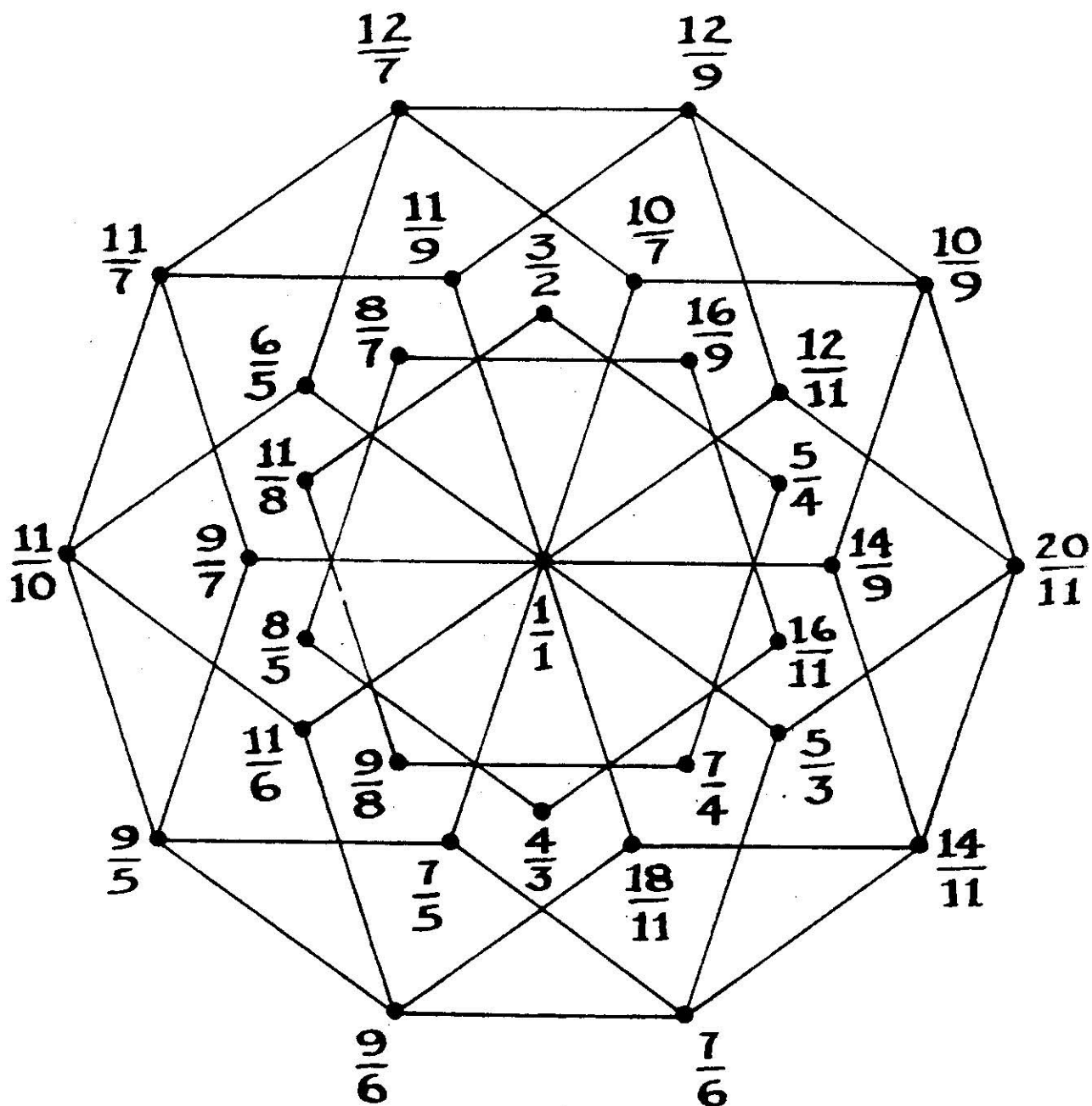


# Xenharmonikôn 3



Spring 1975

ON THE DEVELOPMENT OF INTONATIONAL SYSTEMS  
BY EXTENDED LINEAR MAPPING

1

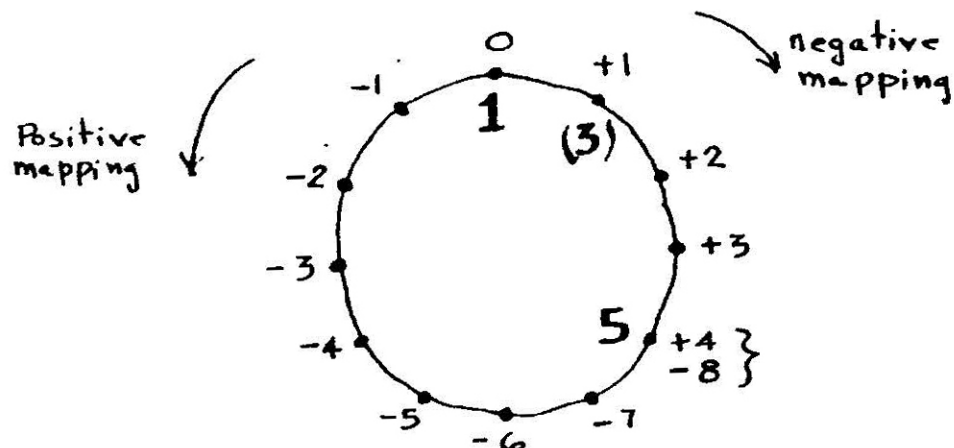
© 1975 by Erv Wilson

There are three important linear mapping templates which are useful in assigning Just materials to the keyboard, and in their subsequent notation. These templates may be also productive in the development of intonational systems. It is interesting to speculate to what extent these templates may reflect some means by which the mind processes information, especially new information in relation that already assimilated. We all know, by now too well, the perils of speculative flight. Timorous with caution, we have become blind to the pitfall of neglecting to speculate. So much for apology--

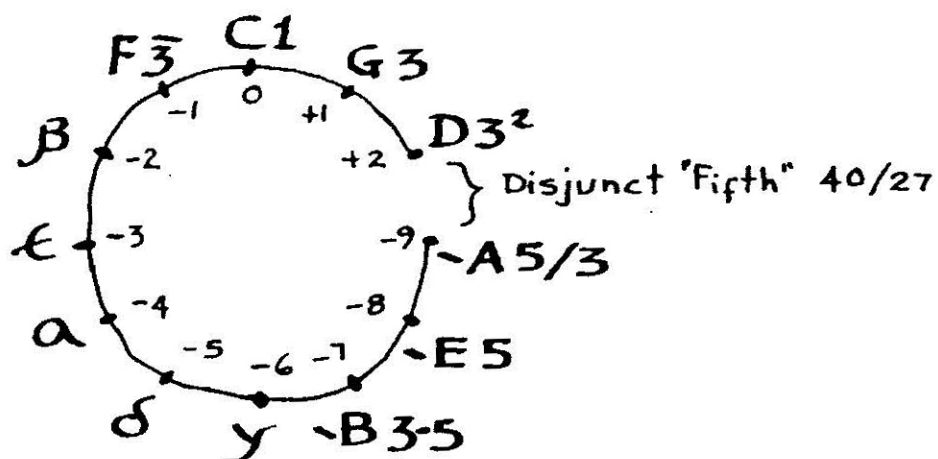
IF we take that certain types of multidimensional scales may mimic the contours of a linear, mono-dimensional scale we have the conditions for mapping. Under such conditions an extended 3-based linear series becomes enriched by acoustic conjunction with new harmonic elements which are assimilated into the linear series, and which modify it. The pattern by which these new elements are assimilated or mapped into the linear series we can call a code or a template. The character of this template may contribute information essential to a comprehension of ones orientation in the tonality environment. These linear mapping codes, if asserted and comprehended, may in a given culture become habitual, and entrenched. And, further, may greatly influence the course upon which the development of their intonational system will embark.

Let us consider, first, the Negative Linear Mapping Template to Modulus 31. (Diagram 1) It is, at the outset, characterized by its association of linear member +4 with the 5-function. This is a cultural habit which is deeply entrenched in European thinking. Indeed it appears to have been elevated to the status of a principle. It requires that the interval 40/27 (and/or similarly small "Fifths") be included within the tolerance of that Typical Fifth which generates the corresponding linear series. Extended linear mapping in accordance with this template produces the expected moments-of-symetry at (1, 2, 3) 5, 7, 12, 19, and 31 members. 31 is the culmination point, and altho the scale continues to be mutable, the number of tones is not likely to increase. (See Diagrams 4 & 7) A progression of linear mapped intonational systems having 5, 7, 12, 19, and 31 members is the course which western music would have to take if it chose to retain the specific information carried and irreversibly associated with its mapping code.

What are the chances that we could re-map? And open ourselves to another, different, progression of linear-related intonational systems? Not so singularly remote as one might imagine. At this particular point in the history of our development a rather curious opportunity to remap in an acoustically advantageous direction presents itself.



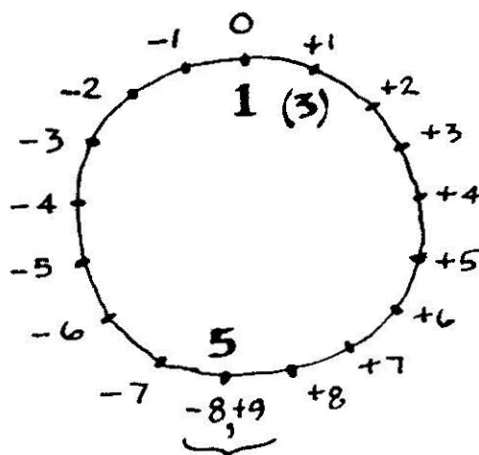
The much maligned Cycle of 12 (and unjustly so) presents that unique coincidence of events whereby may be effected a remapping of the 5-function from the +4 linear position to the -8 linear position. And we may be in the process of doing that. It could not have happened in meantone. But the shifting from meantone, which allows only the +4 mapping of the 5-function, to the cycle of 12, which also allows the -8 mapping; this, combined with practice, in one major school, of avoiding 'diatonic patterns', that is, patterns where the +4 mapping has any effective meaning, suggests that remapping the 5-function to linear -8 is imminent. Likely it has already occurred, and both orders of mapping are competing for dominance. If we re-open the cycle the two orders of mapping must part ways. We can return to negative mapping. Or we can re-open the cycle, thus:



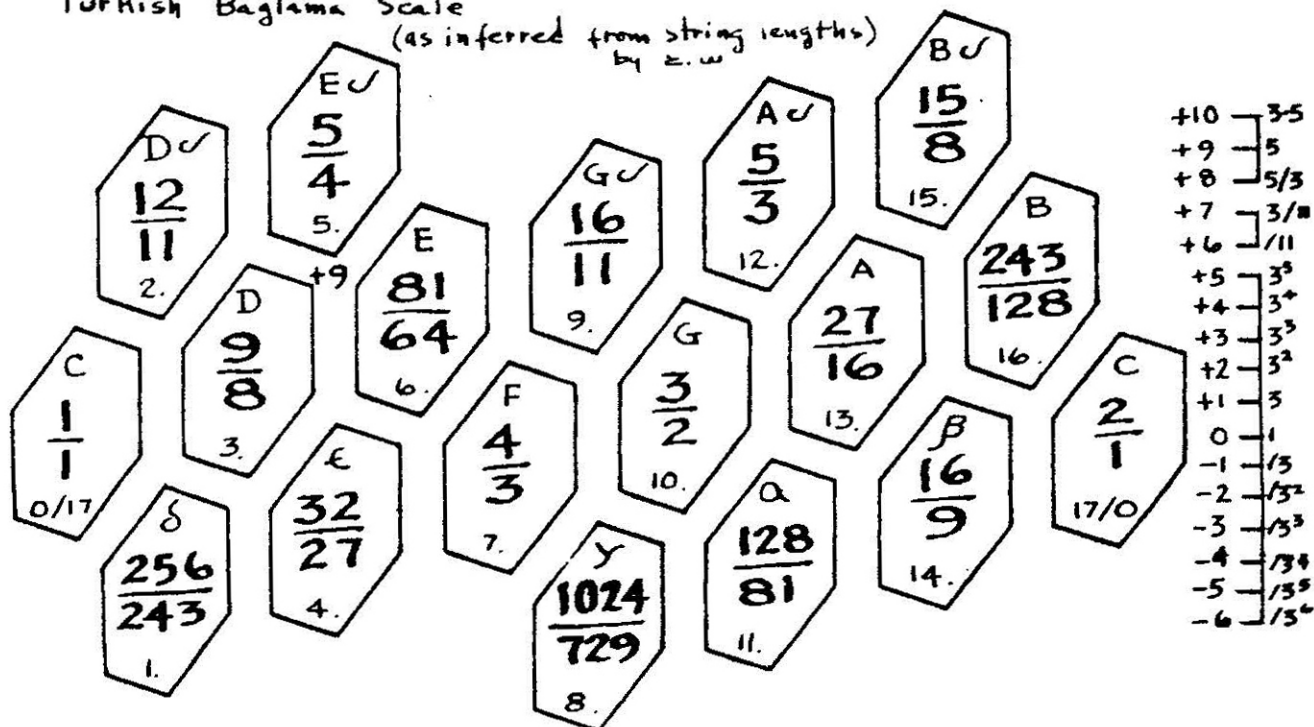
Interestingly, in this 12-member example, the 40/27 is now functioning as the Atypical and Disjunct "Fifth". Instead of sloughing it off, it now needs to be given special attention for its articulation gives to the Positive 12-tone scale depth and definition. This puts us in a new developmental progression whose moments-of-symetry are (1, 2, 3) 5, 7, 12, 17, 29, and 41. (Please refer to Diagrams 2, 5, & 8)

It is this progression which the intonational systems of the Near East might well be expected to follow. If you can locate a copy of Treatise on Music by Abdurakhman Djami, it gives considerable insight into the intended use of the 17-member system by the Persians. That this system is, in fact, fretted up, with commatic distinctions, for popular use (!) must give one cause to pause and reflect. The 17-tone systems are the flower of Near-Eastern acoustical endeavor. Where Turkish theorists have extended the linear series to 24 members, they have gained tones, but lost on integration. They would better have extended the series out to 29 members where the quasi-cyclic properties of the moment-of-symetry are again instated. Or stayed with 17.

At the quasi-cycle of 17 members there arises the possibility of again re-mapping the 5-function, from linear -8 to linear +9.



Turkish Baglama Scale  
(as inferred from string lengths)  
by E.W.



The 17-member Turkish Baglama (Yaz) intonational system may be a mapping in transition. Or in a dual role. I get the 'feeling' that the insertion of " $12/11$ " and " $16/11$ " (where, in positive mapping one would expect " $10/9$ " and " $40/27$ ") is to make a linear bridge between " $243/128$ " and " $5/3$ ", thus putting " $5/4$ " in the +9 linear position. The system, as it stands, is functioning as an elastic 17-member cycle. (And I'm sure any clever acoustician can get some good ideas from it!) In view that the Disjunct Fifth, from " $15/8$ " (linear+10) to " $1024/729$ " (linear -6) has almost got to be considered typical, at least acoustically, this organization may be a stable and enduring one. If the linear plus-9 mapping of the 5-function were to dominate, and the series extend itself accordingly, a development of the system might be made to 22 members. And a possible intersection with Indic intonational practice be effected.

The moments-of-symetry of 22 are (1, 2, 3) 5, 7, 12, 17 and 22. (Please see Diagrams 3, 6, & 9) In spite of the evidence that the acoustic materials of India have strong Pythagorean (3-based) roots, I suspect that by skillful mapping and re-mapping, by unending trial and experimentation, that they may, in fact be doing what they say they are doing; that is, they well may be experiencing their ragas within the environment of a 22-member intonational system. We need more measurements. We cannot, of course trust our ears, because of the risk of re-interpreting what we hear into a context of the negative intonational systems. It does not seem to me that the intonational systems of Europe and India, by any simple developmental progression, are likely to intersect. Let it be hoped that each culture is given the opportunity to pursue its own unique direction, if it so chooses. I fear that certain important and priceless civilizations may be evaporating before our very eyes, and this gives me cause for great concern. It isn't as if we were just "innocent bystanders".

The following diagrams are intended merely to EXEMPLIFY, and surely not to define in any rigid sense, certain plausible patterns of development, of intonational systems. There are endless permutations and variations. Each system is a flux. The intonational flux we finally use has to be determined within the living musical environment. I have only tried to illustrate that there is no mathematical reason why our intonational systems may not be mutable and viable. If there are profound psychic reasons, that is no doing of mine, at least no conscious doing!

Let me call your attention to Harry Partch's scale, as linear mapped to the Bosanquet according to a Positive Template. It really is uncanny how unerringly it follows a linear mapping to modulus 41. In view of his (understandable) bias against "Chronic 3-ism" this correspondence is all the more remarkable. Inflections of two linear members can be resolved with split keys, or off-keyboard in any convenient way. They are no cause to avoid the Bosanquet for Partch.

Diagram 1

# Negative Linear Mapping Template to modulus 31

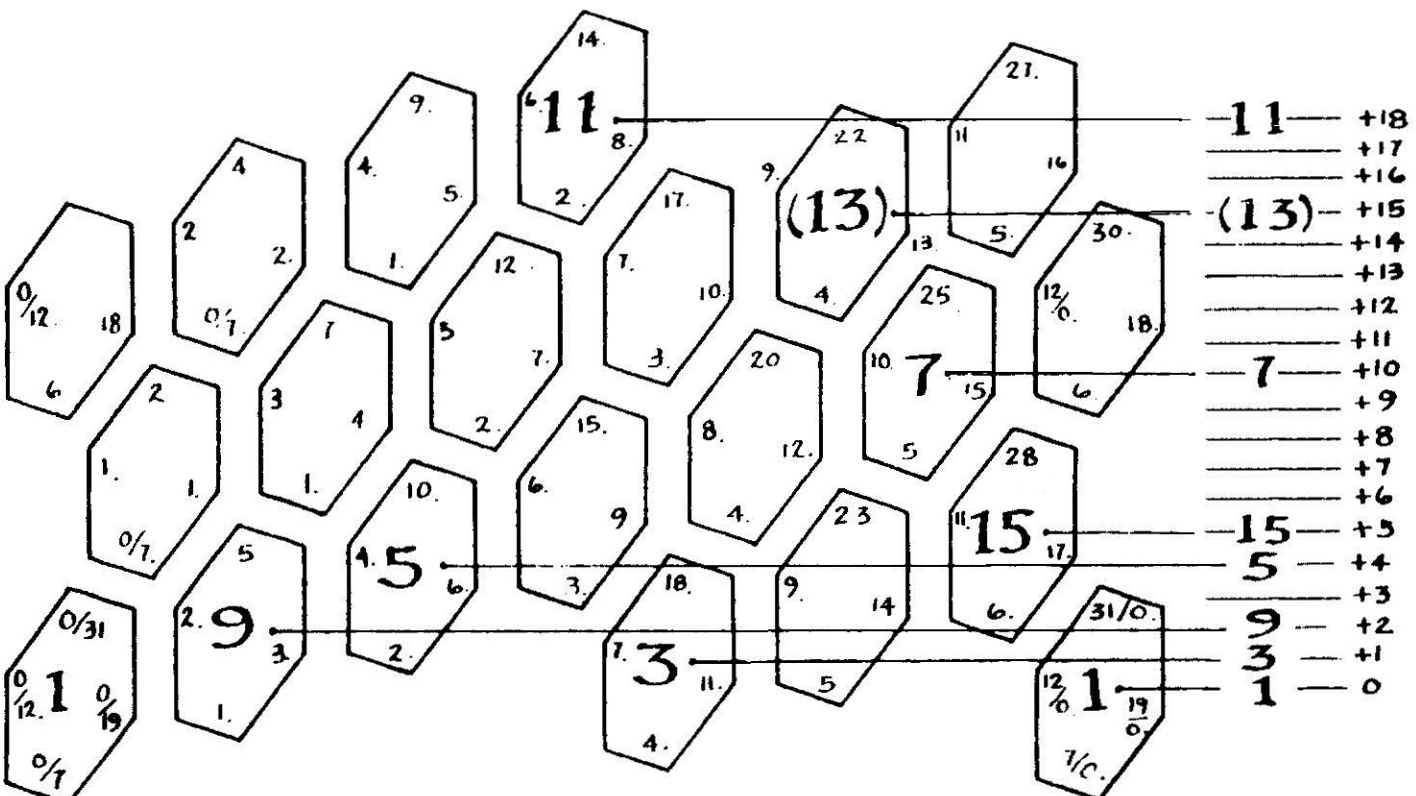
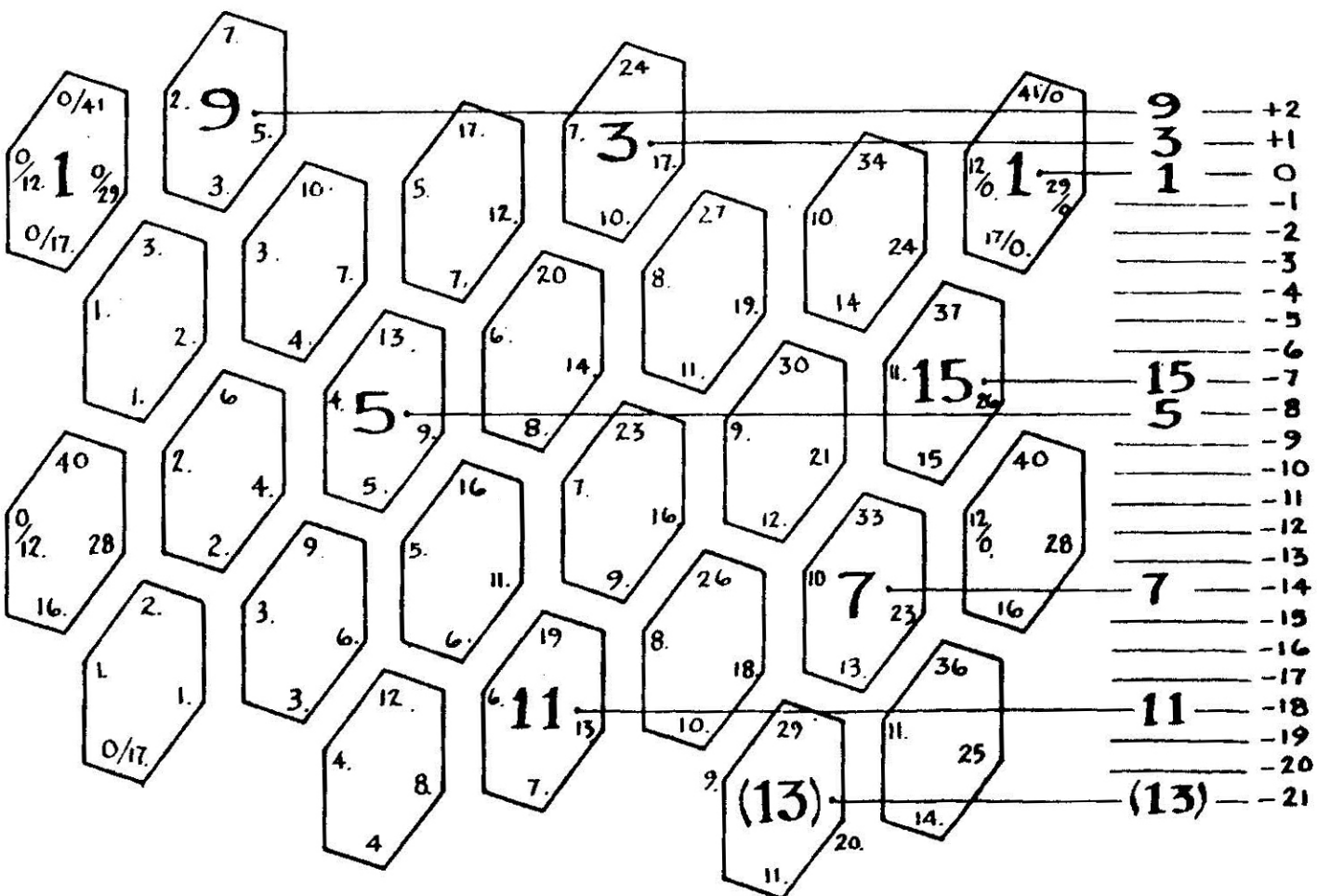
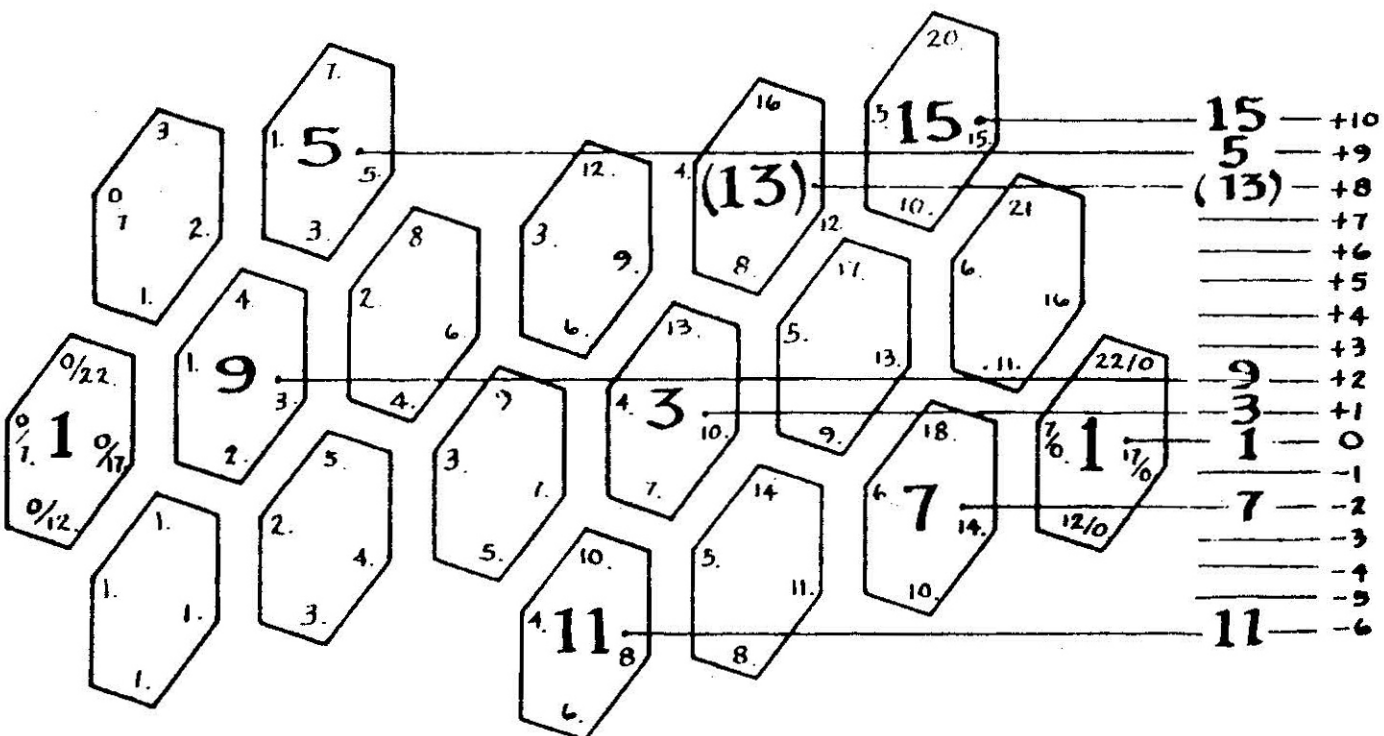


Diagram 2

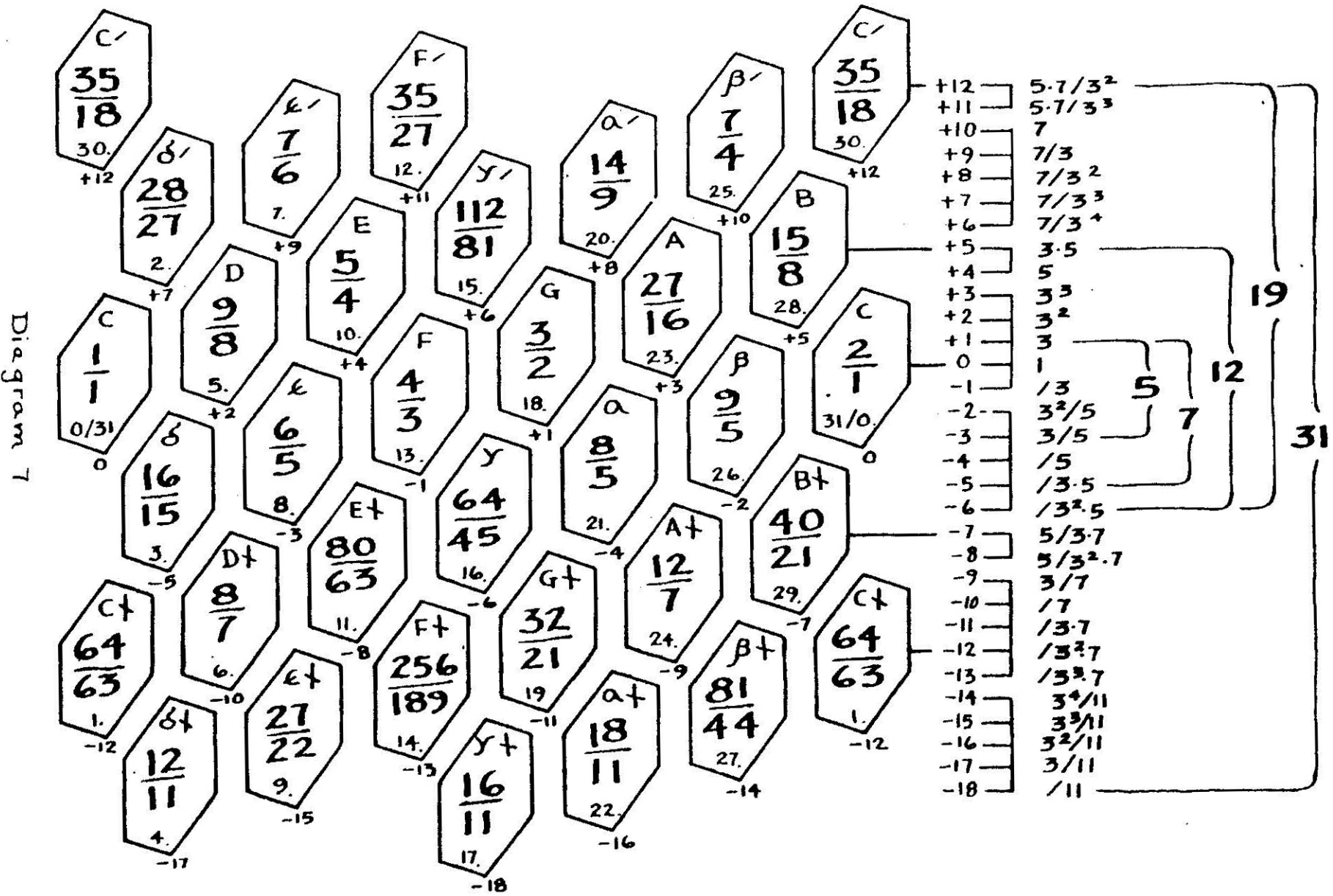
# Positive Linear Mapping Template to modulus 41



# Acute Linear Mapping Template to modulus 22



# 8. Keyboarding Genus 31 (rat diagram 4)

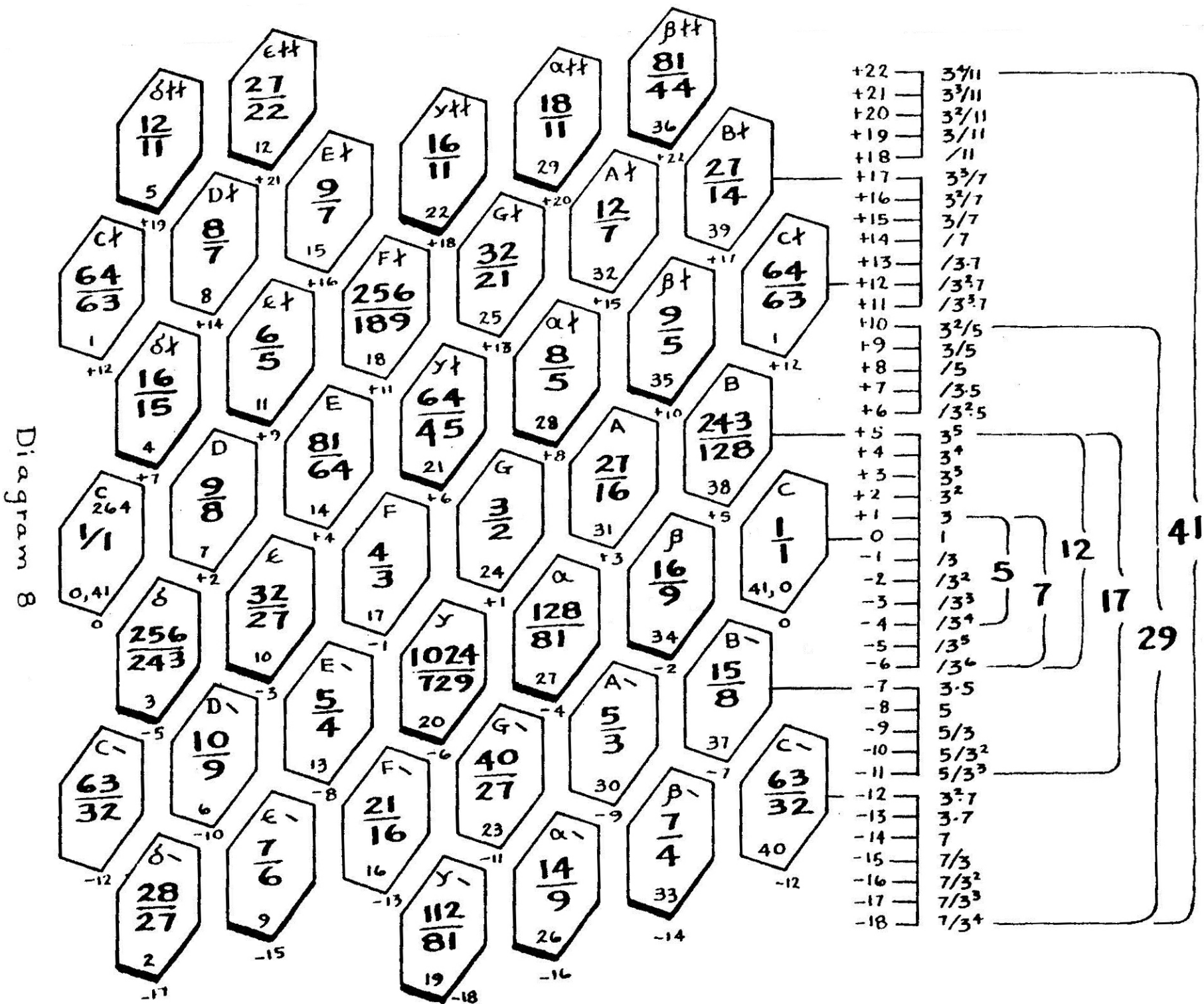


$\frac{6}{5}$					$\frac{10}{9}$					$\frac{9}{8}$					$\frac{6}{5}$					$\frac{10}{9}$					5						
$\frac{16}{15}$		$\frac{9}{8}$			$\frac{10}{9}$					$\frac{9}{8}$					$\frac{16}{15}$		$\frac{9}{8}$			$\frac{10}{9}$					7						
$\frac{16}{15}$		$\frac{135}{128}$		$\frac{16}{15}$		$\frac{25}{24}$		$\frac{16}{15}$			$\frac{16}{15}$		$\frac{135}{128}$			$\frac{16}{15}$		$\frac{135}{128}$		$\frac{16}{15}$		$\frac{25}{24}$		$\frac{16}{15}$			12				
$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$		$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$		$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$		$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$		$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{28}{27}$	$\frac{36}{35}$	19					
$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{45}{44}$	$\frac{33}{32}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{45}{44}$	$\frac{55}{54}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{45}{44}$	$\frac{33}{32}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{45}{44}$	$\frac{33}{32}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	$\frac{45}{44}$	$\frac{55}{54}$	$\frac{64}{63}$	$\frac{49}{48}$	$\frac{36}{35}$	31
$\frac{1}{1}$	$\frac{64}{63}$	$\frac{28}{27}$	$\frac{16}{15}$	$\frac{12}{11}$	$\frac{9}{8}$	$\frac{8}{7}$	$\frac{7}{6}$	$\frac{6}{5}$	$\frac{27}{22}$	$\frac{5}{4}$	$\frac{80}{63}$	$\frac{35}{27}$	$\frac{4}{3}$	$\frac{256}{189}$	$\frac{112}{81}$	$\frac{64}{45}$	$\frac{16}{11}$	$\frac{3}{2}$	$\frac{32}{21}$	$\frac{14}{9}$	$\frac{8}{5}$	$\frac{18}{11}$	$\frac{12}{7}$	$\frac{27}{16}$	$\frac{7}{4}$	$\frac{9}{5}$	$\frac{81}{44}$	$\frac{15}{8}$	$\frac{40}{21}$	$\frac{35}{18}$	$\frac{2}{1}$

A development of negative, linear-mapped  
Intonational systems to modulus 31

Diagram 4

10. Keyboarding Genus 41 (ref diagram 5)



$\frac{32}{27}$												$\frac{9}{8}$												$\frac{9}{8}$												$\frac{32}{27}$												$\frac{9}{8}$												5																																																																																																			
$\frac{256}{243}$				$\frac{9}{8}$								$\frac{9}{8}$				$\frac{9}{8}$								$\frac{256}{243}$				$\frac{9}{8}$								$\frac{9}{8}$				7																																																																																																																							
$\frac{256}{243}$				$\frac{2187}{2048}$				$\frac{256}{243}$				$\frac{2187}{2048}$				$\frac{256}{243}$				$\frac{2187}{2048}$				$\frac{256}{243}$				$\frac{2187}{2048}$				$\frac{256}{243}$				12																																																																																																																											
$\frac{256}{243}$				$\frac{135}{128}$				$\frac{81}{80}$				$\frac{256}{243}$				$\frac{135}{128}$				$\frac{81}{80}$				$\frac{256}{243}$				$\frac{135}{128}$				$\frac{81}{80}$				$\frac{256}{243}$				17																																																																																																																							
$\frac{28}{27}$				$\frac{64}{63}$				$\frac{81}{80}$				$\frac{25}{24}$				$\frac{81}{80}$				$\frac{28}{27}$				$\frac{64}{63}$				$\frac{81}{80}$				$\frac{25}{24}$				$\frac{81}{80}$				$\frac{28}{27}$				$\frac{64}{63}$				29																																																																																																															
$\frac{64}{63}$				$\frac{49}{48}$				$\frac{64}{63}$				$\frac{81}{80}$				$\frac{45}{44}$				$\frac{55}{54}$				$\frac{81}{80}$				$\frac{64}{63}$				$\frac{49}{48}$				$\frac{64}{63}$				$\frac{81}{80}$				$\frac{45}{44}$				$\frac{55}{54}$				$\frac{81}{80}$				$\frac{64}{63}$				$\frac{49}{48}$				$\frac{64}{63}$				41																																																																																											
1/1				64/63				28/27				256/243				16/15				12/11				10/9				8/7				7/6				6/5				32/27				5/4				27/22				9/7				81/64				4/3				256/189				112/81				64/45				1024/729				16/11				40/27				3/2				32/21				14/9				128/81				8/5				18/11				5/3				27/16				12/7				7/4				16/9				9/5				81/44				15/8				243/128				27/14				63/32				2/1			

A development of positive, linear-mapped  
Intonational systems to modulus 41

Diagram 5

12. Keyboarding Genus 22 (ref diagram 6)

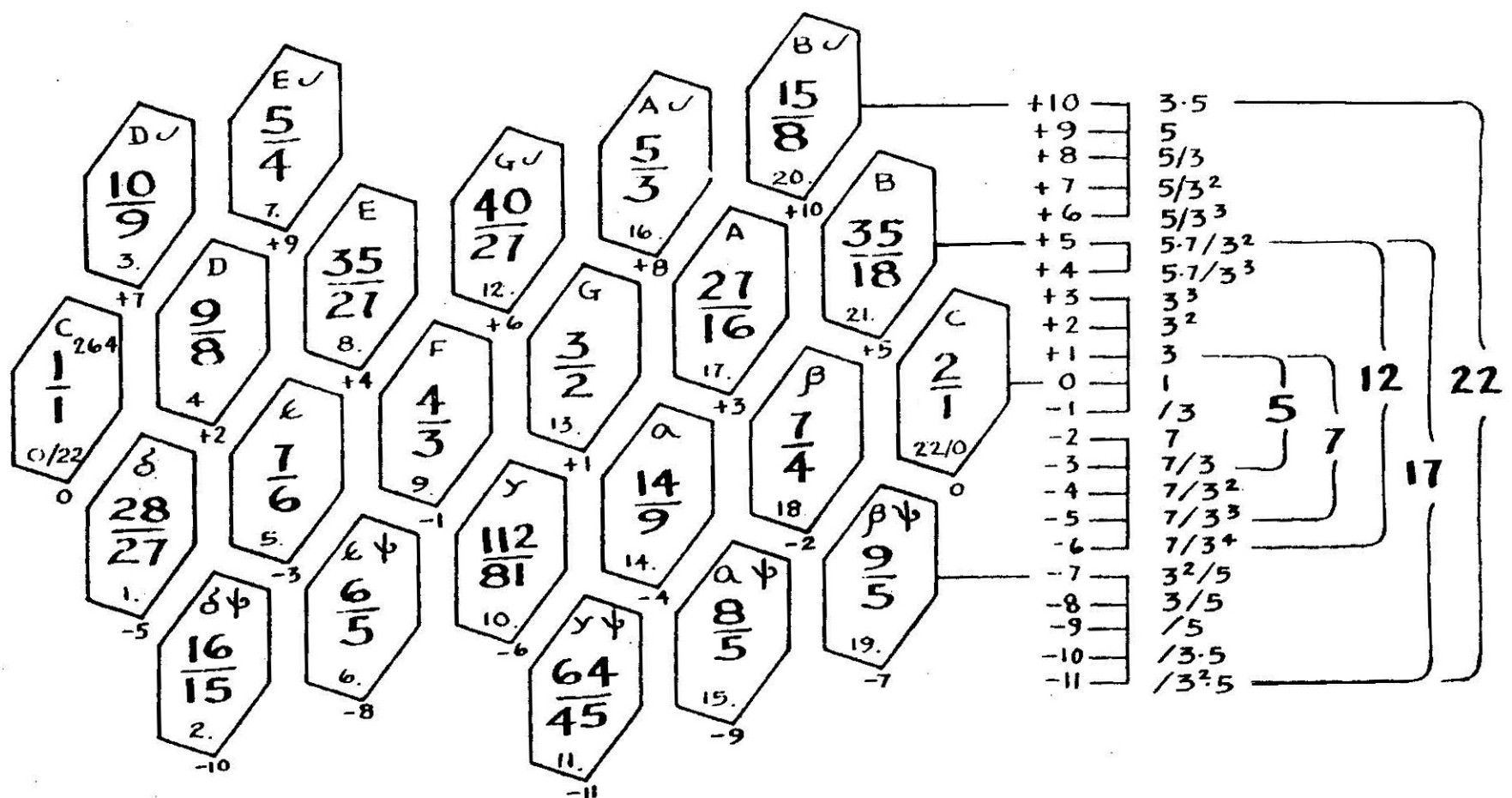


Diagram 9

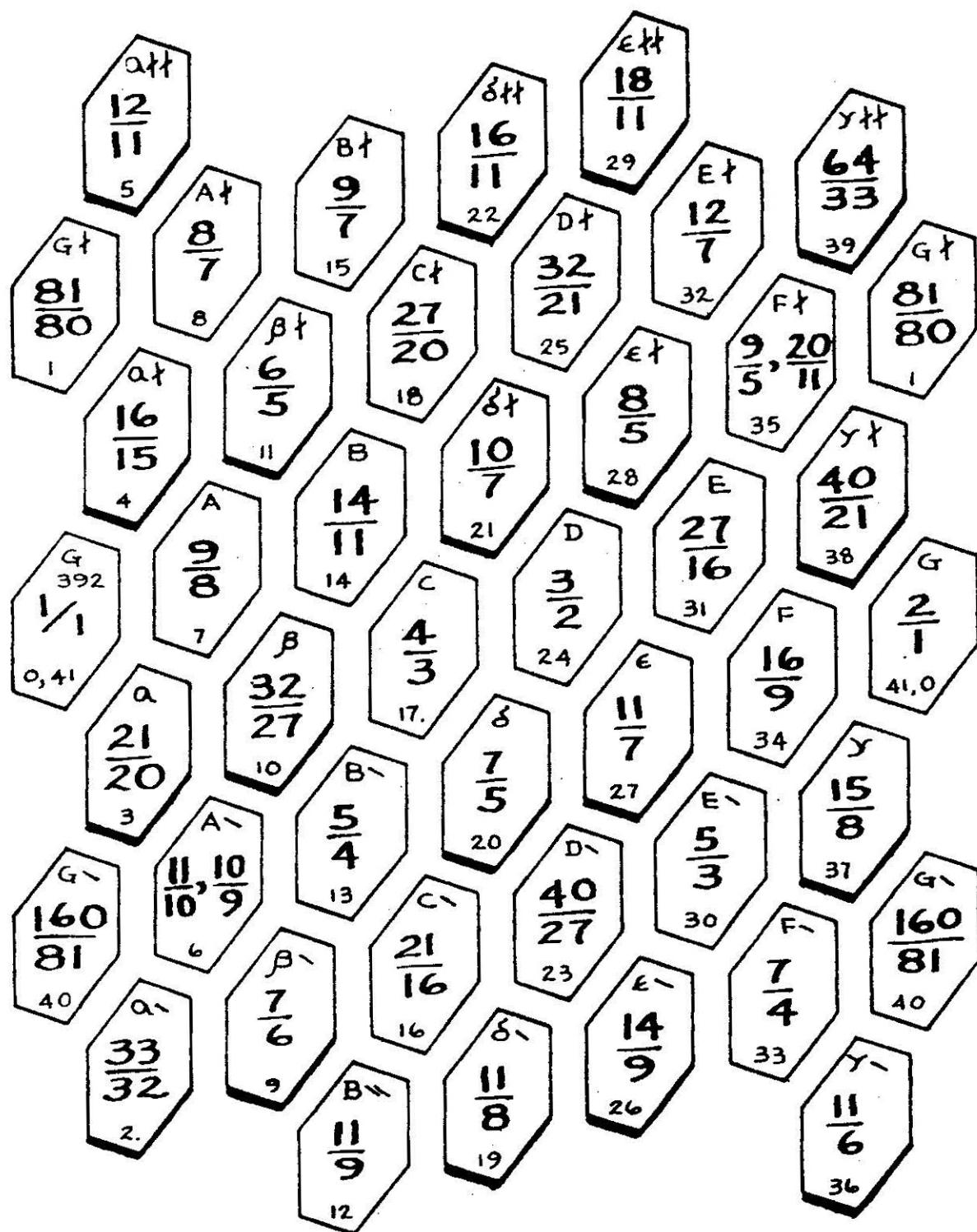
$\frac{7}{6}$				$\frac{8}{7}$				$\frac{9}{8}$				$\frac{7}{6}$				$\frac{8}{7}$				5	
$\frac{28}{27}$	$\frac{9}{8}$			$\frac{8}{7}$			$\frac{9}{8}$			$\frac{28}{27}$	$\frac{9}{8}$			$\frac{8}{7}$			7				
$\frac{28}{27}$	$\frac{243}{224}$	$\frac{28}{27}$	$\frac{10}{9}$		$\frac{36}{35}$	$\frac{28}{27}$	$\frac{243}{224}$	$\frac{28}{27}$	$\frac{243}{224}$	$\frac{28}{27}$	$\frac{10}{9}$		$\frac{36}{35}$	12							
$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{175}{162}$	$\frac{36}{35}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{135}{128}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{175}{162}$	$\frac{36}{35}$	17				
$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{81}{80}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{81}{80}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{81}{80}$	$\frac{28}{27}$	$\frac{36}{35}$	$\frac{25}{24}$	$\frac{28}{27}$	$\frac{36}{35}$	22	
$\frac{28}{27}$	$\frac{16}{15}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{7}{6}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{35}{27}$	$\frac{4}{3}$	$\frac{112}{81}$	$\frac{64}{45}$	$\frac{40}{27}$	$\frac{3}{2}$	$\frac{14}{9}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{27}{16}$	$\frac{7}{4}$	$\frac{9}{5}$	$\frac{15}{8}$	$\frac{35}{18}$	$\frac{2}{1}$

A development of acute, linear-mapped  
Intonational Systems to modulus 22

Diagram 6

# Harry Partch's Scale on the Bosanquet keyboard

by Erv Wilson 1975



# A Keyboard Notation for Harry Partch's Scale<sup>15</sup>

by Em Wilson 1975

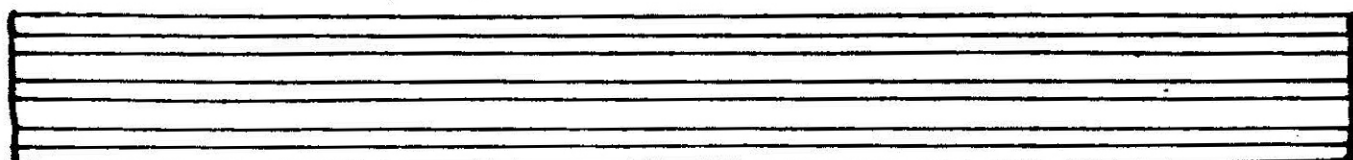
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 G 392 a A  $\beta$

$\frac{32}{27}$   $\frac{6}{5}$   $\frac{11}{9}$   $\frac{5}{4}$   $\frac{14}{11}$   $\frac{9}{7}$   $\frac{21}{16}$   $\frac{4}{3}$   
 $\beta$  B C

$\frac{4}{3}$   $\frac{27}{20}$   $\frac{11}{8}$   $\frac{7}{5}$   $\frac{10}{7}$   $\frac{16}{11}$   $\frac{40}{27}$   $\frac{3}{2}$   
 C  $\delta$  D

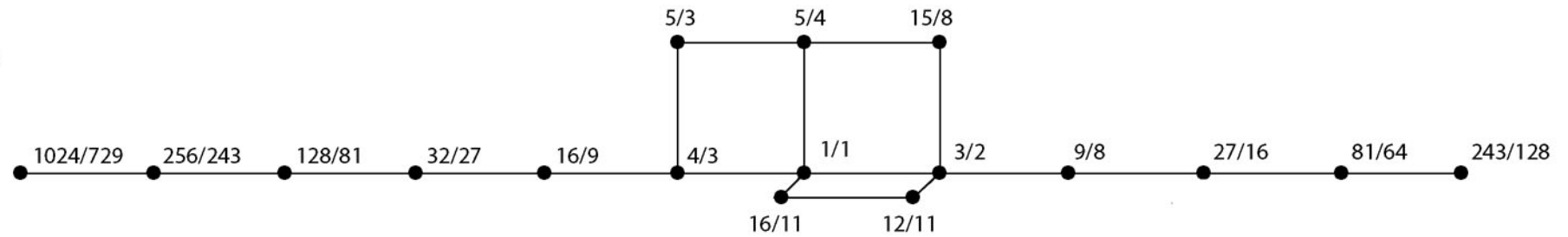
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 D  $\epsilon$  E F

$\frac{16}{9}$   $\frac{9}{5}$   $\frac{20}{11}$   $\frac{11}{6}$   $\frac{15}{8}$   $\frac{40}{21}$   $\frac{64}{33}$   $\frac{160}{81}$   $\frac{2}{1}$   
 F  $\gamma$  G

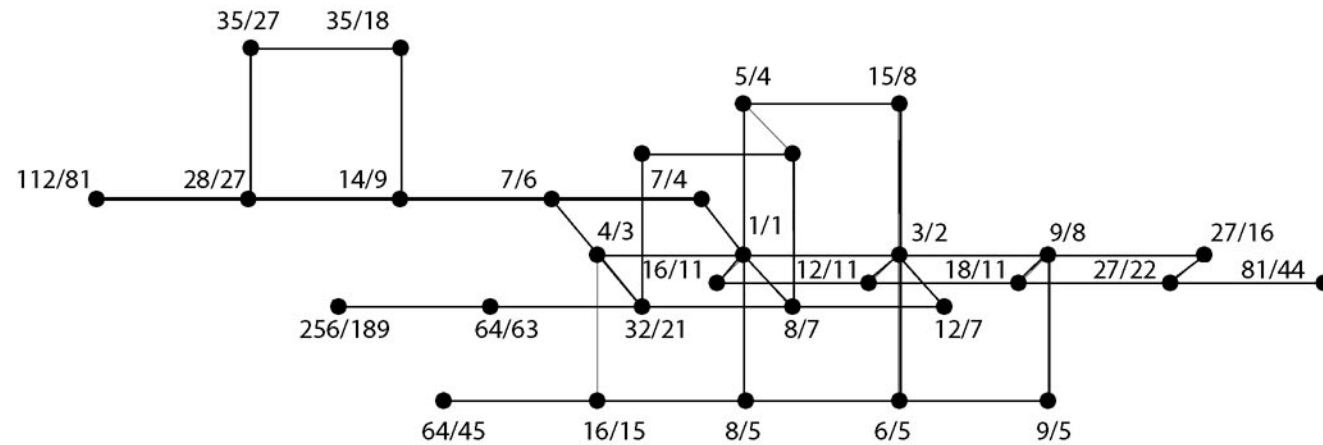


# LATTICES OF SCALES FOUND IN xen3b.PDF

17



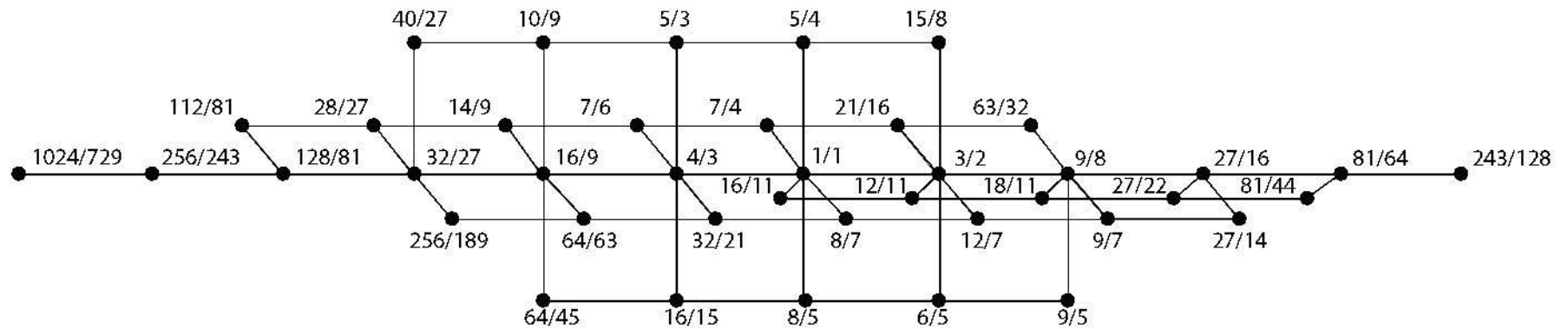
31



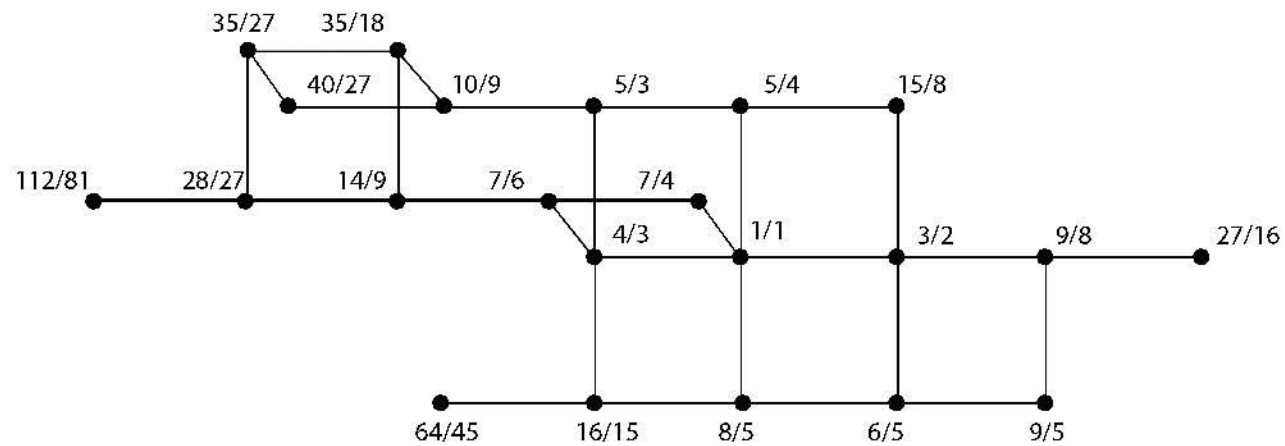
k.grady 5/8/2011

# LATTICES OF SCALES FOUND IN xen3b.PDF

41



22



k.grady 5/8/2011