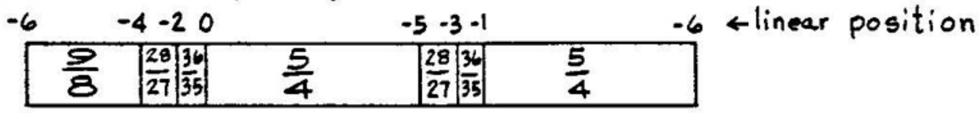
The Purvi Modulations

When the tetrachordal scale begins to exert inertia along it's linear series of Fourths, and the Fourths are variable in size – but the closing Fourth is invariable – a pattern of variable linear modulations and/or parallel, variable linear modes occurs, based on the extrapolation of the generating segment of 6 Fourths by its repetition.

Using the Enharmonic scale of Archytas to illustrate, the linear segment of Fourths is located, and the closing Fourth identified;



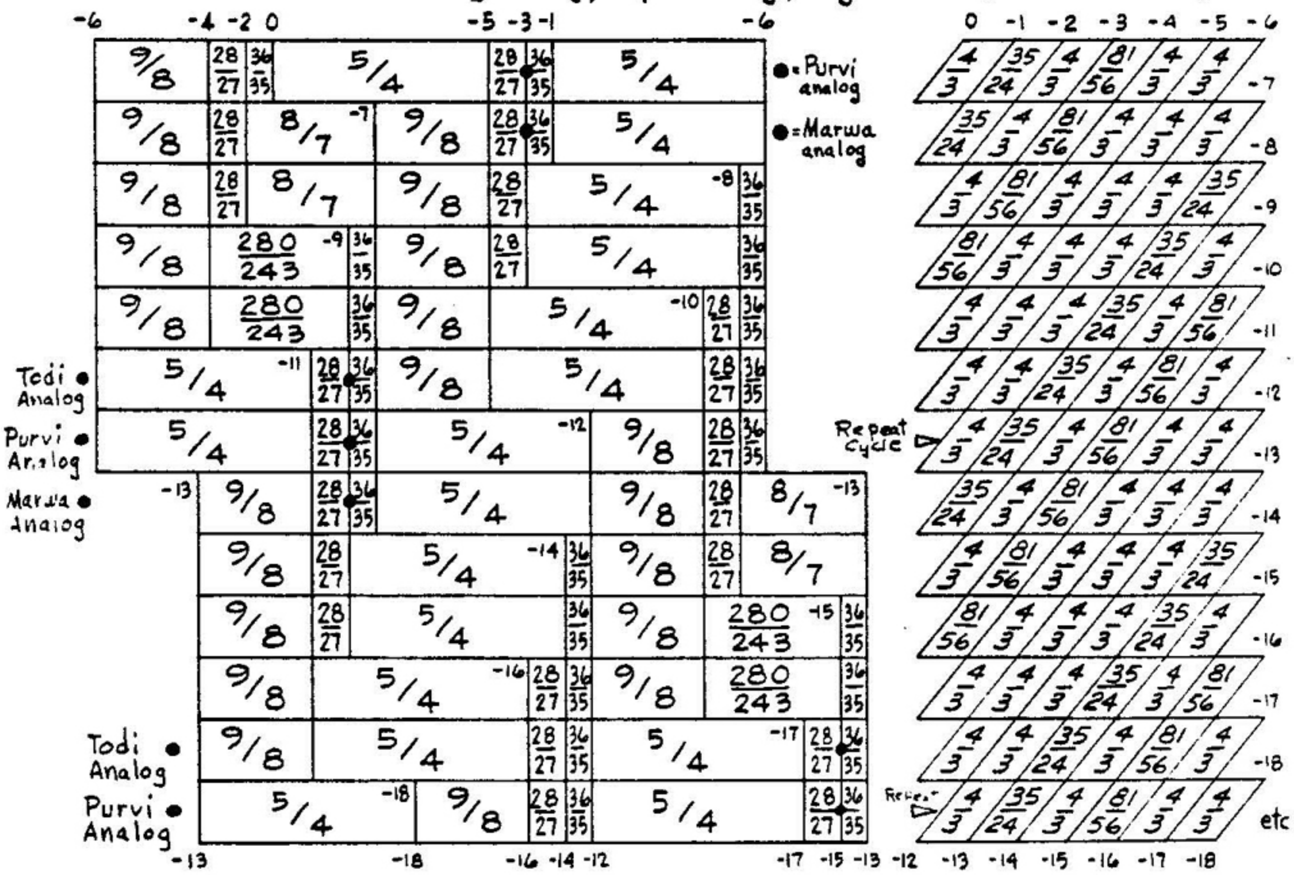
$$\begin{array}{ccccccccc} 0 & -1 & -2 & -3 & -4 & -5 & -6 & & 0 \end{array}$$

$$\underbrace{\frac{4}{3}\ \frac{35}{24}\ \frac{4}{3}\ \frac{81}{56}\ \frac{4}{3}\ \frac{4}{3}}\left(\frac{6}{5}\right)$$ — resultant, Closing Fourth, atypical in size

The Generating Segment of variable Fourths, which is repeated;

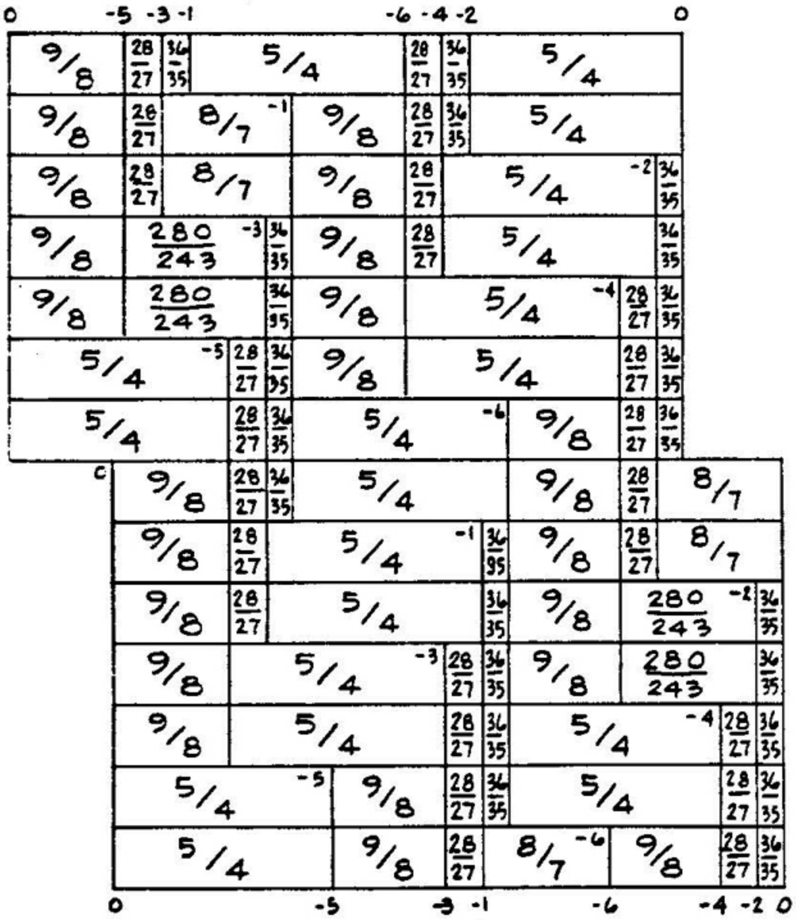
0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	etc
$\frac{4}{3}$	$\frac{35}{24}$	$\frac{4}{3}$	$\frac{81}{56}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{35}{24}$	$\frac{4}{3}$	$\frac{81}{56}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{35}{24}$	$\frac{4}{3}$	$\frac{81}{56}$	$\frac{4}{3}$	$\frac{4}{3}$		indefinitely

A set of variable linear modulations is then constructed from this extended series by taking, sequentially, segments of 6 Fourths;



2 complete cycles are shown. Fig I thru II are a single cycle.

This progression of scales may be expressed in another way, which emphasizes the cyclical (over the linear) and brings out the role that the closing Fourth plays in these changes. This cycle begins to repeat itself on the 42nd change, but in a very remote key, of course. It proceeds like this;



etc

0 -1 -2 -3 -4 -5 -6 0

$\boxed{6/5}$	4/3	35/24	4/3	81/56	4/3	4/3
4/3	$\boxed{6/5}$	35/24	4/3	81/56	4/3	4/3
4/3	35/24	$\boxed{6/5}$	4/3	81/56	4/3	4/3
4/3	35/24	4/3	$\boxed{6/5}$	81/56	4/3	4/3
4/3	35/24	4/3	81/56	$\boxed{6/5}$	4/3	4/3
4/3	35/24	4/3	81/56	4/3	$\boxed{6/5}$	4/3
4/3	35/24	4/3	81/56	4/3	4/3	$\boxed{6/5}$
$\boxed{6/5}$	35/24	4/3	81/56	4/3	4/3	4/3
35/24	$\boxed{6/5}$	4/3	81/56	4/3	4/3	4/3
35/24	4/3	$\boxed{6/5}$	81/56	4/3	4/3	4/3
35/24	4/3	81/56	$\boxed{6/5}$	4/3	4/3	4/3
35/24	4/3	81/56	4/3	$\boxed{6/5}$	4/3	4/3
35/24	4/3	81/56	4/3	4/3	$\boxed{6/5}$	4/3
35/24	4/3	81/56	4/3	4/3	4/3	$\boxed{6/5}$

0 -1 -2 -3 -4 -5 -6 0

etc

And this approach deserves far more attention than given here.

This paper is complementary to The Marwa Permutations which appeared in Xenharmonikon IX. In this use of the words, "permutation" indicates that the Fourths are re-arranged in linear sequence; "modulation" indicates that the closing Fourth is re-located in pitch.

References;

Applied Theory of Indian Music (North), Amiya Dasgupta
1977, California Institute of the Arts, Valencia

The Divisions of the Tetrachord, John Chalmers, unpublished

Sensations of Tone, Additions by the Translator, Helmholtz/Ellis

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Fig 1

Pythagoras ($\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$) all 3 permutations

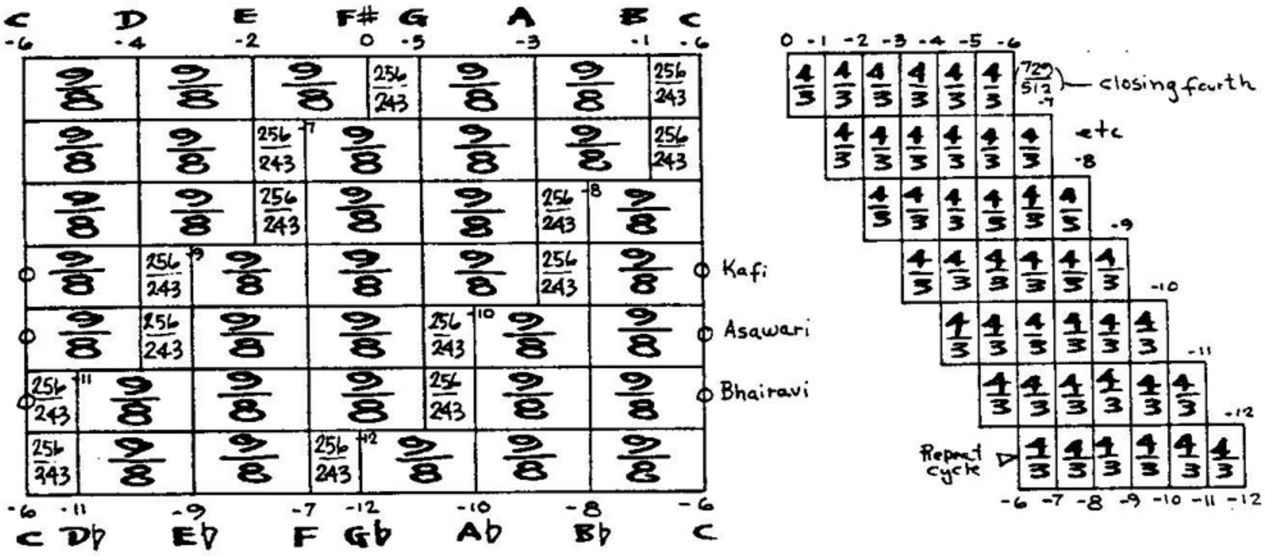

Fig 2a

Helmholtz ($\frac{75}{64}$ $\frac{16}{15}$ $\frac{16}{15}$)*, ($\frac{16}{15}$ $\frac{16}{15}$ $\frac{75}{64}$)+

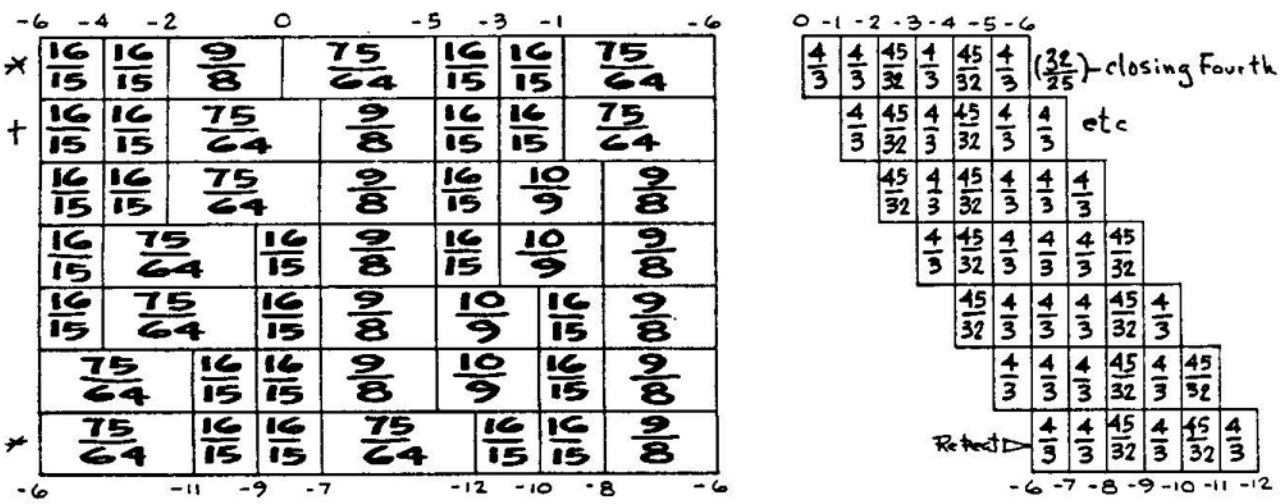

Fig 2b

Helmholtz ($\frac{16}{15}$ $\frac{75}{64}$ $\frac{16}{15}$)*

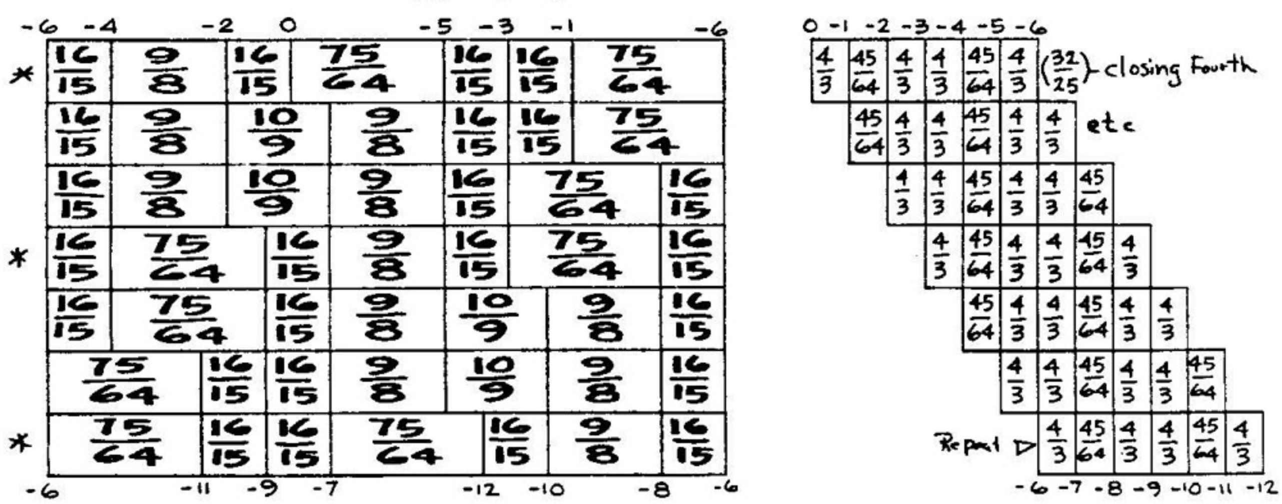

Fig 3a

Al-Farabi $(\frac{8}{7}\ \frac{8}{7}\ \frac{49}{48})^{*}$, $(\frac{49}{48}\ \frac{8}{7}\ \frac{8}{7})^{+}$

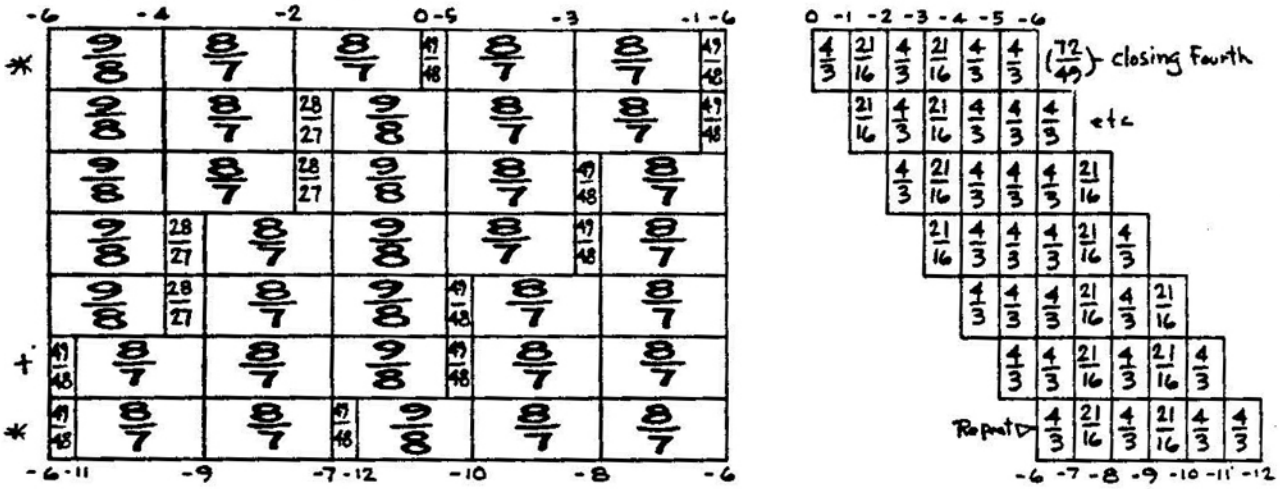

Fig 3b

Al-Farabi $(\frac{8}{7}\ \frac{49}{48}\ \frac{8}{7})^{*}$

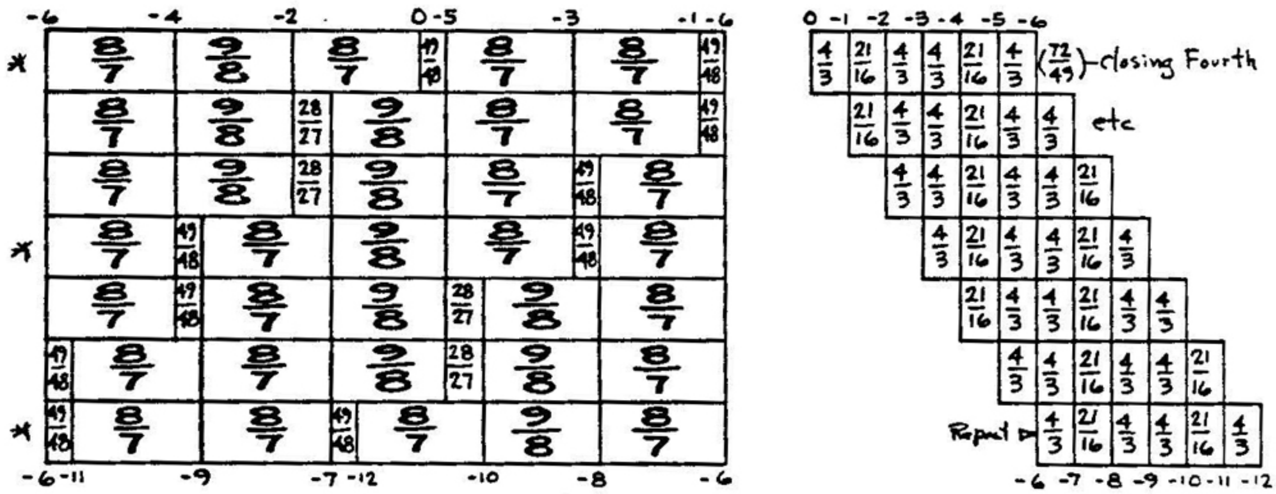

Fig 4

Archytas $(\frac{8}{7}\ \frac{9}{8}\ \frac{28}{27})$ all 6 permutations *

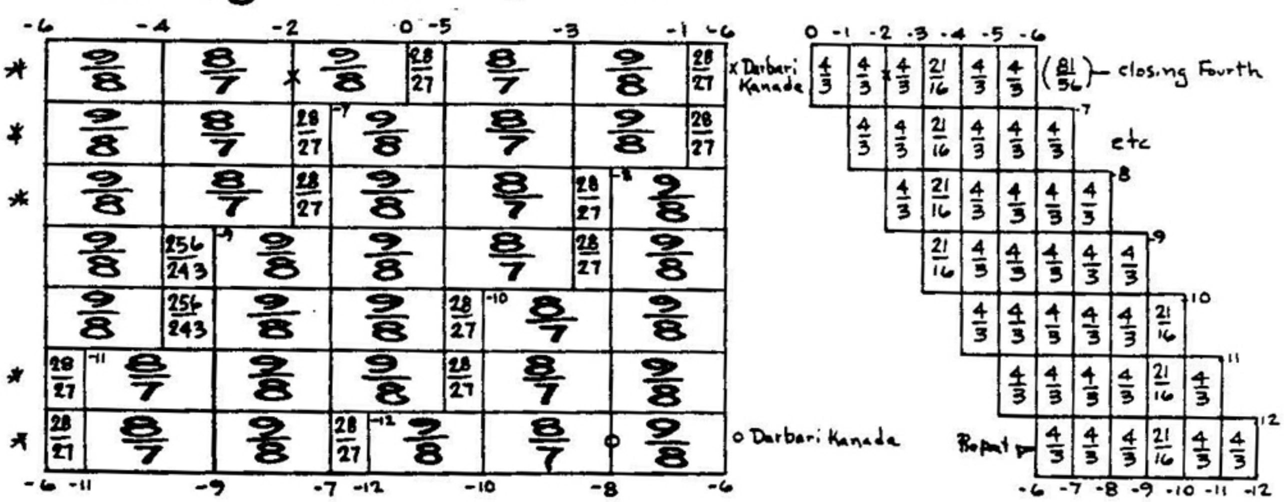

Fig 5

Didymus/Ptolemy $\left(\frac{10}{9}\ \frac{9}{8}\ \frac{16}{15}\right)$ all 6 permutations*

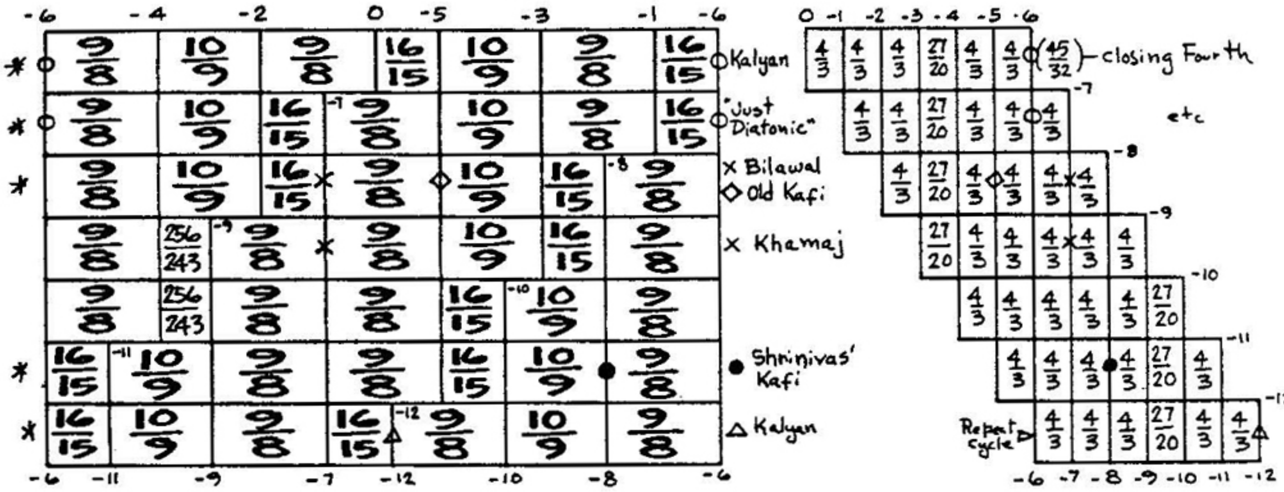

Fig 6a

Al-Farabi $\left(\frac{10}{9}\ \frac{10}{9}\ \frac{27}{25}\right)^*$, $\left(\frac{27}{25}\ \frac{10}{9}\ \frac{10}{9}\right)^+$

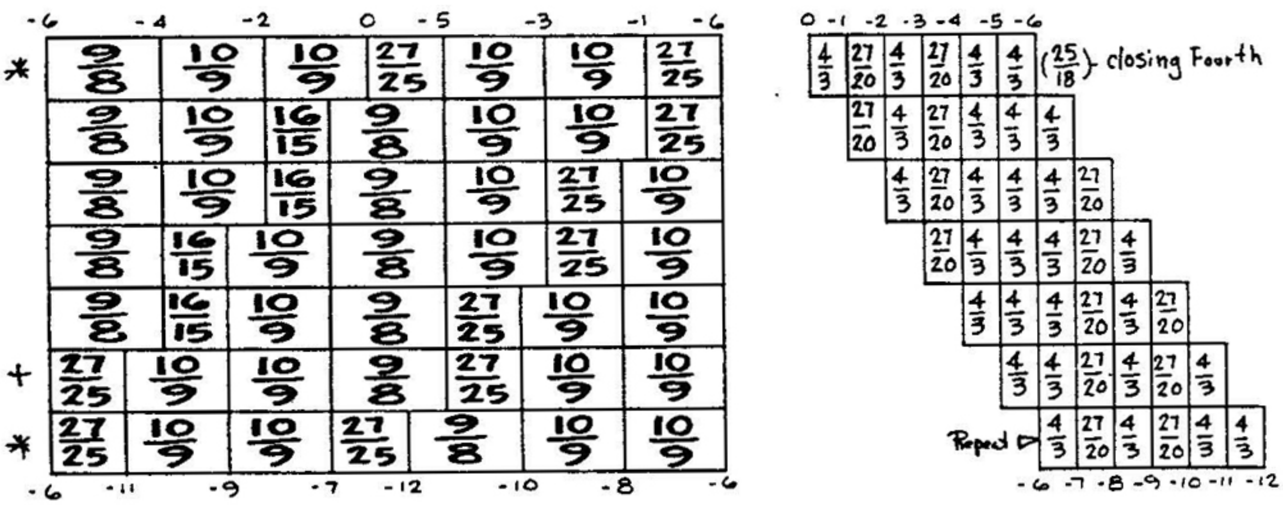

Fig 6b

Al-Farabi $\left(\frac{10}{9}\ \frac{27}{25}\ \frac{10}{9}\right)^*$

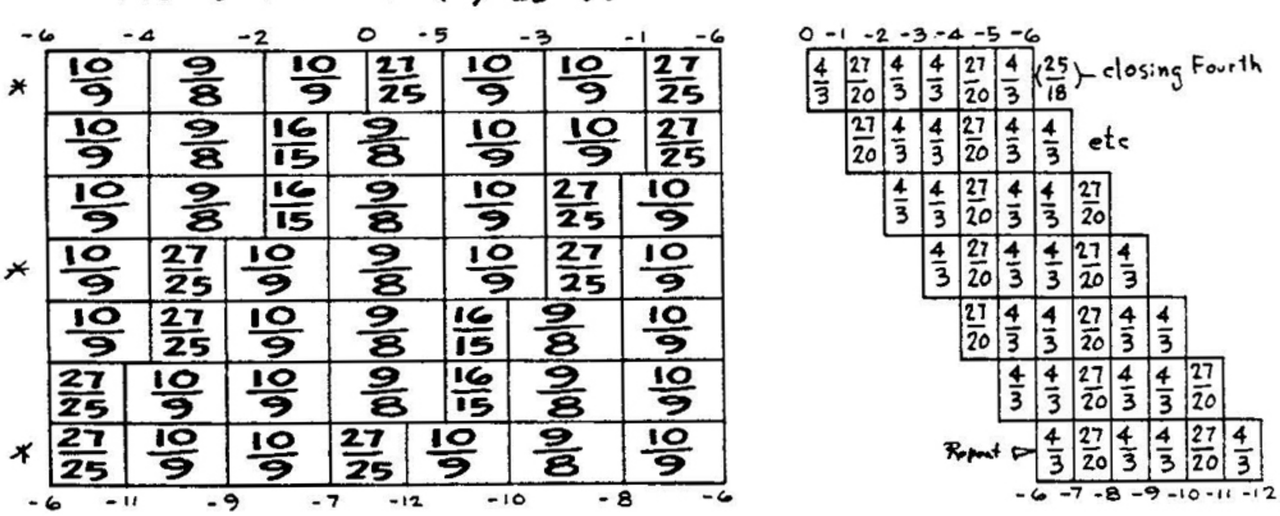

Fig 7a

Kathleen Schlesinger $\left(\frac{13}{12}\,\frac{16}{15}\,\frac{15}{13}\right)^{*}$, $\left(\frac{15}{13}\,\frac{13}{12}\,\frac{16}{15}\right)^{+}$

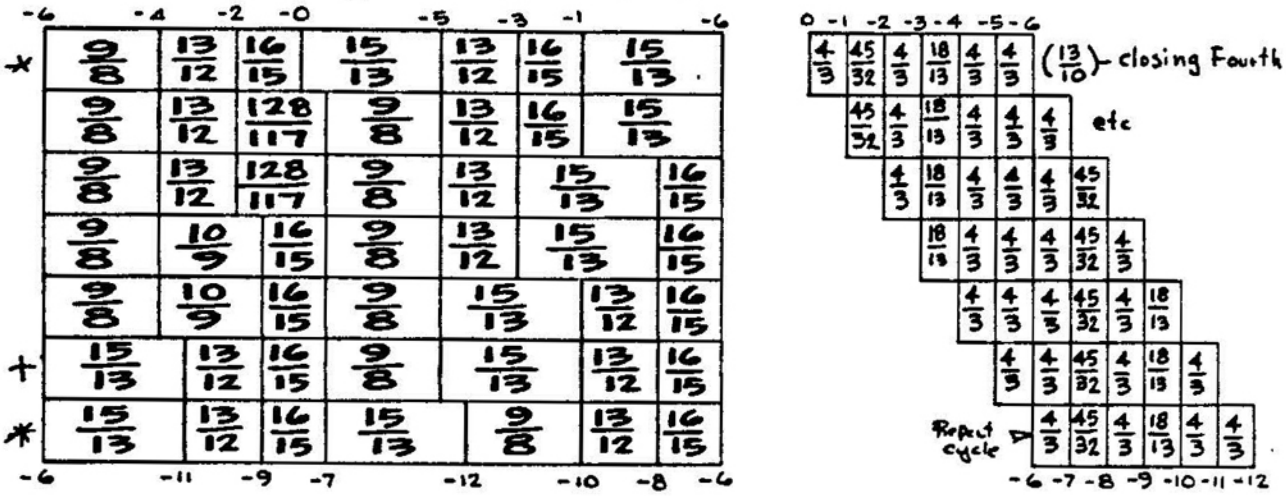

Fig 7b

Kathleen Schlesinger $\left(\frac{15}{13}\,\frac{16}{15}\,\frac{13}{12}\right)^{*}$, $\left(\frac{16}{15}\,\frac{13}{12}\,\frac{15}{13}\right)^{+}$

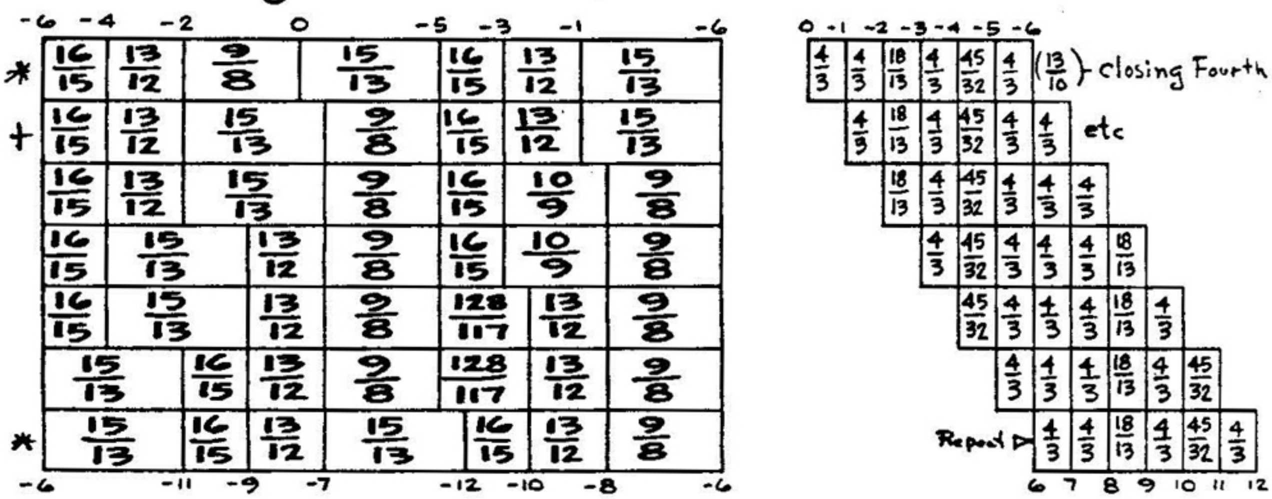

Fig 7c

Kathleen Schlesinger $\left(\frac{16}{15}\,\frac{15}{13}\,\frac{13}{12}\right)^{*}$, $\left(\frac{13}{12}\,\frac{15}{13}\,\frac{16}{15}\right)^{+}$

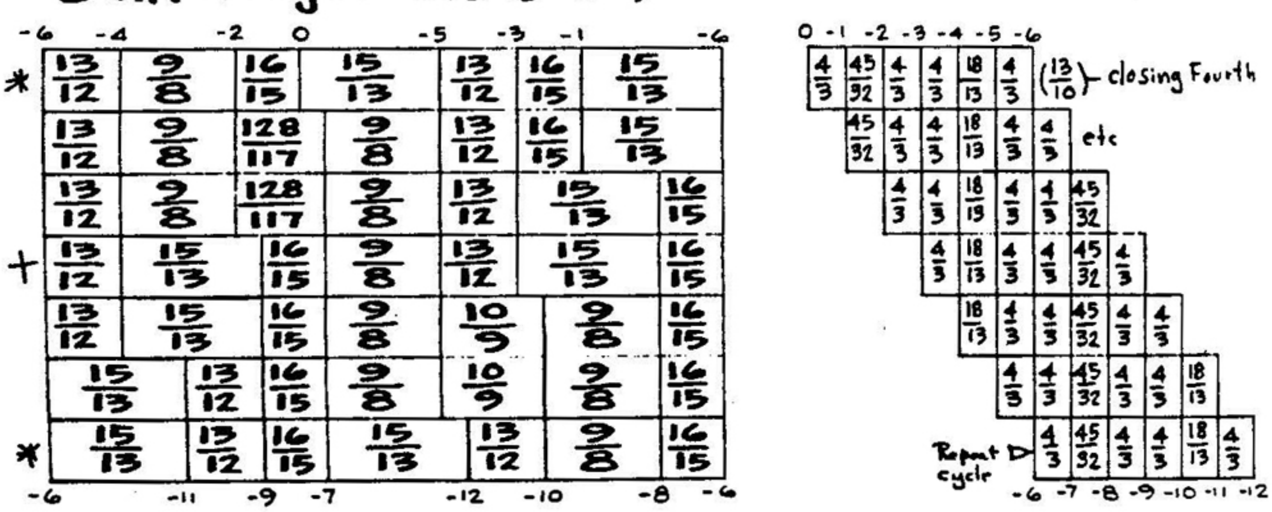

Fig 8a

Ptolemy (12/11 22/21 7/6)*, (7/6 12/11 22/21)+

-6 -4 -2 0 -5 -3 -1 -6

*	9/8	12/11	22/21	7/6	12/11	22/21	7/6
	9/8	12/11	88/81	9/8	12/11	22/21	7/6
	9/8	12/11	88/81	9/8	12/11	7/6	22/21
	9/8	112/99	22/21	9/8	12/11	7/6	22/21
	9/8	112/99	22/21	9/8	7/6	12/11	22/21
+	7/6	12/11	22/21	9/8	7/6	12/11	22/21
*	7/6	12/11	22/21	7/6	9/8	12/11	22/21

-6 -11 -9 -7 -12 -10 -8 -6

0 -1 -2 -3 -4 -5 -6

4/3	63/44	4/3	11/8	4/3	4/3	(9/7) closing Fourth						
	63/44	4/3	11/8	4/3	4/3	4/3						
		4/3	11/8	4/3	4/3	4/3	63/44					
			11/8	4/3	4/3	4/3	63/44	4/3				
				4/3	4/3	4/3	63/44	4/3	11/8			
					4/3	4/3	63/44	4/3	11/8	4/3		
						4/3	63/44	4/3	11/8	4/3	4/3	

-6 -7 -8 -9 -10 -11 -12

Fig 8b

Ptolemy (7/6 22/21 12/11)*, (22/21 12/11 7/6)+

-6 -4 -2 0 -5 -3 -1 -6

*	22/21	12/11	9/8	7/6	22/21	12/11	7/6
+	22/21	12/11	7/6	9/8	22/21	12/11	7/6
	22/21	12/11	7/6	9/8	22/21	112/99	9/8
	22/21	7/6	12/11	9/8	22/21	112/99	9/8
	22/21	7/6	12/11	9/8	88/81	12/11	9/8
	7/6	22/21	12/11	9/8	88/81	12/11	9/8
*	7/6	22/21	12/11	7/6	22/21	12/11	9/8

-6 -11 -9 -7 -12 -10 -8 -6

0 -1 -2 -3 -4 -5 -6

4/3	4/3	11/8	4/3	63/44	4/3	(9/7)- closing Fourth						
	4/3	11/8	4/3	63/44	4/3	4/3						
		11/8	4/3	63/44	4/3	4/3	4/3					
			4/3	63/44	4/3	4/3	4/3	11/8				
				63/44	4/3	4/3	4/3	11/8	4/3			
					4/3	4/3	4/3	11/8	4/3	63/44		
					Repeat ▷	4/3	4/3	11/8	4/3	63/44	4/3	

-6 -7 -8 -9 -10 -11 -12

Fig 8c

Ptolemy (22/21 7/6 12/11)*, (12/11 7/6 22/21)+

-6 -4 -2 -0 -5 -3 -1 -6

*	12/11	9/8	22/21	7/6	12/11	22/21	7/6
	12/11	9/8	88/81	9/8	12/11	22/21	7/6
	12/11	9/8	88/81	9/8	12/11	7/6	22/21
+	12/11	7/6	22/21	9/8	12/11	7/6	22/21
	12/11	7/6	22/21	9/8	112/99	9/8	22/21
	7/6	12/11	22/21	9/8	112/99	9/8	22/21
*	7/6	12/11	22/21	7/6	12/11	9/8	22/21

-6 -11 -9 -7 -12 -10 -8 -6

0 -1 -2 -3 -4 -5 -6

4/3	63/44	4/3	4/3	11/8	4/3	(9/7)- closing Fourth						
	63/44	4/3	4/3	11/8	4/3	4/3						
		4/3	4/3	11/8	4/3	4/3	63/44					
			4/3	11/8	4/3	4/3	63/44	4/3				
				11/8	4/3	4/3	63/44	4/3	4/3			
					4/3	4/3	63/44	4/3	4/3	11/8		
					Repeat ▷	4/3	63/44	4/3	4/3	11/8	4/3	

-6 -7 -8 -9 -10 -11 -12

Fig 9a

Hawkins $(\frac{135}{128}\ \frac{16}{15}\ \frac{32}{27})^{*}, (\frac{32}{27}\ \frac{135}{128}\ \frac{16}{15})^{+}$

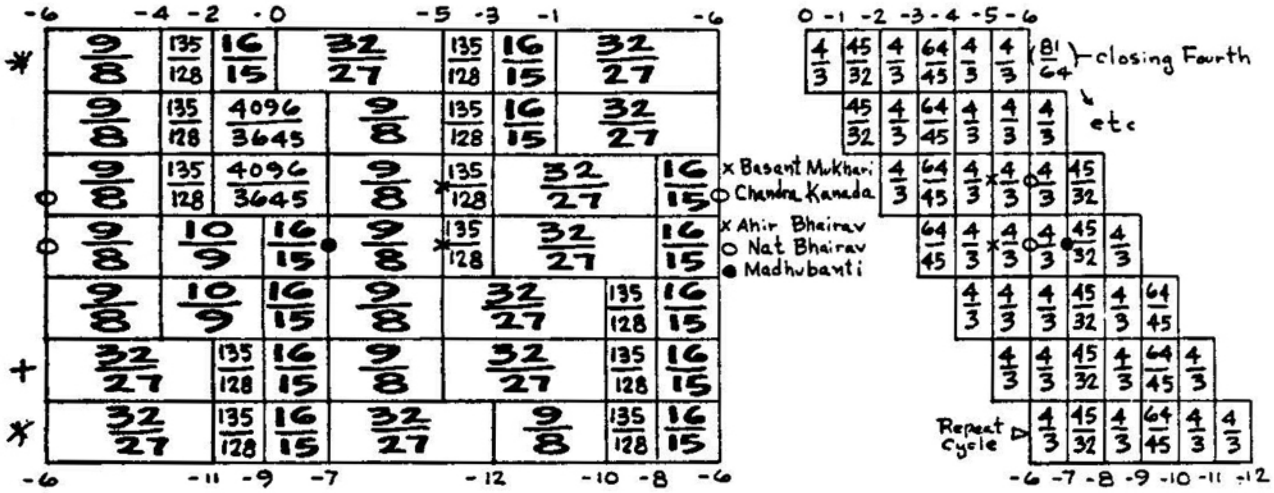

Fig 9b

Hawkins $(\frac{32}{27}\ \frac{16}{15}\ \frac{135}{128})^{*}, (\frac{16}{15}\ \frac{135}{128}\ \frac{32}{27})^{+}$

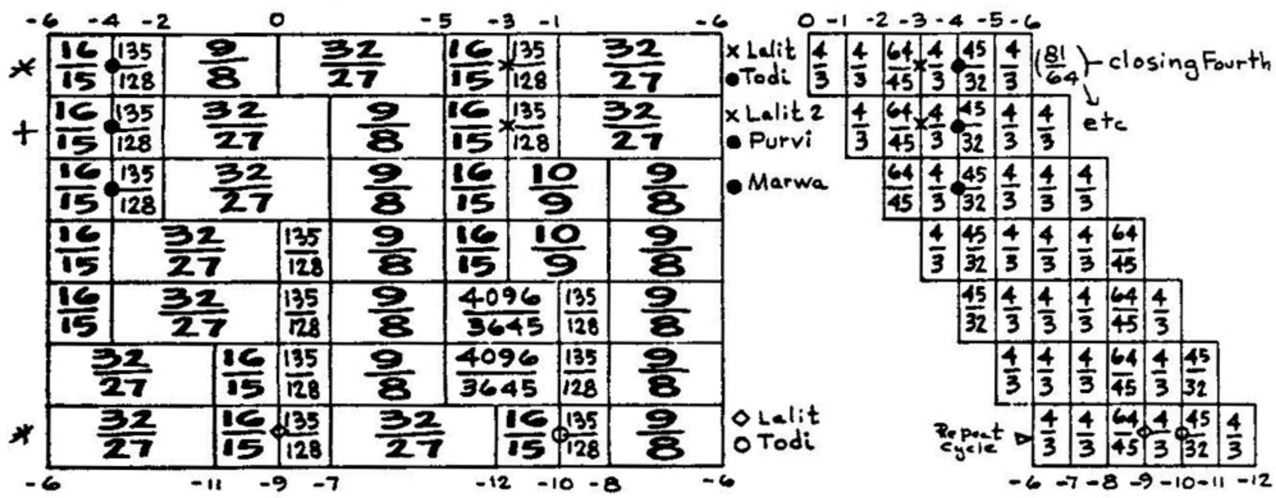

Fig 9c

Hawkins $(\frac{135}{128}\ \frac{32}{27}\ \frac{16}{15})^{*}\ (\frac{16}{15}\ \frac{32}{27}\ \frac{135}{128})^{+}$

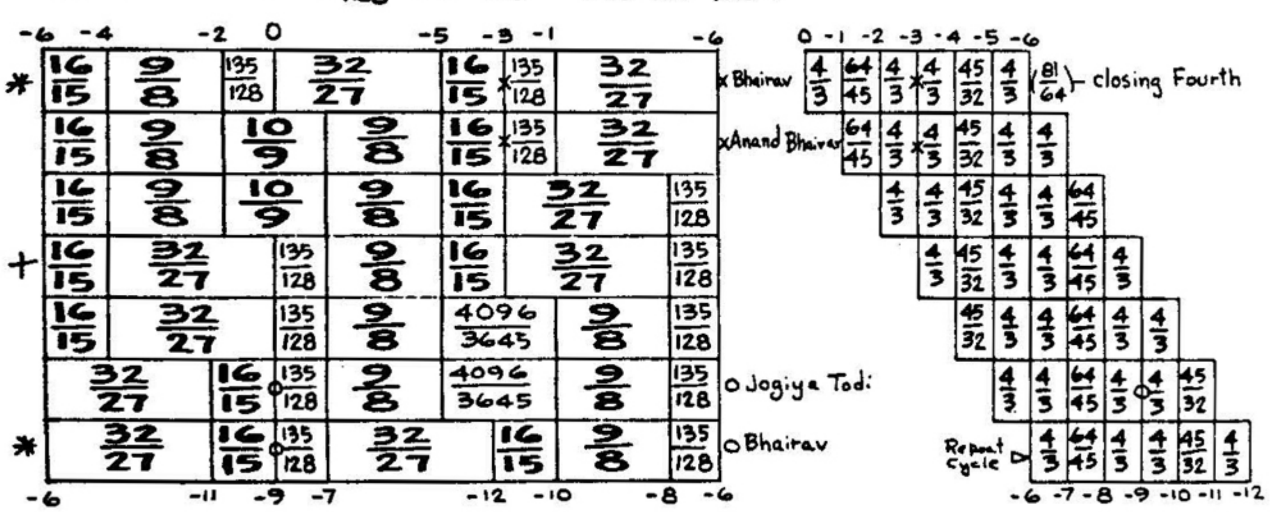

Fig 10a

Didymus $\left(\frac{16}{15}\ \frac{25}{24}\ \frac{6}{5}\right)^{*}, \left(\frac{6}{5}\ \frac{16}{15}\ \frac{25}{24}\right)^{+}$

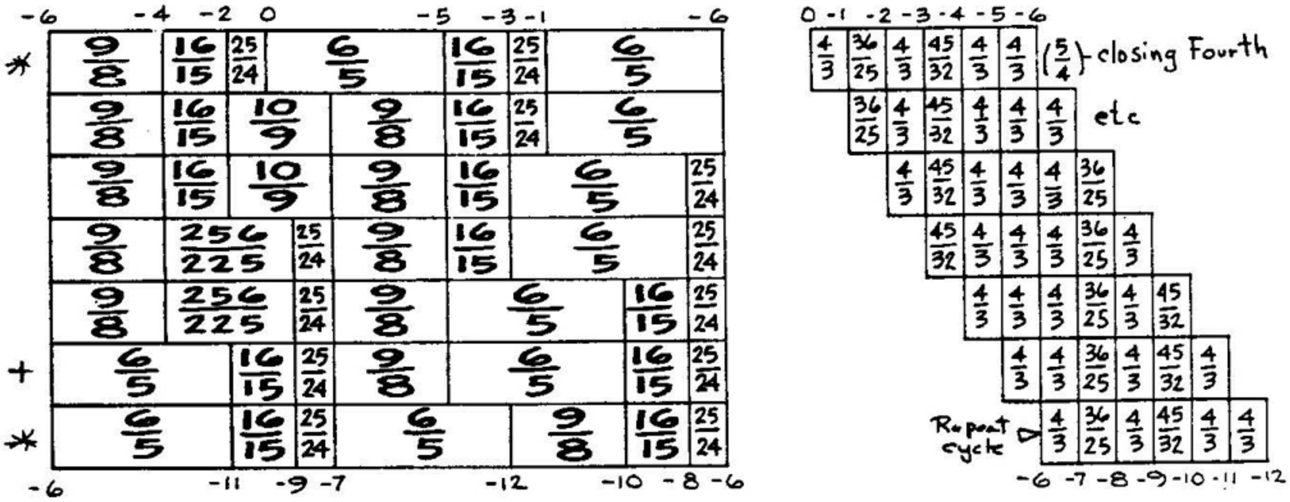

Fig 10b

Didymus $\left(\frac{6}{5}\ \frac{25}{24}\ \frac{16}{15}\right)^{*}, \left(\frac{25}{24}\ \frac{16}{15}\ \frac{6}{5}\right)^{+}$

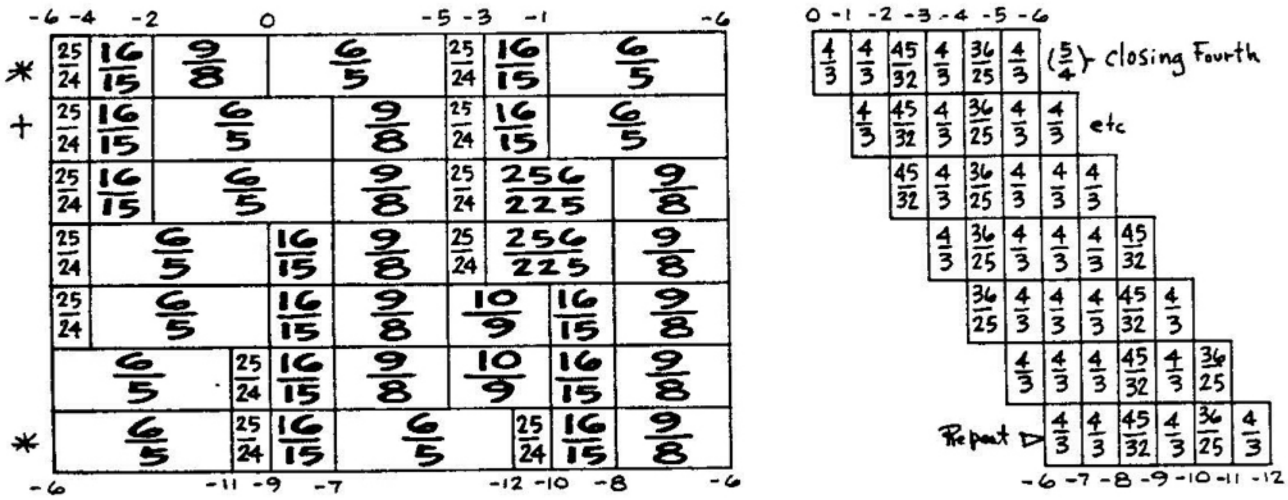

Fig 10c

Didymus $\left(\frac{25}{24}\ \frac{6}{5}\ \frac{16}{15}\right)^{*}, \left(\frac{16}{15}\ \frac{6}{5}\ \frac{25}{24}\right)^{+}$

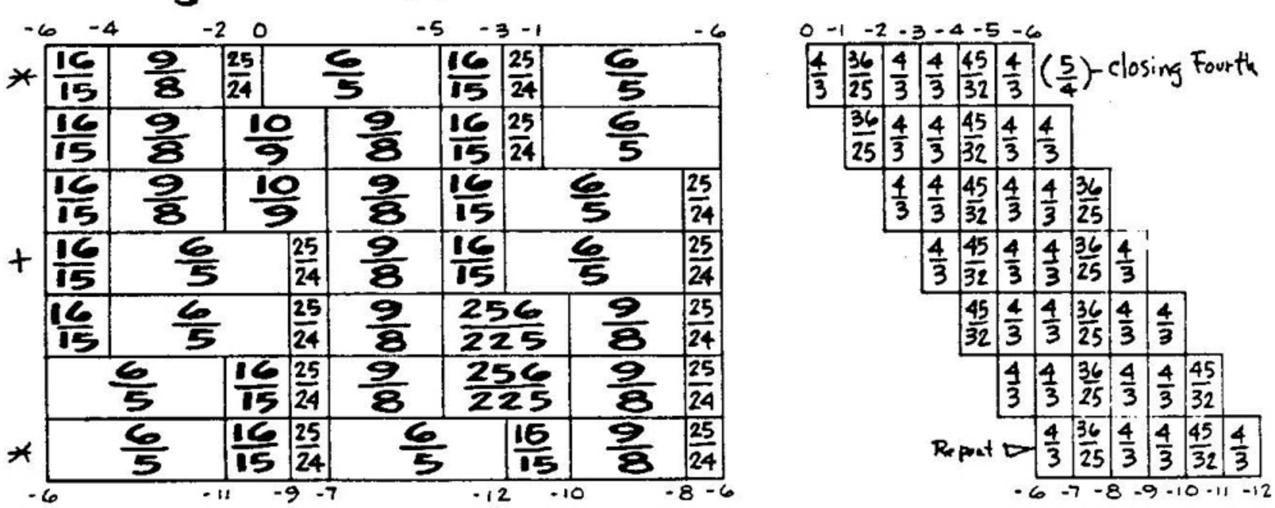

Fig 11a

Archytas $\left(\frac{28}{27}\ \frac{36}{35}\ \frac{5}{4}\right)^{*}, \left(\frac{5}{4}\ \frac{28}{27}\ \frac{36}{35}\right)^{+}$

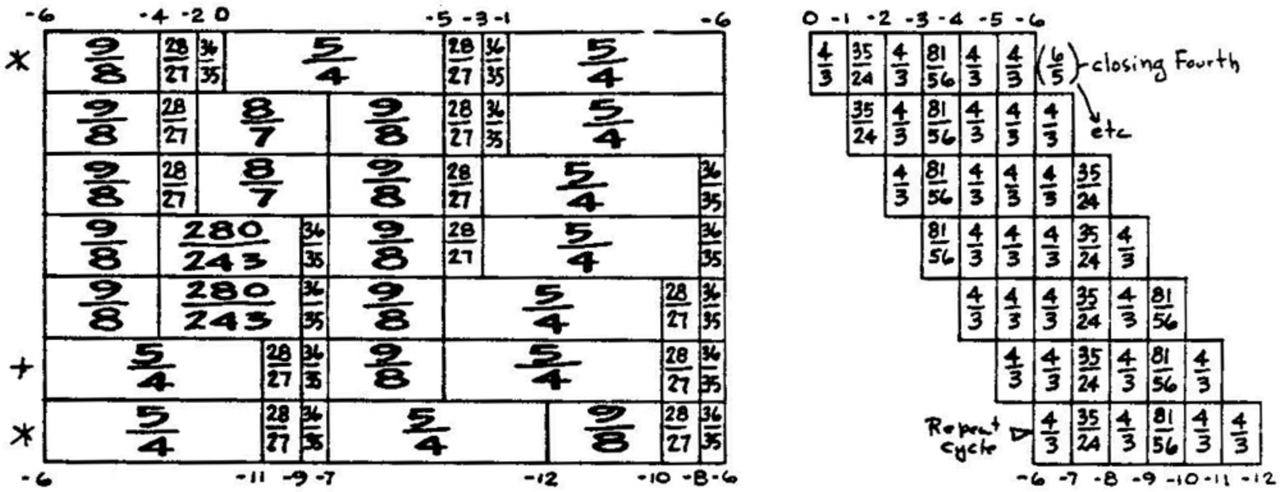

Fig 11b

Archytas $\left(\frac{5}{4}\ \frac{36}{35}\ \frac{28}{27}\right)^{*}, \left(\frac{36}{35}\ \frac{28}{27}\ \frac{5}{4}\right)^{+}$

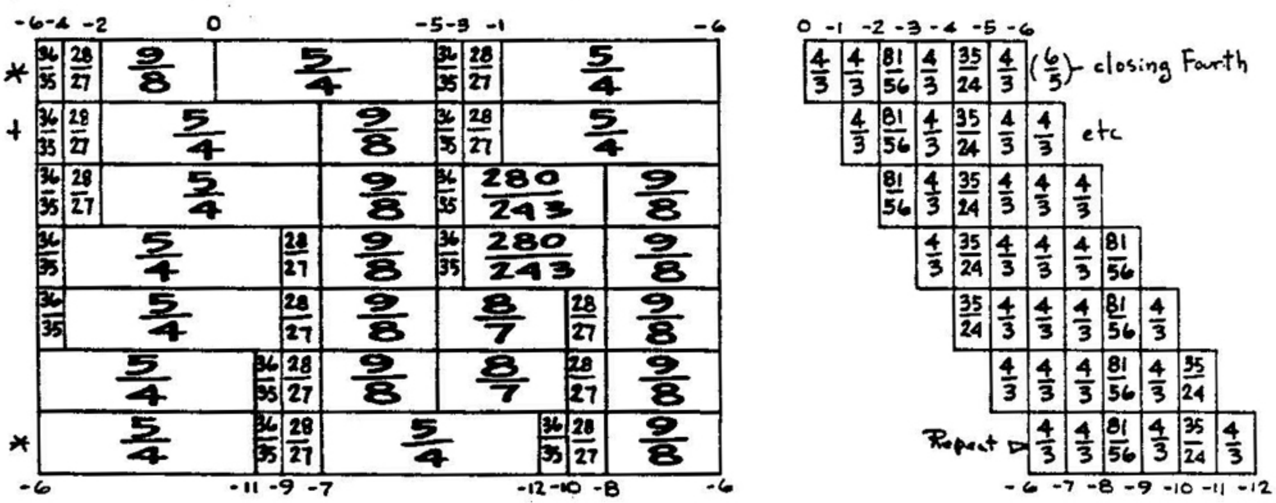

Fig 11c

Archytas $\left(\frac{28}{27}\ \frac{5}{4}\ \frac{36}{35}\right)^{*} \left(\frac{36}{35}\ \frac{5}{4}\ \frac{28}{27}\right)^{+}$