shrunk 41-tone octave.

I get the above comparisons with the help of Scale-Tree, which is nearly ready for publication. I see no reason why you should hold up on publishing your Taxonomy.

Yours Sincerely,

Erv Wilson

Cuando se precisa una escala, por ejemplo, 1, 3/2, 2/1, sería suficiente con anotar 3/2, 2/1, que en sí ya establecen el fundamental; no obstante, para mayor claridad, emplearemos dicha repetición.

Al expresar una escala no se indica altura de sonidos, únicamente sus relaciones; en la práctica se elevan a la altura debida para producir música.

Después del 2/1, corresponde por su sencillez al metro musical 3/2 establecer el segundo grupo de escalas fundamentales:

Primera,	1	3/2				
Segunda,	1	5/4	3/2			
Tercera,	1	7/6	4/3	3/2		
Cuarta,	1	9/8	5/4	11/8	3/2	
Quinta,	1	11/10	6/5	13/10	7/5	3/2
etc.						

Con igual ordenamiento, el metro 4/3 establece las escalas fundamentales siguientes:

Primera,	1	4/3				
Segunda,	1	7/6	4/3			
Tercera,	1	10/9	11/9	4/3		
Cuarta,	1	13/12	7/6	5/4	4/3	
Quinta,	1	16/15	17/15	6/5	19/15	4/3
etc.						

Siendo nuestro metro musical 5/3, sus respectivas escalas fundamentales serían expresadas en esta forma:

Primera,	1	5/3				
Segunda,	1	4/3	5/3			
Tercera,	1	11/9	13/9	5/3		
Cuarta,	1	7/6	4/3	3/2	5/3	
Quinta,	1	17/15	19/15	7/5	23/15	5/3
etc.						

La relación 2/1 nos servirá, igualmente, de punto de referencia para la clasificación de intervalos *abiertos* o *cerrados*. Serán considerados abiertos, cuando pasen los extremos de esta relación, y cerrados, cuando no lleguen a ella.

Asimismo, este metro nos muestra, dentro de sus escalas fundamentales, los más indispensables intervalos en música. Si a éstos se agrega el 2/1, obtendremos campos armónicos más abiertos.

Para evitar posibles confusiones, aclararemos que sumar intervalos equivale a multiplicar quebrados. Por ejemplo, $4/3 \times 3/2 = 2/1$. Al reducir a un símbolo los dos sonidos que expresan todo quebrado, denominamos al 4/3, una cuarta; al 3/2, una quinta; al 2/1, una octava. Entonces, decimos: una cuarta más una quinta es igual a una octava.

La primera relación abierta más sencilla es el resultado de multiplicar el 2/1 por su mismo valor: $2/1 \times 2/1 = 4/1$. Sus respectivas escalas fundamentales son las siguientes:

Primera,	1	4/1				
Segunda,	1	5/2	4/1			
Tercera,	1	2/1	3/1	4/1		
Cuarta,	1	7/4	5/2	13/4	4/1	
Quinta,	1	8/5	11/5	14/5	17/5	4/1
etc.						

En igual forma obtenemos el 3/1: $2/1 \times 3/2 = 3/1$; siendo sus escalas fundamentales:

Primera,	1	3/1				
Segunda,	1	2/1	3/1			
Tercera,	1	5/3	7/3	3/1		
Cuarta,	1	3/2	2/1	5/2	3/1	
Quinta,	1	7/5	9/5	11/5	13/5	3/1
etc.						

Del 8/3, $2/1 \times 4/3 = 8/3$, se obtienen las escalas fundamentales siguientes:

Primera,	1	8/3				
Segunda,	1	11/6	8/3			
Tercera,	1	14/9	19/9	8/3		
Cuarta,	1	17/12	11/6	9/4	8/3	
Quinta,	1	4/3	5/3	2/1	7/3	8/3
etc.						

$\frac{2}{2}$ $\frac{3}{2}$

$\frac{5}{4}$

$\frac{7}{6}$ $\frac{8}{6}$

$\frac{9}{8}$ $\frac{12}{10}$ $\frac{13}{10}$ $\frac{11}{8}$

$\frac{11}{10}$ $\frac{16}{14}$ $\frac{19}{16}$ $\frac{17}{14}$ $\frac{18}{14}$ $\frac{21}{16}$ $\frac{19}{14}$ $\frac{14}{10}$

$\frac{13}{12}$ $\frac{20}{18}$ $\frac{25}{22}$ $\frac{23}{20}$ $\frac{26}{22}$ $\frac{31}{26}$ $\frac{29}{24}$ $\frac{22}{18}$ $\frac{23}{18}$ $\frac{31}{24}$ $\frac{34}{26}$ $\frac{29}{22}$ $\frac{27}{20}$ $\frac{30}{22}$ $\frac{25}{18}$ $\frac{17}{12}$

$\frac{3}{3}$ $\frac{4}{3}$

$\frac{7}{6}$

$\frac{10}{9}$ $\frac{11}{9}$

$\frac{13}{12}$ $\frac{17}{15}$ $\frac{18}{15}$ $\frac{15}{12}$

$\frac{16}{15}$ $\frac{23}{21}$ $\frac{27}{24}$ $\frac{24}{21}$ $\frac{25}{21}$ $\frac{29}{24}$ $\frac{26}{21}$ $\frac{19}{15}$

$\frac{1}{1}$ $\frac{4}{1}$

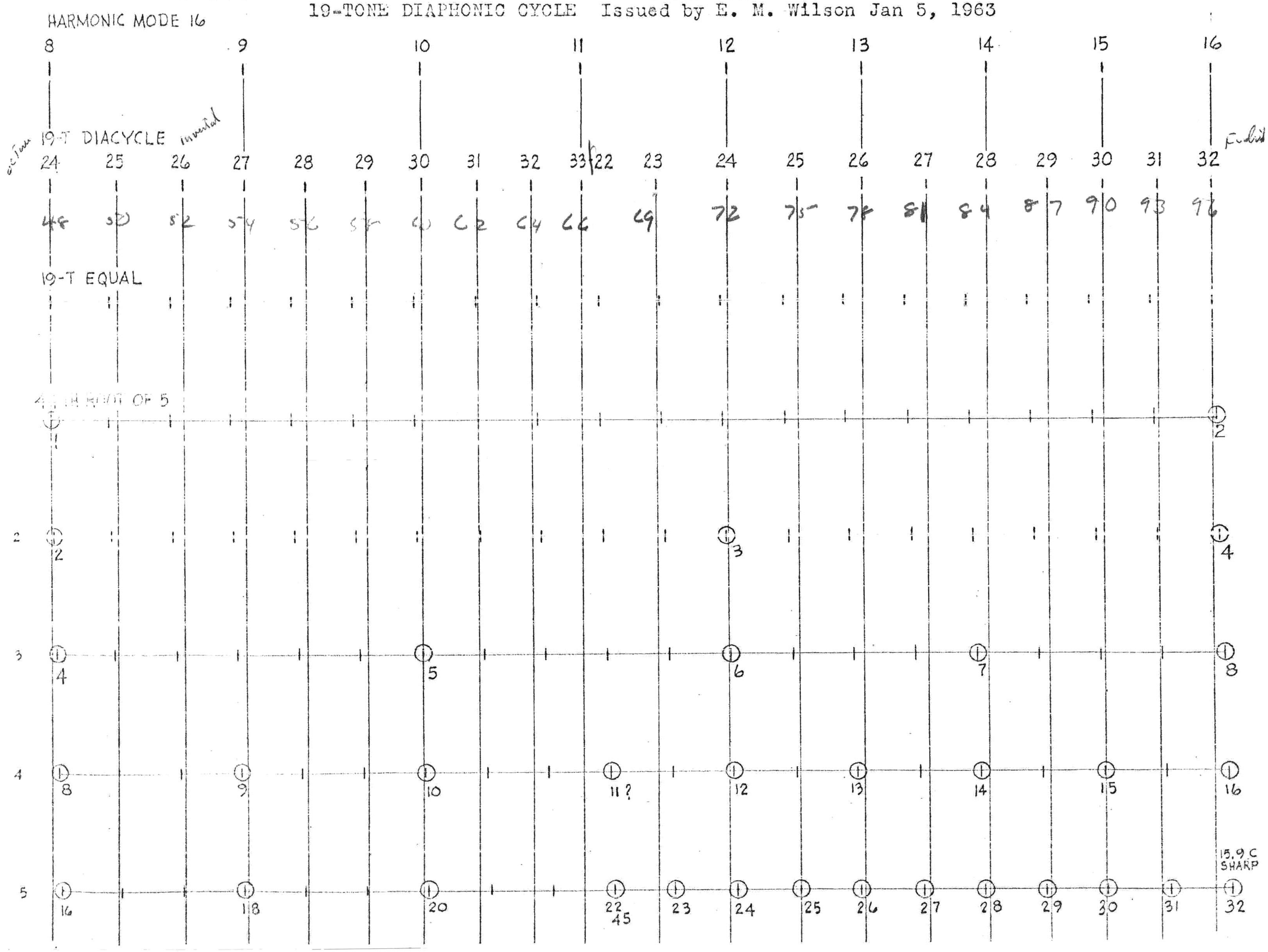
$\frac{5}{2}$

$\frac{6}{3}$ $\frac{9}{3}$

$\frac{7}{4}$ $\frac{11}{5}$ $\frac{14}{5}$ $\frac{13}{4}$

$\frac{8}{5}$ $\frac{13}{7}$ $\frac{17}{8}$ $\frac{16}{7}$ $\frac{19}{7}$ $\frac{23}{8}$ $\frac{22}{7}$ $\frac{17}{5}$

THE SCALE OF THE 44TH ROOT OF 5 PLOTTED AGAINST THE HARMONIC SERIES & THE 19-TONE DIAPHONIC CYCLE Issued by E. M. Wilson Jan 5, 1963



$$\begin{pmatrix} \frac{1}{4} & \frac{2}{7} & \frac{1}{3} \end{pmatrix} \times \frac{4}{1} = \begin{pmatrix} \frac{4}{4} & \frac{8}{7} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{3}{5} & \frac{2}{3} \end{pmatrix} \times \frac{2}{1} = \begin{pmatrix} \frac{2}{2} & \frac{6}{5} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{4} & \frac{4}{5} & \frac{1}{1} \end{pmatrix} \times \frac{4}{3} = \begin{pmatrix} \frac{12}{12} & \frac{16}{15} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{1} & \frac{5}{4} & \frac{4}{3} \end{pmatrix} \times \frac{1}{1} = \begin{pmatrix} \frac{1}{1} & \frac{5}{4} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} & \frac{2}{1} \end{pmatrix} \times \frac{2}{3} = \begin{pmatrix} \frac{6}{6} & \frac{10}{9} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{1} & \frac{7}{2} & \frac{4}{1} \end{pmatrix} \times \frac{1}{3} = \begin{pmatrix} \frac{3}{3} & \frac{7}{6} & \frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{4} & \frac{2}{7} & \frac{1}{3} \end{pmatrix} \times \frac{3}{1} = \begin{pmatrix} \frac{3}{4} & \frac{6}{7} & \frac{3}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{3}{5} & \frac{2}{3} \end{pmatrix} \times \frac{3}{2} = \begin{pmatrix} \frac{3}{4} & \frac{9}{10} & \frac{6}{6} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{4} & \frac{4}{5} & \frac{1}{1} \end{pmatrix} \times \frac{1}{1} = \begin{pmatrix} \frac{3}{4} & \frac{4}{5} & \frac{1}{1} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{1} & \frac{5}{4} & \frac{4}{3} \end{pmatrix} \times \frac{3}{4} = \begin{pmatrix} \frac{3}{4} & \frac{15}{16} & \frac{12}{12} \end{pmatrix}$$

TRIPLET SETS WITHIN THE SCALE TREE

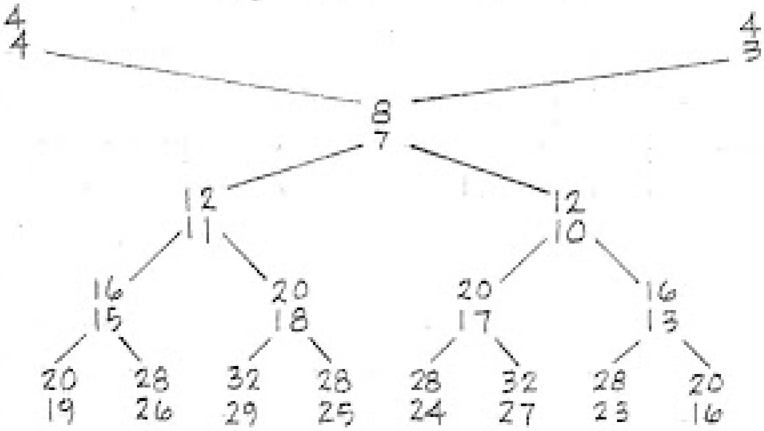
RAISED TO SEED TREE WITHIN A FOURTH

$\left(\frac{4}{4}, \frac{4}{3}\right)$ Scale-Tree, and $\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangle

* "$\frac{c-a}{c-b} = \frac{b}{a}$", Neo-Pythagorean School/Ghyka 1977

Scale-Tree

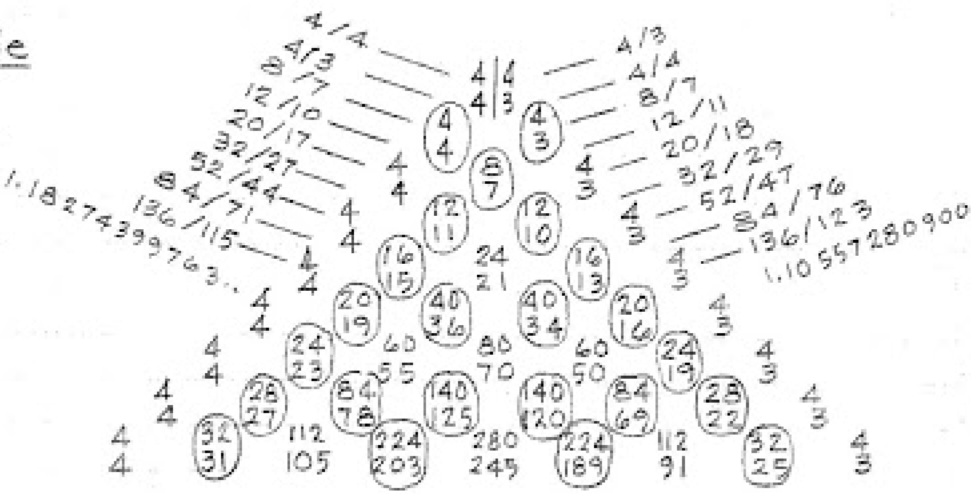
(Stern-Brocot, Peirce)



reduced:	1/1	20/19	16/15	14/13	12/11	32/29	10/9	28/25	8/7	7/6	20/17	32/27	6/5	28/23	16/13	5/4	4/3
epimoria:	20/19	76/75	105/104	78/77	88/87	145/144	126/125	50/49	49/48	120/119	136/135	81/80	70/69	92/91	65/64	16/15	

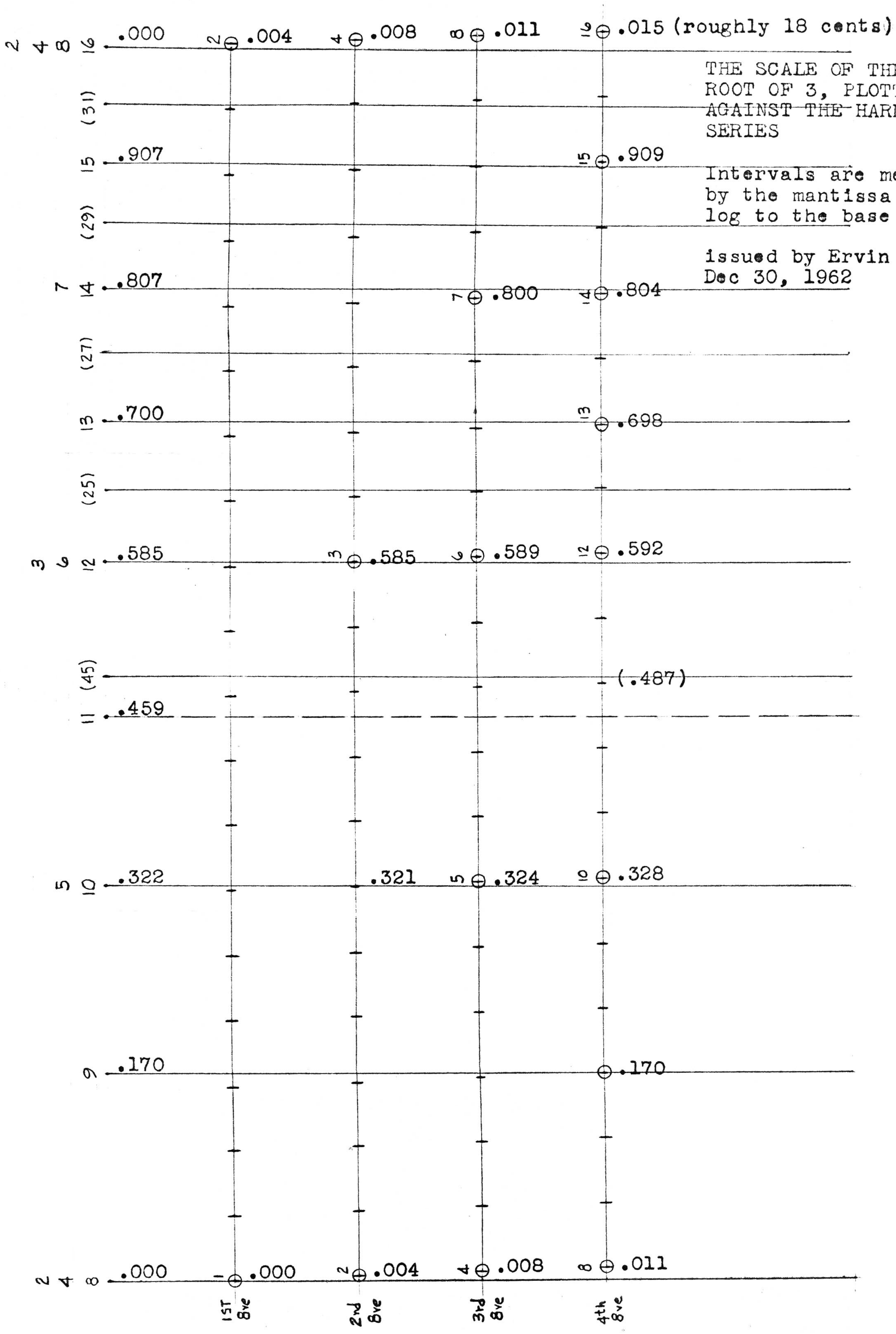
Triangle

(Pascal, Meru)



reduced;	4/4	32/31	28/27	24/23	20/19	16/15	14/13	12/11	32/29	10/9	28/25	8/7	7/6	20/17	32/27	6/5	28/23	16/13	5/4	24/19	14/11	32/25	4/3
epimoria;	32/31	217/216	162/161	115/114	76/75	105/104	78/77	88/87	145/144	126/125	50/49	49/48	120/119	136/135	81/80	70/69	92/91	65/64	96/95	133/132	176/175	25/24	

* Example; $\frac{8-4}{8-4} = \frac{4}{4}$, and $\frac{7-3}{7-4} = \frac{4}{3}$.



THE SCALE OF THE 30TH ROOT OF 3, PLOTTED AGAINST THE HARMONIC SERIES

Intervals are measured by the mantissa of their log to the base 2

issued by Ervin M. Wilson
Dec 30, 1962

$5\,(13/11)$'s plus $4\,(15/13)$'s

$= 2$ Octaves

(.0308 44007 shrink)

$5\,(15/13)$'s plus $4\,(13/11)$'s

$= 2$ octaves

(.003713215 (stretch)

The Golden Section of the 1/4 1/5 series is approximated by 121393/560597 = .216542364

It has Moments of symetry at 1, 2, 3, 4, 5, 9, 14, 23, 37 etc

	$Logs_2$
0.	.000000
1.	.216542
2.	.433085
3.	.649627
4.	.866169
5.	.082712
6.	.299254
7.	.515796
8.	.732339
9.	.948881
10.	.165424
11.	.381966
12.	.598508
13.	.815051
14.	.031593
15.	.248135
16.	.464678
17.	.681220
18.	.897762
19.	.114305
20.	.330847
21.	.547390
22.	.763932
23.	.980474
24.	.197017
25.	.413559
26.	.630101
27.	.846643
28.	.063186
29.	.279728
30.	.496271
31.	.712813
32.	.929355
33.	.145898
34.	.362440
35.	.578983
36.	.795525
37.	8.012067

Stretched Pelog

15¢ stretch = 2.017 40396758, Log_2 = 1.0125
Fourth = 1.363 53 143761, Log_2 = .447347963445
close to (15/11)

Generator
.441825149077
1/14 pattern

←	2	.264
→	3	.797
←	1	.254
→	3	.935
←	1	.068
	14	.523
	1	.910

0/1 1/1 1/2 1/3 2/5 3/7 4/9 7/16 11/25 15/34 19/43 34/77 53/120 14 places

264 × 1.363 = 359.972299529
− 264
= 95.9722995290
× 2.017.
= 193.614897848

Proof

Wilson's stretched pelog
circa pre oct,93

REF SENSATIONS OF TONE, P. 517, 74. NEW--- INDIAN CHROMATIC SCALE (INFERRED BY ELLIS FROM TAGORE)

0	49	99	151	204	259	316	374	435	498	543	589	637	685	736	787	841	896	952	1011	1070	1135	1200
.000	.041	.083	.126	.170	.216	.263	.312	.362	.415	.452	.491	.531	.571	.613	.656	.701	.747	.795	.842	.892	.946	1.000
36	35	34	33	32	31	30	29	28	27/39	38	37	36	35	34	33	32	31	30	29	28	27	26

DIAPHONIC ANALYSIS

403/333 → Factored $\frac{13 \cdot 31}{3^2 \cdot 37}$, Complexity value = 87 (sum of factored less powers of 27)

31/29 complexity value = 60

37/26, CV = 50

73. OLD (INFERRED) THIS IS VERY GOOD

0	51	102	153	204	$264\frac{2}{3}$	$325\frac{1}{3}$	386	442	498	549	600	651	702	753	804	855	906	$966\frac{2}{3}$	$1027\frac{1}{3}$	1088	1144	1200
.000	.042	.085	.128	.170	.221	.271	.322	.368	.415	.458	.500	.542	.585	.628	.670	.712	.755	.806	.856	.907	.953	1.000
1/1				9/8			5/4		4/3				3/2				27/16			15/8		2/1
32	33	34	35	27/36	28	29	30	31	32	33	34	35	36	37	38	39/26	27	28	29	30	31	32
1	33	17	35	9	(7/6)		5		(4/3)	11		(35/24)	3			13	27	7	29	15	31	2

DIAPHONIC IMPLICATION, WILSON

HARMONICS, REF.

INDIAN PARTIAL SCALES AS PLAYED BY RAJAH RÁM PÁL SINGH, WHICH HAVE LITTLE RESEMBLANCE TO THE ABOVE SYSTEMS

76. 1st

0	183	342	533	685	871	1074	1230
.000	.152	.285	.444	.571	.726	.895	1.025

77. 4th

0	174	350	477	697	908	1070	1181
.000	.145	.292	.398	.581	.757	.892	.984

78. 2nd

0	183	271	534	686	872	983	1232
.000	.152	.226	.445	.572	.727	.819	1.027

79. 3rd

0	111	314	534	686	828	1017	1198
.000	.092	.262	.445	.572	.690	.848	.998

80. 5th

0	90	366	493	707	781	1080	1187
.000	.075	.305	.411	.589	.651	.900	.989

Part of this flexibility might be explained by the fact that in the diaphonic-oriented species 6 units, for example may vary from 32/26 to 39/33; and 5 units would vary from 31/26 to 39/34. There is a slight cross-over as 5 units of 31/26 is larger than 6 units of 39/33. In 73. OLD this type of cross-over is subtly avoided.

PREPARED BY ERV WILSON, 16 NOV 65

$\sqrt[39]{(77/32)}$ .0324817

$\times 14 = .4547439$ $-.0046877$ (-5.62c)

$11/8 = .4594316$

$\times 18 = .5846707$ $-.0002918$ $(-.35\text{c})$

$3/2 = .5849625$

$\times 25 = .8120427$ $+.0046878$ $(+5.62\text{c})$

$\times 45 = 1.4616768$ $+.0022452$ $+(2.69\text{c})$

$\sqrt[49]{3}$

.0323462

×10 =	.3234617	+.0015336	(1.84c)
5/4 =	.3219281		
10/31 =	.3225806	+.0006525	(.78¢)
×18 =	.5822311	−.0027314	(−3.28c)
3/2 =	.5849625		
18/31 =	.5806452	−.0043173	(−5.18)
×25 =	.8086543	+.0012994	(1.56c)
7/4 =	.8073549		
25/31 =	.8064516	−.0009033	(−1.08)
×28 =	.9056929	−.0011977	(−1.44c)
15/8 =	.9068906		*
28/31 =	.9032258	−.0036648	(−4.40)
×36 =	1.1644622	−.0054628	(−6.55)
9/4 =	1.169925		
36÷31 =	1.1612903	−.0086347	(−10.36)
×45 =	1.4555778	−.0038538	(−4.62)
11/4 =	1.4594316		
45÷31 =	1.4516129	−.0078187	(−9.38)
×54 =	1.7466934	−.0081941	(9.83)
27/8 =	1.7548875		
54/31 =	1.7419355	−.012952	(−15.54)
×56 =	1.8113857	+.0040308	(4.84)
7/2	1.8073549		
56÷31 =	1.8064516	−.0009033	(1.08)
×62 =	2.0054628	+.0054628	(+6.55)
4/1 =	2.0000000		

$\overline{.0337664}$

− .0283036

.0398466 for equal

$\sqrt[18]{(3/2)}$

.0324979

×10 =	.3249792	+.0030511	(3.66c)
×18 =	.5849625	±.0000000	
×25 =	.8124479	+.005093	(6.11)
×28 =	.9099417	+.0030511	(3.66)
×36 =	1.169925	±.000000	
×45 =	1.4624063	+.0029747	(3.57)
×54 =	1.7548875	±.0000000	
×56 =	1.8198833	+.0125284	(15.03)
×62 =	2.0148708	+.0148708	(17.84)

$\overline{.0415691}$

− .0266983

× 5 =	.1624896	−.0074354	(8.92)
× 14 =	.4549708	−.0044608	(−5.35)
× 31 =	1.0074354	+.0074354	(8.92)

$\sqrt[67]{9/2} = 1.0227028$ 8ve = 2.005546

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0.	528.00
1.	539.99
2.	552.25
3.	564.78
4.	577.60
5.	590.72
6.	604.13
7.	617.84
8.	631.87
9.	646.22
10.	660.89
11.	675.89
12.	691.24
13.	706.93
14.	722.98
15.	739.39
16.	756.18
17.	773.35
18.	790.90
19.	808.86
20.	827.22
21.	846.00
22.	865.21
23.	884.85
24.	904.94
25.	925.49
26.	946.50
27.	967.98
28.	989.96
29.	1012.44
30.	1035.42
31.	1058.93
32.	1082.97
33.	1107.55
34.	1132.70
35.	1158.42
36.	1184.71
37.	1211.61
38.	1239.12
39.	1267.25
40.	1296.02
41.	1325.44
42.	1355.53
43.	1386.31
44.	1417.78
45.	1449.97
46.	1482.89
47.	1516.55
48.	1550.98
49.	1586.19
50.	1622.20
51.	1659.03
52.	1696.70
53.	1735.22
54.	1774.61
55.	1814.90
56.	1856.10
57.	1898.24
58.	1941.34
59.	1985.41
60.	2030.49
61.	2076.58
62.	2123.73
63.	2171.94
64.	2221.25
65.	2271.68
66.	2323.25
67.	2376.00

Why do the 5 & 7 sound good in 12 ET ?

0/12 = 400

4/12 = 503.96842

10/12 = 712.71897

×2 = 1007.9368

} 208.7 shrunk

295.21787 streched by 27.8 ¢

a not uncommon strech

Study on $\sqrt[67]{9/2}$

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$\sqrt[55]{55/16}$ =.0323884

$\sqrt[43]{21/8}$ =.0323795

Virtually identical

* $\sqrt[70]{77/16}$ .0323827

$\sqrt[67]{9/2}$ .0323869

($\sqrt[28]{15/8}$ .0323889) ≠ +.00002 (.0024¢)

×31 = 1.0039951 +.0039951 (+4.79)

×62 = 2.0079903 +.0079903 (+9.59¢)

×36 = 1.1659299 −.0039951 (−4.79¢)

9/4 = 1.169925

×5 = .1619347 −.0079903 (−9.59¢)

* ×45 = 1.4572199 −.0022117 (−2.65)

×45 = 1.4574123 −.0020193 (−2.42)

11/4 = 1.4594316

×76 = 2.4614075 +.0019759 (+2.37)

×10 = .3238694 +.0019413 (+2.334)

5/4 = .3219281

×18 = .5829649 −.0019976 (−2.40¢) *

3/2 = .5849625

×49 = .5869601 +.0019976 (+2.40) *

* ×25 = .8095666 +.0022117 (2.65)

×25 = .8096735 +.0023186 (+2.78)

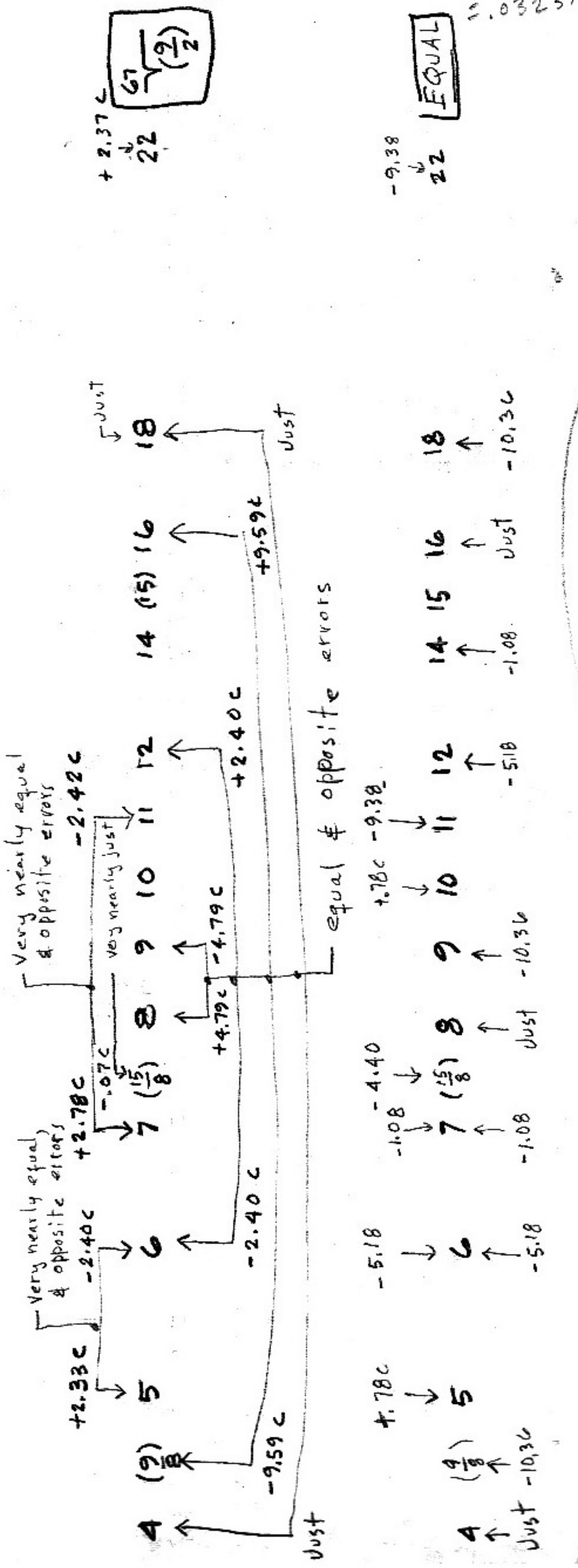
7/4 = .8073549

×28 = .9068343 −.0000563 (.07¢)

15/8 = .9068906

×54 = 1.7488948 −.0059927 (−7.19)

27/8 = 1.7548875



In $\sqrt[70]{77/16}$ the 11/4 & 7/4 have Equal & Opposite errors.

In $\sqrt[28]{15/8}$ the (5/4 & 3/2) have equal and opposite discrepancies.